Topics in Gauge-Gravity Duality - I

I. Introduction

II. Holographic tools

III. Holographic dictionary

IV. Holography and fluid dynamics

V. Holography and condensed matter physic

References: provided

Exercises: optional

Web: possible

Lecture notes: currently unavailable

I. Introduction

1. Duality

Example 1: harmonic oscillator

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2 \hat{x}^2}{2}$$

with $[\hat{p}, \hat{x}] = -i\hbar$ $\quad (\hbar = 1 \text{ now})$

is self-dual under

$$D: \quad \hat{x} \to \hat{X} = \hat{p}/m\omega, \quad \hat{p} \to \hat{P} = -m\omega\hat{x}$$

Note: this is a canonical transformation,

$$[\hat{P}\,\hat{X}] = [\hat{p}\,\hat{x}] = -i$$

Also, $D^2 = P$ (parity) : $\hat{x} \to -\hat{x}$, $\hat{p} \to -\hat{p}$.

The ground state wave function and its Fourier transform are mapped into one another by the action of D:

Fourier transform of a Gaussian is a Gaussian

Example 2: Kramers - Wannier duality (1941)

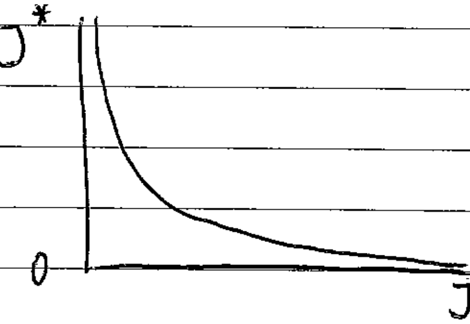
Ising model in $d = 2$ (square lattice) nearest neighbour inter.

$$Z[J] = \sum_{\{s=\pm 1\}} \exp\left(J \sum_{\langle ij \rangle} s_i s_j\right)$$

The partition function $Z[J]$ can be written as (introducing dual lattice):

$$Z = C(J)\, Z^*(J^*)$$

where $J^* = -\frac{1}{2} \ln \tanh J$:



$$Z[J] = \frac{1}{2}(\cosh J \sinh J)^N \sum_{\{s^* = \pm 1\}} \exp\left(J^* \sum_{\langle ij \rangle} s_i^* s_j^*\right)$$

(N is the number of sites)

__Note:__ $\sinh 2J \sinh 2J^* = 1$

Phase trans. to ferromagn. state at $J = J_c$:

(if unique, must be $J_c = J_c^*$, i.e.

$$\sinh 2J_c = 1$$

$$J_c = -\frac{1}{2} \ln(\sqrt{2} - 1) \approx 0.4407$$

Onsager, 1944 ; exact solution

Note: symmetry group: $\mathbb{Z}_2$

Note: in other dimensions models may not be self-dual:

$d = 3$: $\quad Z_{Ising}(J) = \tilde{C}(J)\, Z^{*}_{gauge}(J^{*})$

($\mathbb{Z}_2$-gauge theory in $d=3$)

Variables: $\quad S_i, \quad A_{i,\mu}, \quad B_{i,\mu\nu} \quad \ldots$

p: number of directional indices (μ, $\mu\nu$ etc)

then $\quad p^{*} = d - p - 2$

In particular, models with $p^{*} = p = \frac{d-2}{2}$ are self-dual (e.g. $p = p^{*} = 0$ for Ising in $d = 2$)

Remark: duality in lattice stat. mech. models with Abelian groups ($\mathbb{R}$, S^1, $\mathbb{Z}$, $\mathbb{Z}_N$) =

$\equiv$ Fourier analysis on Abelian groups

Crucial property: characters of an Abelian group form an Abelian group

<u>Note</u>: non-Abelian groups: Tannaka - Krein duality, Hopf algebras, non-commutative geometry (?) (Nicos knows recent ref.)

See Savit, Polyakov

Example 3: sine-Gordon - Thirring duality

$d = 2$

$$S_{SG} = \int d^2x \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{\alpha}{\beta^2} (\cos \beta\phi - 1) \right)$$

Excitations: mesons $M_{mes} = \sqrt{\alpha}$

solitons $M_{sol} = \frac{8\sqrt{\alpha}}{\beta^2}$

(see Rajaraman)

Coupling const: β^2

$M_{sol} \gg M_{mes}$ for $\beta^2 \ll 1$.

Thirring model:

$$S_T = \int d^2x \left(\bar{\psi} i \gamma_\mu \partial^\mu \psi + m \bar{\psi}\psi - \frac{g}{2} \bar{\psi}\gamma^\mu \psi \bar{\psi}\gamma_\mu \psi \right)$$

SG = Thirring (bosonization):

$$\frac{\beta^2}{4\pi} = \frac{1}{1 + g/\pi}$$

SG		Thirring
~~Meson~~ Soliton	$\longleftrightarrow$	Fundam. fermion
Meson	$\longleftrightarrow$	Fermion - antifermion bound state

Lessons:

1) Duality can relate weak and strong coupling

2) Duality interchanges fundamental quanta with solitons and Noether charges with topological charges

3) Involves geometrical concepts such as Fourier transform / Harmonic analysis

- Can be considered for $d=4$ QFTs with gauge groups (Montonen - Olive duality), especially for QFTs with SUSY.
- Electric / magnetic d.o.f., elem. excit. $\leftrightarrow$ monopoles.
- Seiberg - Witten $\mathcal{N}=2$ SYM
- String models generalization
- Hitchin on monopoles
- Langlands program

Our goal is more modest:

can we formulate a duality for theories involving gravitational d.o.f. ?

Original motivation for exploring dualities in QFTs: gauge theories at strong coupling (QCD): find convenient change of variables suitable for strong coupling