Elements of string theory (continued)

Type IIB low energy ($E \ll m_s \sim 1/l_s$) e.o.m. follow (apart from the self-duality condition $\tilde{F}_5 = * \tilde{F}_5$ which should be imposed by hand) from the action

$$S_{IIB}^{\mathcal{E}_s} = S_{NS} + S_R + S_{CS} ,$$

$$S_{NS} = \frac{1}{2\varkappa_{10}^2} \int d^{10}x \sqrt{-g} \left(R - \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - \right.$$
$$\left. - \frac{1}{2} e^{-\phi} |H_3|^2 \right)$$

$$S_R = - \frac{1}{4\varkappa_{10}^2} \int d^{10}x \sqrt{-g} \left(e^{2\phi} |F_1|^2 + \right.$$
$$\left. + e^{\phi} |\tilde{F}_3|^2 + \frac{1}{2} |\tilde{F}_5|^2 \right)$$
$$\longrightarrow \frac{1}{2} \tilde{F}_5 \wedge * \tilde{F}_5 \overset{\text{on shell}}{=} 0$$

$$S_{CS} = - \frac{1}{4\varkappa_{10}^2} \int C_4 \wedge H_3 \wedge F_3$$

Plus fermionic terms to complete type IIB sugra

<u>Remark</u>: there are corrections to the e.o.m. coming from „stringy" terms with $\alpha' \neq 0$

e.g.

$$R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4 \cdot 5!}(\tilde{F}_5)^2 + \ldots + \gamma e^{-\frac{3}{2}\phi} W + \ldots$$

$$\gamma = \frac{1}{8}\zeta(3)\alpha'^3$$

$$W = C^{hmnk} C_{pmnq} C_h{}^{rsp} C^q{}_{rsk} + \text{similar terms}$$

$$W_{abcd} = R_{abcd} - \frac{2}{d-2}\left(g_{a[c} R_{d]b} - g_{b[c} R_{d]a}\right) + \frac{2}{(d-1)(d-2)} R\, g_{a[c}\, g_{d]b}$$

is the Weyl tensor.

<u>Remark</u>: most solutions to the low-energy e.o.m. receive α' corrections. But some (usually very symmetric ones such as Mink. or $AdS_5 \times S^5$ do not).

Recall that $G_{10} \sim l_p^8 \sim g_s^2 l_s^8$

Any classical solution characterized by a scale L (e.g. Schwarzschild radius) will get (generically) stringy and quantum grav. corrections unless

$$L \gg \ell_s \quad \text{and} \quad L \gg \ell_p$$

In AdS/CFT, $\dfrac{L^4}{\ell_s^4} \sim g_{YM}^2 N_c \to \infty$

(classical geom. valid) and

$$L^4 \gg \ell_p^4 \sim g_s \ell_s^4 \quad \text{i.e.} \quad \frac{L^4}{\ell_s^4} \gg g_s \sim g_{YM}^2$$

$$\Rightarrow g_{YM}^2 N_c \gg g_{YM}^2 \Rightarrow N_c \gg 1$$

So, $N_c \to \infty$ suppresses quantum grav. corrections to classical geometry.

Consider some solutions of type IIB eom: let all fields except $g_{\mu\nu}$, F_5 and $\varphi = \text{const}$ be zero. This is a self-consistent choice for e.o.m. which now become:

$$R_{\mu\nu} = \frac{1}{96} F_{\mu\rho\sigma\lambda\varkappa} F_{\nu}{}^{\rho\sigma\lambda\varkappa}$$

$$F_{(5)} = * F_{(5)}$$

($d * F_{(5)} = 0$ is satisfied as a consequence of Bianchi $dF_{(5)} = 0$ and self-duality condition.)

The solution is:

$$ds^2_{10} = H^{-1/2}(r)\left[-f(r)\,dt^2 + dx^2 + dy^2 + dz^2\right] + H^{1/2}(r)\left(\frac{dr^2}{f} + r^2 d\Omega_5^2\right),$$

where
$$H(r) = 1 + L^4/r^4,$$
$$f(r) = 1 - r_0^4/r^4,$$ and the

Ramond - Ramond form:

$$F_5 = -\frac{4L^2}{H^2 r^5}\sqrt{r_0^4+L^4}\,(1+*)\,dt\wedge dx\wedge dy\wedge dz\wedge dr$$

For future ref, consider the near-horizon limit $r \ll L$: the metric becomes

$$ds_{10}^2 = \frac{(\pi T L)^2}{u}\left(-f(u)\,dt^2 + dx^2 + dy^2 + dz^2\right)$$

$$+ \frac{L^2}{4u^2 f}\,du^2 + L^2 d\Omega_5^2$$

where $u \equiv r_0^2/r^2$, $f = 1-u^2$, $T = \frac{r_0}{\pi L^2}$

is the Hawking temperature.

The horizon is at $u=1$, the spatial infinity - at $u=0$.

Remark: more generally, solutions to the eom (type IIB with RR forms A_{p+1}

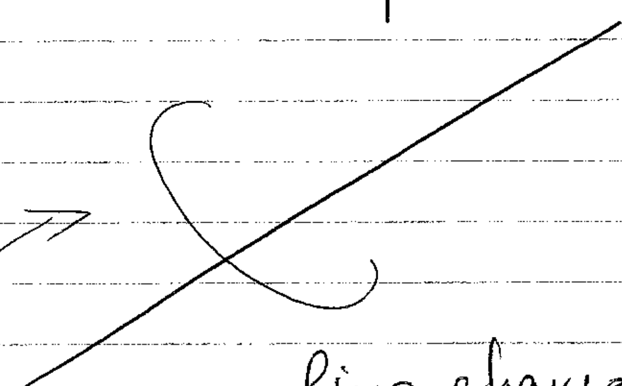
with p odd and type IIA with p even)
with $g_{\mu\nu}$, A_{p+1}, ϕ were considered
in 1988 - 1991 by Gibbons, Maeda; Garfinkle, Horowitz, Strominger; Horowitz, Strominger
See e.g. K. Stelle, Lectures on supergravity p-branes, hep-th/9701088.

Schematically, one is looking for charged extended objects with mass per unit vol. M and charge (under A_{p+1}) Q:

$$ds^2 = A(\rho)\left(-f dt^2 + \sum_{i=1}^{p} dx^i dx^i\right) +$$

$$+ B(\rho)\, d\rho^2 + C(\rho)\, d\Omega^2_{8-p}$$

$$\int_{S^{8-p}} * F_{p+2} = Q$$



line charge

Solutions have $M \geq Q$ (absence of naked singularity)

Solutions with $M = Q$ are called extremal.

$A_H \to 0 \Rightarrow S \to 0$

Non-extremal black p-branes have

$S > 0$, $T_H > 0$.

<u>Extremal p-brane solutions</u>:

$$ds^2 = H^{-1/2}(r)\left(-dt^2 + \sum_{i=1}^{p} dx^i dx^i\right) + H^{1/2} r^2 d\Omega^2_{8-p}$$

$$e^{\phi} = g_s H^{\frac{3-p}{4}}(r)$$

$$H(r) = 1 + \frac{L^{7-p}}{r^{7-p}}, \quad L^{7-p} = d_p g_s Q l_s^{7-p},$$

$$d_p = 2^{5-p} \pi^{\frac{5-p}{2}} \Gamma\left(\frac{7-p}{2}\right)$$

<u>Remark</u>: Q is quantized (Dirac quantization condition).

<u>Remark</u>: the metric above describes geometry <u>outside</u> the horizon (see AGMOO for details).

<u>Remark</u> : the inequality $M \geqslant Q$ is also a BPS condition