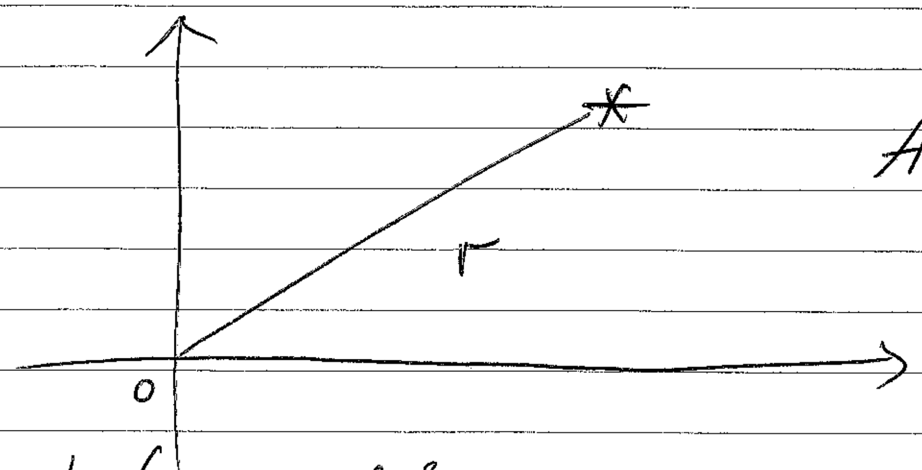
Luminosity distance
$d_L(z)$

Flux of energy from stars / supernovae

$$\Phi = \frac{L_{em}}{4\pi l^2}$$

Here $A = 4\pi l^2$: area at a distance l from the star.

In curved FRW metric $A_{proper} = 4\pi r^2 a^2(t_{obs})$ where r is co-moving distance from the source (star) to observer at $r=0$ at $t = t_{obs}$. (may be $t_{obs} = t_0$ = now.)



$$A = 4\pi r^2 a^2(t_{obs})$$

Photons: $ds^2 = 0$ (radial geodesics) $\qquad -c^2 dt^2 + a^2 \frac{dr^2}{1-kr^2} = 0$

②

$$c\int_{t_{em}}^{t_{obs}} \frac{dt}{a} = -\int_{r}^{0} \frac{dr}{\sqrt{1-kr^2}}$$ $\Rightarrow$ can continue $\forall k$ but let $k=0$ for simpl.

$$\Rightarrow r = -c\int_{t_{obs}}^{t_{em}} \frac{dt}{a} = c\int_{t_{em}}^{t_{obs}} \frac{dt}{a}$$

Now, recall: $\dfrac{\nu_{obs}}{\nu_{em}} = \dfrac{1}{1+z}$

$$\frac{\delta t_{obs}}{\delta t_{em}} = \frac{a(t_{obs})}{a(t_{em})} = 1+z$$

(see Set 4-5 solutions).

$\Rightarrow L_{obs} = \dfrac{L_{em}}{(1+z)^2}$, since $L = \dfrac{dE}{dt}$,

$E \sim h\nu$.

$$\Rightarrow \Phi_{obs} = \frac{L_{obs}}{4\pi r^2 a^2(t_{obs})} = \frac{L_{em}}{4\pi r^2 a^2(t_{obs})(1+z)^2} =$$

$$\equiv \frac{L_{em}}{4\pi d_L^2},$$

where $d_L^2 \equiv r^2 a^2(t_{obs})(1+z)^2$.

This is the luminosity distance

$$d_L = r\, a(t_{obs})(1+z)$$

To find $d_L(z)$, one can express the lhs of

$$c\int_{t_{em}}^{t_{obs}} \frac{dt}{a} = \int_0^r \frac{dr}{\sqrt{1-kr^2}}$$

via z using Friedmann eq. to convert dt to da and then

$$\frac{a(t_{obs})}{a(t)} = 1+z$$ to switch to dz.

Then solve for $r = r(z)$ and get $d_L(z)$.
(Do this!)
The exact formula can then be expanded for small z.

Alternatively, this approx. result one can get using the expansion

$$a(t) = a(t_0) + \dot{a}(t_0)(t-t_0) +$$

$$+ \frac{\ddot{a}(t_0)}{2}(t-t_0)^2 + \dots =$$

$$= a(t_0)\left[\underbrace{1 + H_0(t-t_0)}_{\epsilon \ll 1} + \frac{\ddot{a}(t_0)(t-t_0)^2}{2a(t_0)} + \dots\right] =$$

$$= a(t_0)\left[1 + H_0(t-t_0) - \frac{1}{2}q_0 H_0^2 (t-t_0)^2 + \dots\right]$$

where $q = -\frac{a\ddot{a}}{\dot{a}^2}$ (deceleration parameter)

Now we use (can put $t_{obs} = t_0$ if we want).

$$z = \frac{a(t_{obs})}{a(t_{em})} - 1 = \frac{1}{1 + H_0(t_{em}-t_0) + \dots} - 1 =$$

$$= H_0(t_0 - t_{em}) + \dots$$

On the other hand,

$$r \sim \int_{t_{em}}^{t_0} \frac{dt}{1 + H_0(t-t_0) + \dots} = t_0 - t_{em} + \dots$$

Combining the two series, we find

$$r = \frac{cz}{H_0 a(t_0)} + \dots$$

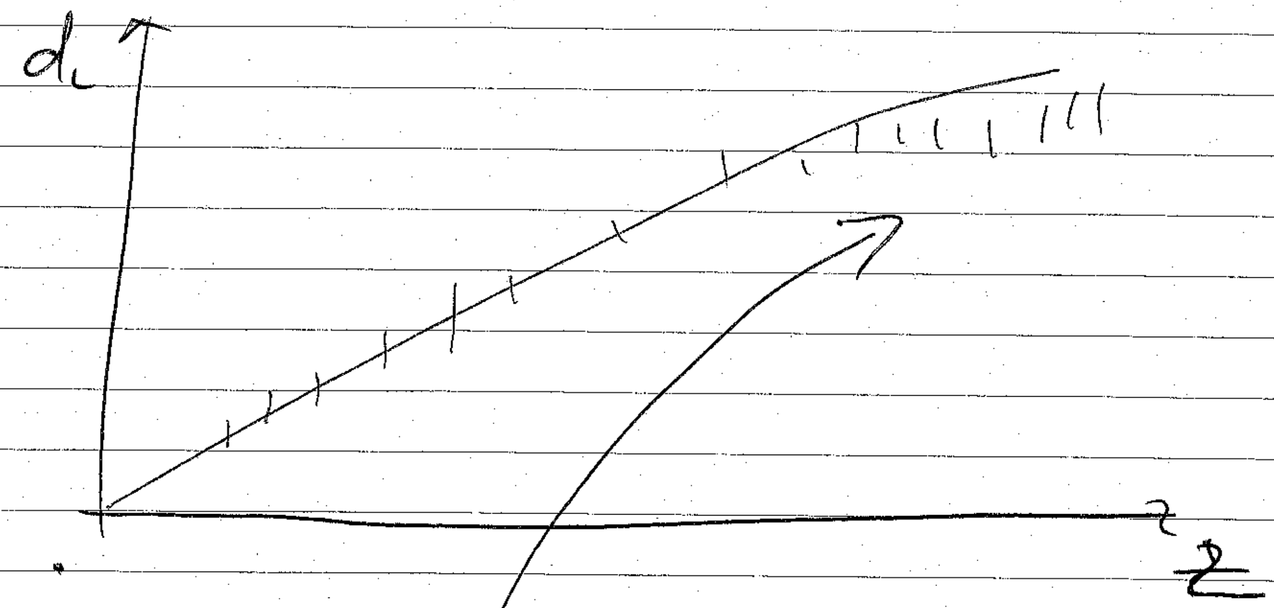
$$\Rightarrow d_L = \frac{cz}{H_0} + \dots$$

(recall $1+z$ factor would add to $O(z^2)$).

Keeping the next order, should find

$$\boxed{d_L = \frac{c}{H_0}\left[z + \frac{1}{2}(1-q_0) z^2 + \dots \right]}.$$

From observations $d_L(z)$:



full expression allows to find Ω_i's.