Advanced Quantum Mechanics

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Problem Sheet II
Non-relativistic potential scattering in 3d

Note: some problems are optional, problems with one or more stars are more difficult and can be treated as optional as well, although an ambitious student should attempt all problems.

1. Find the s-wave phase shifts δ_0 for the potentials

a)

$$U(r) = \begin{cases} \infty, & r < a\ , \\ 0, & r > a\ . \end{cases}$$

b)

$$U(r) = \begin{cases} -U_0, & r < a\ , \\ 0, & r > a\ . \end{cases}$$

c) $U(r) = -U_0\, e^{-r/a}$. *Hint: Change of variables* $x = \exp{(-r/2a)}$ *reduces the radial Schrödinger equation to Bessel equation.*

In all cases, find the cross-sections for slow particles *(optional for cases b) and c))*.

2. Find the phase shifts $\delta_l(k)$ for the potential $U(r) = \alpha/r^2$, $\alpha > 0$.

For $m\alpha/\hbar^2 \ll 1$, compute the sum over l using the generating function for Legendre polynomials

$$(1 - 2xz + x^2)^{-1/2} = \sum_{l=0}^{\infty} x^l P_l(z)$$

and find the differential cross section.

3. In the Born approximation, find the scattering amplitude and the cross section for the following potentials:

a) $U(r) = \alpha\delta(r - a)$

b) $U(r) = U_0 e^{-r/a}$

c) $U(r) = \frac{\alpha}{r}\, e^{-r/a}$

d) $U(r) = \alpha/r^2$

e) $U(r) = U_0 e^{-r^2/a^2}$

f)

$$U(r) = \begin{cases} U_0, & r \le a\ , \\ 0, & r > a. \end{cases}$$