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A(4) Family Symmetry and Quark-Lepton Unification

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S.F.King, M.M., ArXiv: hep-ph/0610250

S. Antusch, S.F.King, M.M., work in progress

Outline

- General comments on SUSY flavour models
- Hints from the neutrino sector
- Sample SO(3) x Pati-Salam model
- A(4) flavour symmetry and quark-lepton unification

General comments on (SUSY) flavour models

In (MS)SM, the flavour physics is governed by the Yukawa couplings (and SSB sector)

Matter multiplets: $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

$$Q_L^i = \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix}_L^i = (3, 2, +1/3) \qquad L_L^i = \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L^i = (1, 2, -1)$$

$$U^{c}{}_L^i = \begin{pmatrix} u_1^c & u_2^c & u_3^c \end{pmatrix}_L^i = (\overline{3}, 1, -4/3) \qquad N^{c}{}_L^i = \begin{pmatrix} \nu^c \end{pmatrix}_L^i = (1, 1, 0)$$

$$D^{c}{}_L^i = \begin{pmatrix} d_1^c & d_2^c & d_3^c \end{pmatrix}_L^i = (\overline{3}, 1, +2/3) \qquad E^{c}{}_L^i = \begin{pmatrix} l^c \end{pmatrix}_L^i = (1, 1, +2)$$

Yukawa interactions:

$$\mathcal{L}_Y \ni Y_U^{ij} Q_L^{i\,T} C^{-1} U^{c}{}_L^j H_u + Y_D^{ij} Q_L^{i\,T} C^{-1} D^{c}{}_L^j H_d +$$
$$+ Y_N^{ij} L_L^{i\,T} C^{-1} N^{c}{}_L^j H_u + Y_E^{ij} L_L^{i\,T} C^{-1} E^{c}{}_L^j H_d + h.c.$$

Majorana sector:

$$\mathcal{L}_M \ni Y_\Delta^{ij} L_L^{i\,T} C^{-1} L_L^j \Delta + M^{ij} \nu^{c}{}_L^{i\,T} C^{-1} \nu^{c}{}_L^j + h.c.$$

Quark and charged lepton masses:

$$M_u^{ij} \propto Y_U^{ij} v_u, \qquad M_d^{ij} \propto Y_D^{ij} v_d, \qquad M_l^{ij} \propto Y_E^{ij} v_d$$

Seesaw mechanism:

$$m_\nu \doteq Y_\Delta \langle \Delta \rangle - Y_N^T M^{-1} Y_N v_u^2$$

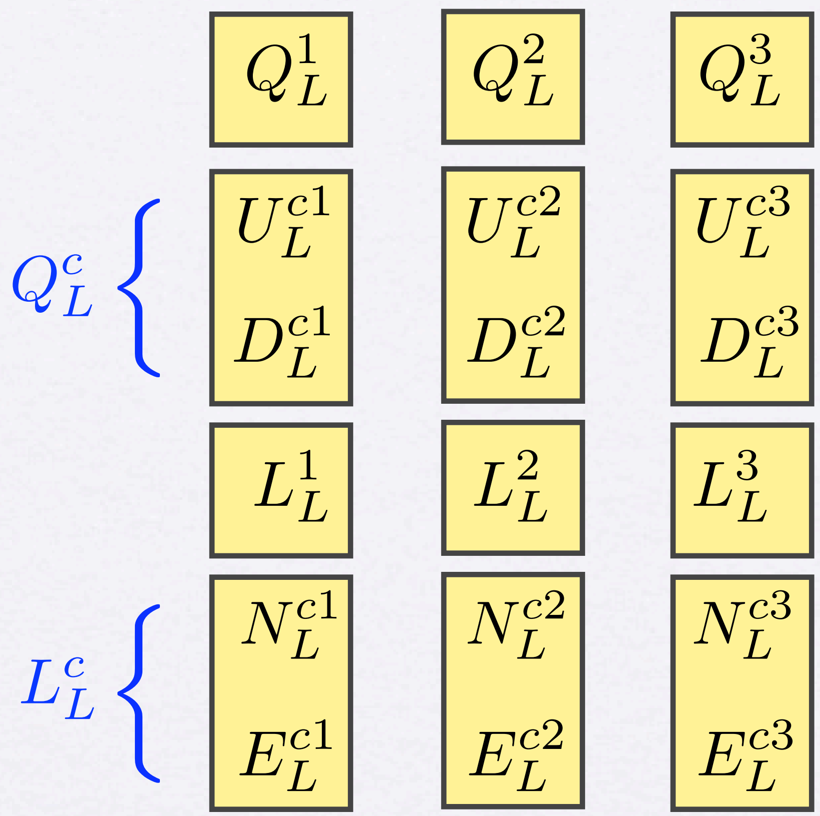
type-II type-I

General comments on (SUSY) flavour models

Extra symmetries proposed to constrain the Yukawa (and SSB) structures

Extended gauge symmetries

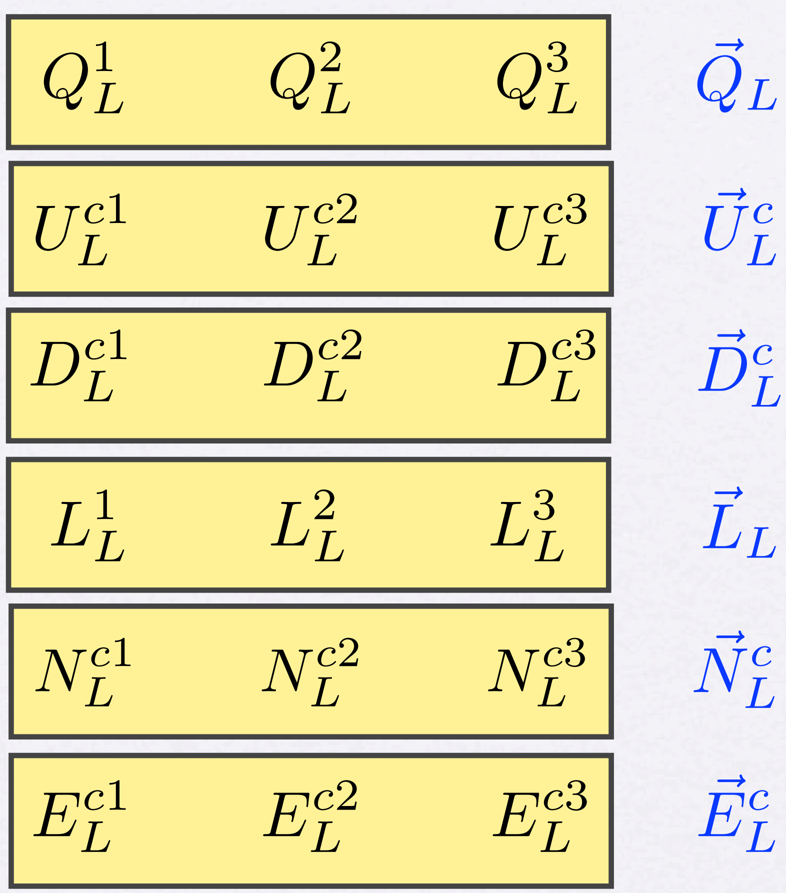
$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}, \ldots$$



Yukawa matrices correlated

Horizontal symmetries

$$SU(3),\ SO(3)\ldots$$



Yukawa entries correlated

In both cases, the extra symmetries must be badly broken at the electroweak scale

General comments on (SUSY) flavour models

The flavour symmetry breaking (usually driven by flavour symmetry Higgs fields - flavons) must be well under control to lead to correct spectra and mixing patterns.

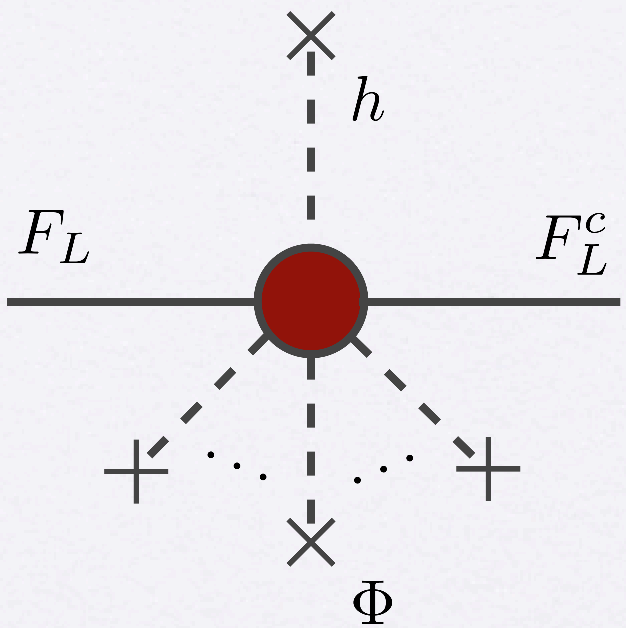
It is typically transmitted to the matter sector via higher order vertices with flavour symmetry breaking flavon VEVs:

spectra

Typically driven by flavour symmetry breaking scale(s)

mixing

Typically driven by vacuum alignment



Realistic model = flavour symmetry + FS breaking + vacuum alignment mechanism

General comments on (SUSY) flavour models

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$$U(1),\ SU(2),\ SU(3),\ SO(3)$$

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$$D(3),\ D(4),\ D(5),\ S(4),\ A(4)\ \ldots$$

Discrete symmetries fine for the vacuum alignment

In SUSY, there is an extra piece of information coming from FCNC and CPV which get naturaly enhanced due to the off-diagonalities in the soft sector

⇓ SUSY flavour problem

$\mu \to e\gamma,\ b \to s\gamma,\ \ldots$

⇓ SUSY CP problem

EDMs, $K-\overline{K}$ mixing,...

Usually, the larger the flavour symmetry multiplets the better the control over the Kähler potential and the soft sector.

Hints from the neutrino sector

Tri-bimaximal mixing in the lepton sector:

P.F.Harrison,D.Perkins,W. G. Scott, Phys.Lett.B530, 167 (2002)

$$U \sim \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

indicates correlations among all three lepton families.

Similar correlations arise for the neutrino Yukawa entries. If RH neutrinos happen to be sufficiently hierarchical and the charged lepton sector mixing negligible (in the basis in which RH neutrinos are diagonal), the sequential dominance mechanism gives the tri-bimaximal pattern automatically for

S. F. King, Nucl. Phys. B576 (2000) 85
S. F. King, JHEP 09 (2002) 011

$$Y^{\nu}_{LR} \sim \begin{pmatrix} 0 & b & . \\ a & b & . \\ -a & b & c \end{pmatrix} \qquad \frac{a^2}{M_1} \gg \frac{b^2}{M_2} \gg \frac{c^2}{M_3}$$

Both SUSY constraints and lepton mixing tend to call for maximal flavor symmetries !

A sample SO(3) x Pati-Salam model

For example $PS = SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$

S. F. King, JHEP 08 (2005) 105
S. F. King, M.M., JHEP 11 (2006) 071

$$W_Y^{leading} = \frac{1}{M} y_{23} \vec{F}.\vec{\phi}_{23} F_1^c h + \frac{1}{M} y_{123} \vec{F}.\vec{\phi}_{123} F_2^c h + \frac{1}{M} y_3 \vec{F}.\vec{\phi}_3 F_3^c h + \ldots$$

$(3;4,1,2)$

$$\vec{F} \equiv \left\{ \begin{array}{ccc} Q_L^1 & Q_L^2 & Q_L^3 \\ L_L^1 & L_L^2 & L_L^3 \end{array} \right. \quad \leftarrow PS \rightarrow$$

$\leftarrow SO(3) \rightarrow$

$3 \times (1;\overline{4},2,1)$

$$F_1^c,\ F_2^c,\ F_3^c \equiv \left\{ \begin{array}{ccc} U_L^{c1} & U_L^{c2} & U_L^{c3} \\ D_L^{c1} & D_L^{c2} & D_L^{c3} \\ N_L^{c1} & N_L^{c2} & N_L^{c3} \\ E_L^{c1} & E_L^{c2} & E_L^{c3} \end{array} \right.$$

leads to the desired neutrino Yukawa matrix provided

$$\langle \vec{\phi}_{23} \rangle \sim \begin{pmatrix} 0 \\ v \\ -v \end{pmatrix}, \quad \langle \vec{\phi}_{123} \rangle \sim \begin{pmatrix} \tilde{v} \\ \tilde{v} \\ \tilde{v} \end{pmatrix}, \quad \langle \vec{\phi}_3 \rangle \sim \begin{pmatrix} . \\ . \\ V \end{pmatrix}$$

Issues to be addressed:

RH neutrino sector should be essentially diagonal

Charged lepton sector should not spoil the T-B lepton mixing

The quark and lepton mass hierarchies and CKM mixing should be accommodated

The vacuum alignment mechanism

A sample SO(3) x Pati-Salam model

field	$SU(4)\otimes SU(2)_L\otimes SU(2)_R$	$SO(3)$	$U(1)$	Z_2
$\vec{F}$	$(4,2,1)$	3	0	+
F_1^c	$(\overline{4},1,2)$	1	+2	−
F_2^c	$(\overline{4},1,2)$	1	+1	+
F_3^c	$(\overline{4},1,2)$	1	−3	−
h	$(1,2,2)$	1	0	+
H, $\overline{H}$	$(4,1,2)$, $(\overline{4},1,2)$	1	±3	+
H', $\overline{H'}$	$(4,1,2)$, $(\overline{4},1,2)$	1	∓3	+
Σ	$(15,1,3)$	1	-1	−
$\vec{\phi}_3$	$(1,1,1)$	3	+3	−
$\vec{\phi}_{23}$	$(1,1,1)$	3	−2	−
$\vec{\phi}_{123}$	$(1,1,1)$	3	−1	+
$\vec{\phi}_{12}$	$(1,1,1)$	3	0	+
$\vec{\tilde{\phi}}_{23}$	$(1,1,1)$	3	0	−

$$\sigma \equiv \langle\Sigma\rangle/M_f$$

$$\varepsilon_x^f \equiv |\langle\vec{\phi}_x\rangle|/M_f$$

$$y_x \sim O(1)$$

$$C^{u,d,l,\nu} = -2, 1, 3, 0$$

are the Clebsches to accommodate the proper mass hierarchies

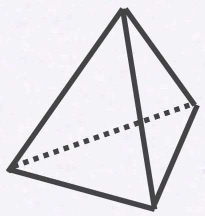
$$\varepsilon_{23}, \varepsilon_{123}, \varepsilon_{12} \ll \tilde{\varepsilon}_{23} < \varepsilon_3 \sim O(1)$$

$$Y_{LR}^f = \begin{pmatrix} 0 & y_{123}\varepsilon_{123}^f & y_{12}\varepsilon_{12}^f\varepsilon_3^f \\ y_{23}\varepsilon_{23}^f & y_{123}\varepsilon_{123}^f + C^f y_{GJ}\tilde{\varepsilon}_{23}^f\sigma & \tilde{y}_{23}(\tilde{\varepsilon}_{23})^2\varepsilon_3 \\ -y_{23}\varepsilon_{23}^f & y_{123}\varepsilon_{123}^f + C^f y_{GJ}\tilde{\varepsilon}_{23}^f\sigma & y_3\varepsilon_3^f \end{pmatrix}$$

The model works quite well, but:

the standard vacuum alignment mechanism very complicated and quite expensive !!!

A(4) x Pati-Salam model



A(4) is the symmetry group of tetraheadron, i.e. a discrete subgroup of SO(3)

Equivalently, it is a group of even permutations of 4 objects.

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L.Lavoura,H.Kuhbock, hep-ph/0610050.
G. Altarelli,F. Feruglio,Y.Lin, hep-ph/0610165.

Benefits of a discrete subgroup in the game:

Invariants of the continuous case remain intact and new terms are allowed

The extra terms break explicitly the original continuous symmetry

Example: SO(3) and A(4) invariants that can be built out of triplets:

	SO(3)	A(4)
quadratic:	$\phi.\phi$	$\phi.\phi$
cubic:	$(\phi\times\chi).\psi$	$(\phi\times\chi).\psi,\ (\phi * \chi).\psi$
quartic:	$(\phi.\phi)^2, (\phi\times\psi)^2, \ldots$	$(\phi.\phi)^2, \sum_{i=1}^{3}\phi_i\phi_i\phi_i\phi_i, (\phi\times\psi)^2, \ldots$

Discrete symmetries can help with the huge vacuum degeneracy of the continuous case !!!

A(4) x Pati-Salam model

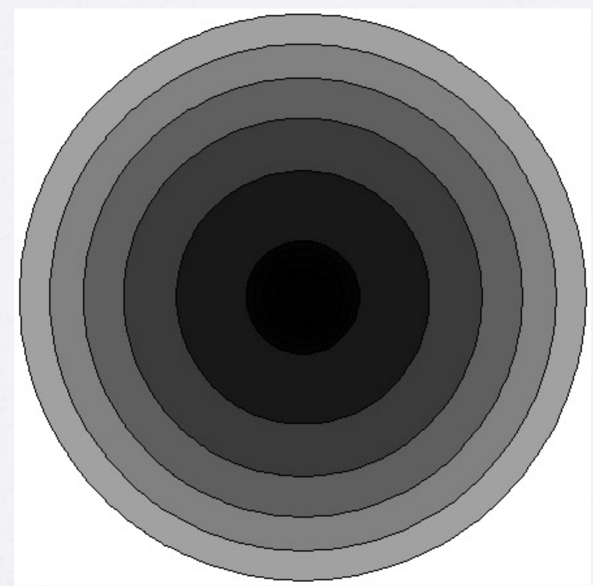
Example:

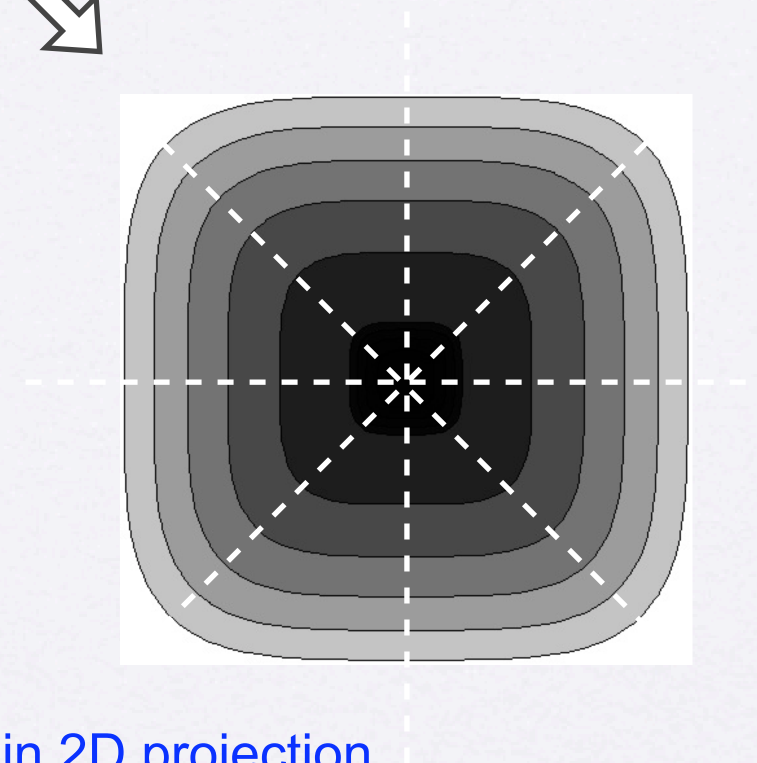
The (single) flavon scalar potential can in A(4) case contain terms like:

$$V \ni -M_\phi^2(\phi^\dagger\phi) + \Lambda(\phi^\dagger\phi)^2 + \lambda\, \phi_i^\dagger \phi_i \phi_i^\dagger \phi_i + \ldots$$

$$\lambda > 0 \,:\, \langle\vec{\phi}\rangle \sim \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$\lambda < 0 \,:\, \langle\vec{\phi}\rangle \sim \begin{pmatrix}0\\0\\1\end{pmatrix}$$ or perms.





Isocurves in 2D projection

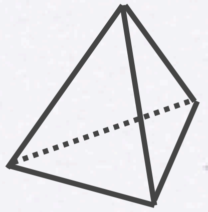
$\vec{\phi}_3,\ \vec{\phi}_{123}$ VEVs almost for free !!!

On the other hand, it might be difficult to get such simple structures from the F-terms.

However, higher order D-terms can naturally lead to a set of such extra quartic terms in the effective potential.

I. de Medeiros Varzielas, S. F. King, and G. G. Ross, hep-ph/0607045.

A(4) x Pati-Salam model



Slight modification of the previous SO(3) model:

field	$SU(4)\otimes SU(2)_L \otimes SU(2)_R$	A_4	$U(1)$	Z_2
F	$(4,2,1)$	3	0	$+$
F_1^c	$(\overline{4},1,2)$	1	$+2$	$-$
F_2^c	$(\overline{4},1,2)$	1	$+1$	$+$
F_3^c	$(\overline{4},1,2)$	1	-3	$-$
h	$(1,2,2)$	1	0	$+$
H, $\overline{H}$	$(4,1,2)$, $(\overline{4},1,2)$	1	± 3	$+$
H', $\overline{H'}$	$(4,1,2)$, $(\overline{4},1,2)$	1	∓ 3	$+$
Σ	$(15,1,3)$	1	-1	$-$
ϕ_1	$(1,1,1)$	3	$+4$	$+$
ϕ_2	$(1,1,1)$	3	0	$+$
ϕ_3	$(1,1,1)$	3	$+3$	$-$
ϕ_{23}	$(1,1,1)$	3	-2	$-$
$\tilde{\phi}_{23}$	$(1,1,1)$	3	0	$-$
ϕ_{123}	$(1,1,1)$	3	-1	$+$

Vacuum:

$$\langle\vec{\phi}_1\rangle \sim \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix} \qquad \langle\vec{\phi}_2\rangle \sim \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix}$$

$$\langle\vec{\phi}_2\rangle \sim \begin{pmatrix} 0 \\ 0 \\ V_3 \end{pmatrix} \qquad \langle\vec{\phi}_{123}\rangle \sim \begin{pmatrix} v \\ v \\ v \end{pmatrix}$$

As we have seen these structures are easy to get !

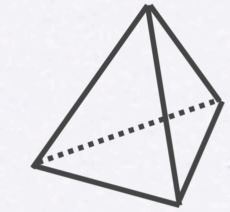
$$\langle\vec{\tilde{\phi}}_{23}\rangle \sim \begin{pmatrix} 0 \\ V_{23} \\ -V_{23} \end{pmatrix}$$

$$\langle\vec{\phi}_{23}\rangle \sim \begin{pmatrix} 0 \\ v_{23} \\ -v_{23} \end{pmatrix}$$

How to get these ?

The Yukawa sector remains quite similar to that of the SO(3) case, but the vacuum alignment is very simple !

A(4) x Pati-Salam model



Obtaining $\langle\vec{\phi}_{23}\rangle \sim \begin{pmatrix} 0 \\ v \\ -v \end{pmatrix}$:

S. F. King, M.M., hep-ph/0610250

$$V \ni -M_{23}^2|\phi_{23}|^2 + \lambda_{123}|\phi_{123}^\dagger.\phi_{23}|^2 + \lambda_1|\phi_1^\dagger.\phi_{23}|^2 + \dots$$

⇩ Minimized for $\langle\vec{\phi}_{23}\rangle$ orthogonal to $\langle\vec{\phi}_{123}\rangle$

⇩ Minimized for $\langle\vec{\phi}_{23}\rangle$ orthogonal to $\langle\vec{\phi}_{1}\rangle$

Virtues of the vacuum alignment in the discrete case :

Simplicity

Extra constraints on the Kähler potential

⇩

Handle on the soft SUSY breaking sector (?)

Work in progress...

Conclusions

- SUSY flavour models strongly constrained in both Yukawa and soft sectors
- SUSY flavour and CP problems call for maximal symmetries
- Models with discrete subgroups of continuous symmetries provide for simple vacuum alignment mechanisms
- Work in progress

Thanks for your kind attention !