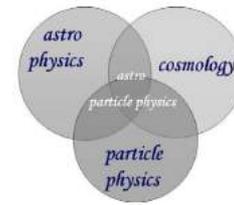




Hilary 2021



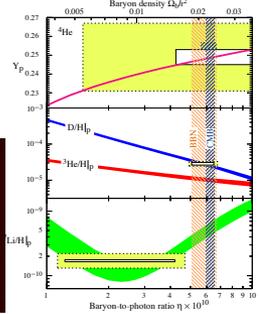
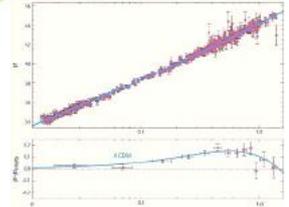
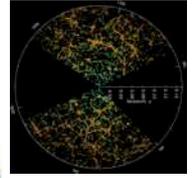
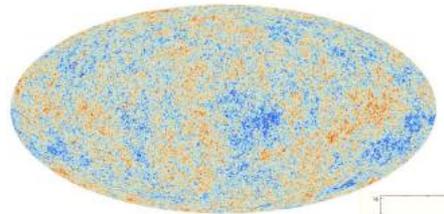
Oxford Master Course in Mathematical and Theoretical Physics

- ✧ The universe observed
- ✧ Relativistic world models
- ✧ Reconstructing the thermal history
- ✧ Big bang nucleosynthesis
- ✧ **Dark matter: astrophysical observations**
 - ✧ Dark matter: relic particles
 - ✧ Dark matter: direct detection
 - ✧ Dark matter: indirect detection
 - ✧ Cosmic rays in the Galaxy
 - ✧ Antimatter in cosmic rays
 - ✧ Ultrahigh energy cosmic rays
 - ✧ High energy cosmic neutrinos
- ✧ The early universe: constraints on new physics
- ✧ The early universe: baryo/leptogenesis
- ✧ The early universe: inflation & the primordial density perturbation
 - ✧ Cosmic microwave background & large-scale structure

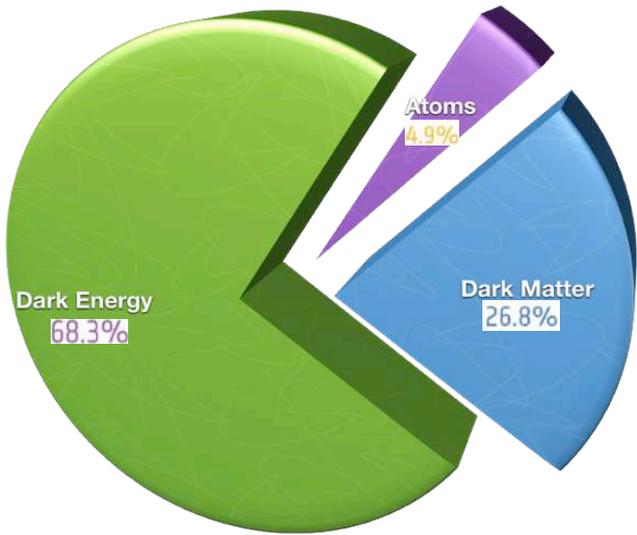
WHAT IS THE WORLD MADE OF?

Mainly geometrical evidence:
 $\Lambda \sim O(H_0^2)$, $H_0 \sim 10^{-42}$ GeV
 ... dark energy is *inferred* from
 the 'cosmic sum rule':

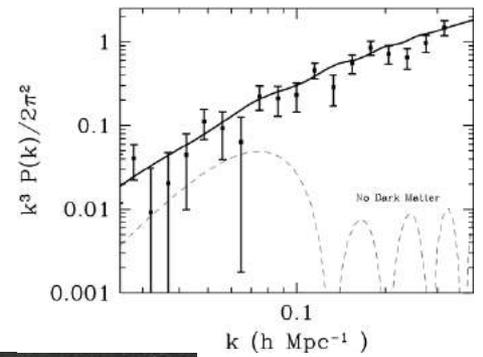
$$\Omega_m + \Omega_k + \Omega_\Lambda = 1$$



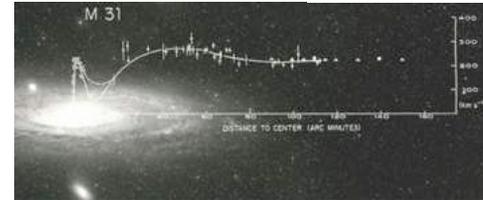
Baryons
 (but *no*
 anti-
 baryons)

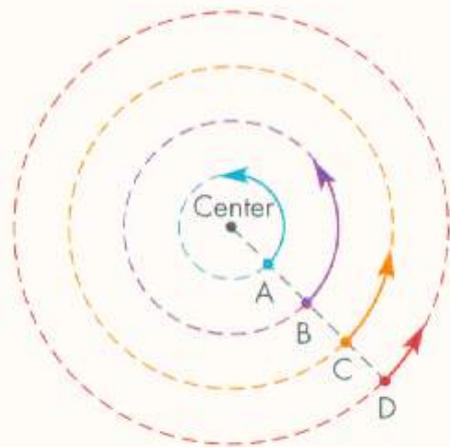


Both geometrical
 and dynamical
 evidence (assuming
 GR to be valid)

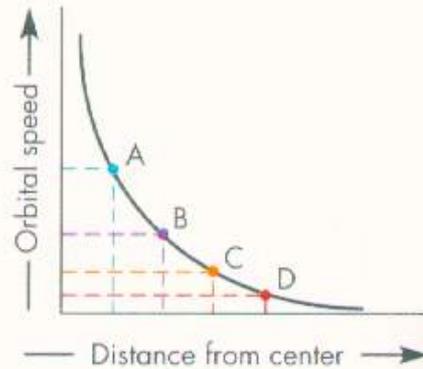


Both the baryon asymmetry and dark matter
 require that there be *new* physics beyond the
 Standard $SU(3)_c \times SU(2)_L \times U(1)_Y$ Model
 ... dark energy is even more mysterious (but as
 yet lacks compelling *dynamical* evidence)





Planet-like rotation

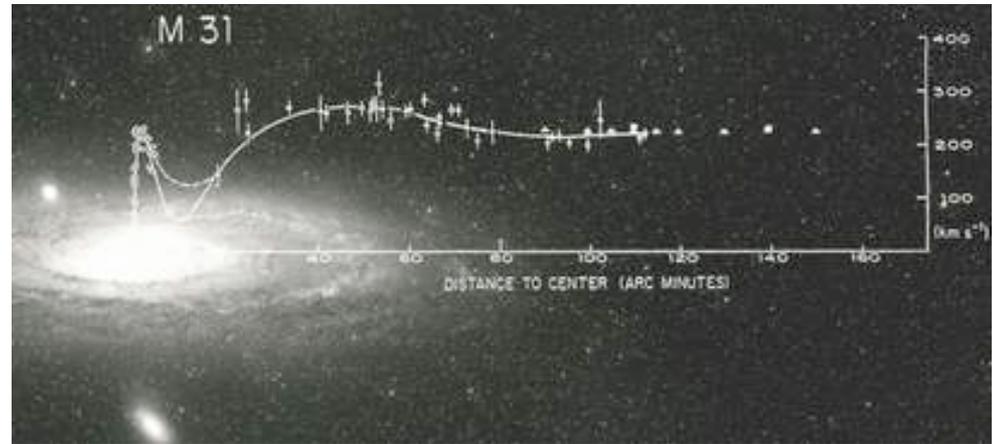


Rotation curve for planet-like rotation

At large distances from the centre, beyond the edge of the visible galaxy, the velocity should fall as $1/\sqrt{r}$ if most of the matter is in the optical disc

$$v_{\text{circ}} = \sqrt{\frac{G_N M(< r)}{r}}$$

... but e.g. Rubin & Ford (ApJ 159:379,1970) observed that the rotational velocity remains \sim constant in Andromeda – interpreted *later* as implying the existence of an extended (dark) ‘corona’ or halo

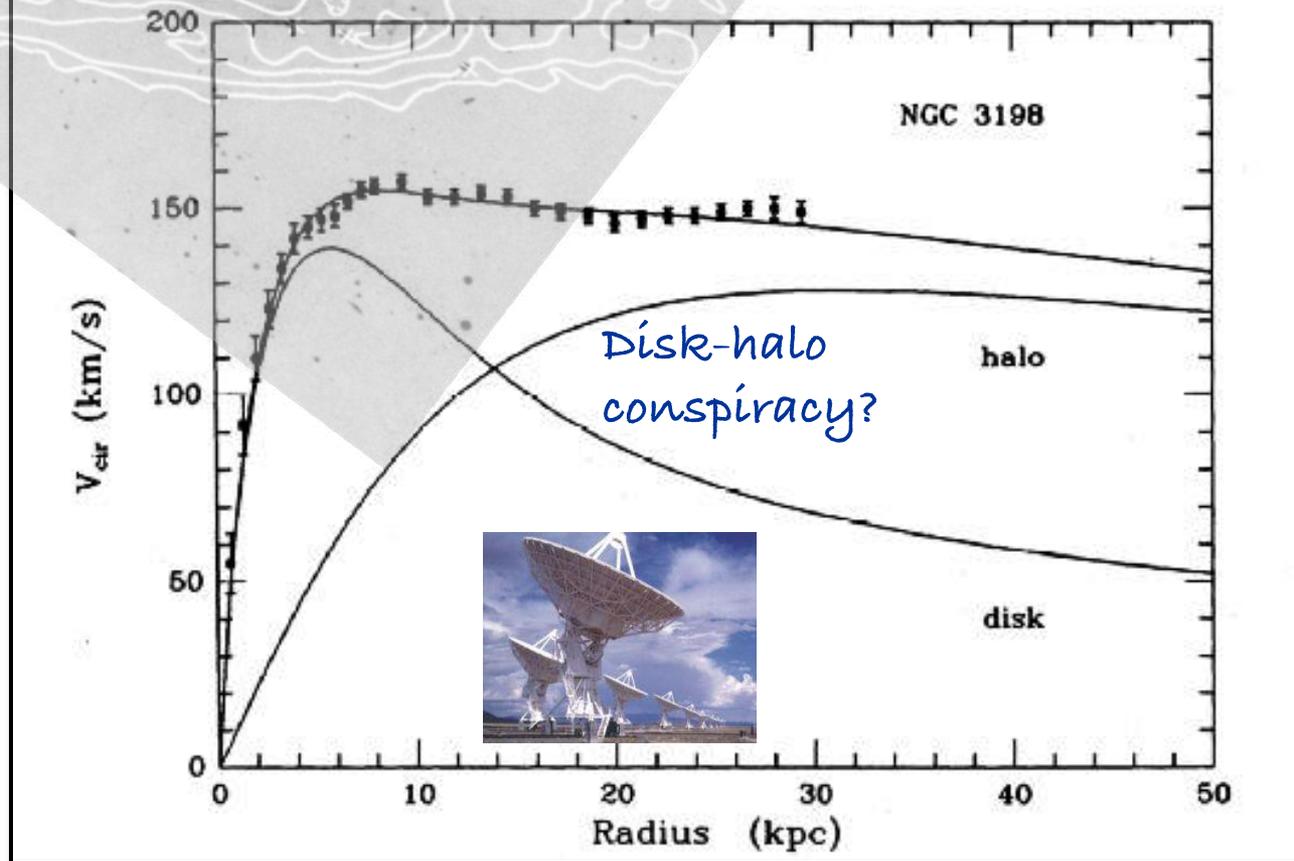


$$v_{\text{circ}} \sim \text{constant} \quad \Rightarrow \quad M(< r) \propto r \quad \Rightarrow \quad \rho \propto 1/r^2$$

The really compelling evidence for **extended halos of dark matter** came much later from observations of 21 cm line emission from neutral hydrogen (orbiting around the Galaxy at \sim constant velocity) well *beyond* the visible disk

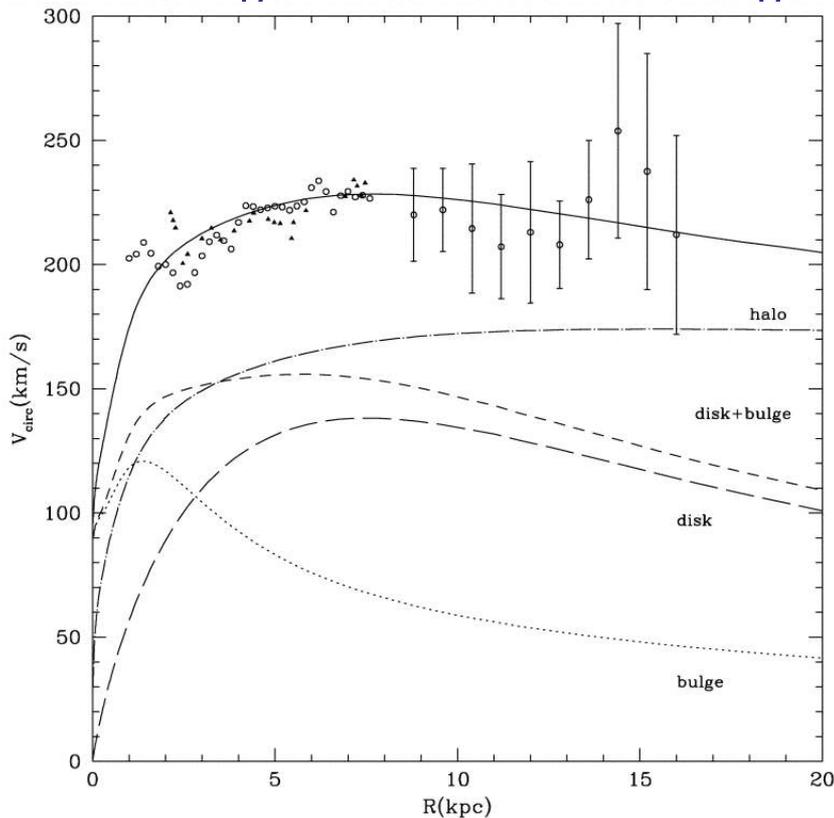


VAN ALBADA *ET AL.* (ApJ 295:305,1985)



MORE SOPHISTICATED MODELLING ACCOUNTS FOR MULTIPLE COMPONENTS AND THE COUPLING BETWEEN BARYONIC & DARK MATTER

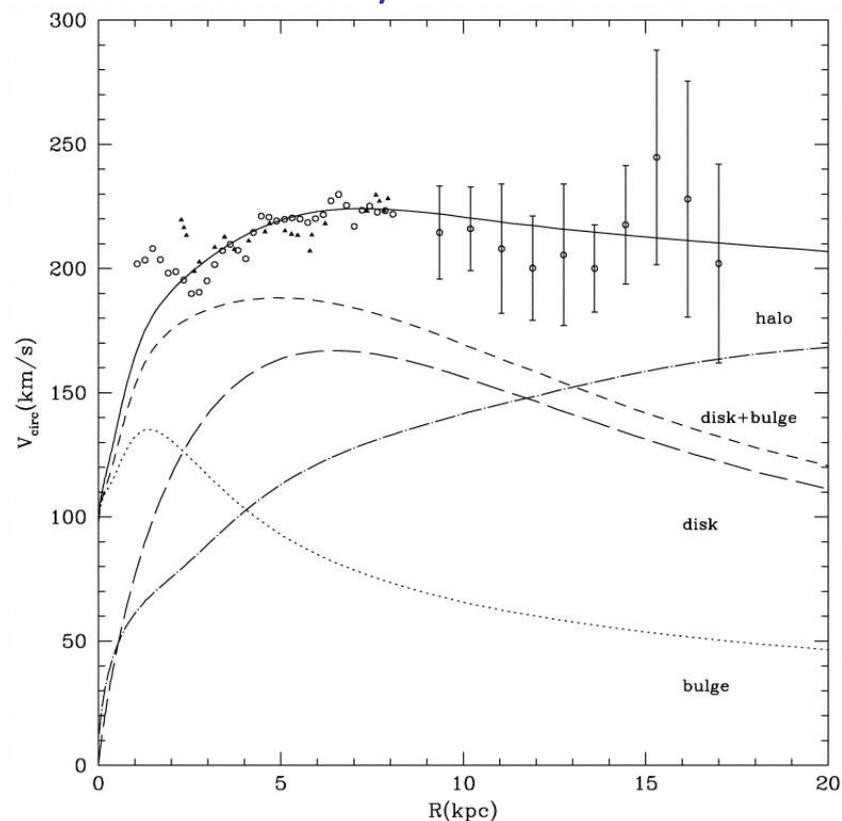
Adiabatic compression of baryons
- no angular momentum exchange



$$G[M_b(r) + M_{\text{dm}}(r)]r = GM_{\text{halo}}(r_i)r_i,$$

$$M_{\text{halo}}(r_i) = \frac{M_{\text{dm}}(r)(\Omega_b + \Omega_{\text{dm}})}{\Omega_{\text{dm}}}.$$

With angular momentum exchange
between baryons & dark matter



$$j = rV_c = \sqrt{G[M_b(r) + M_{\text{dm}}(r)]r}$$

$$dJ_b = dM_b \left[\left(V_c + \frac{dV_c}{dr} dr \right) (r + dr) - V_c r \right]$$

The *local* halo dark matter density is inferred to be $\sim 0.3 \text{ GeV cm}^{-3}$ (uncertainty x2)

With the $1/r^2$ density profile, the solution of the collisionless Boltzmann equation is the ‘Maxwellian distribution’:

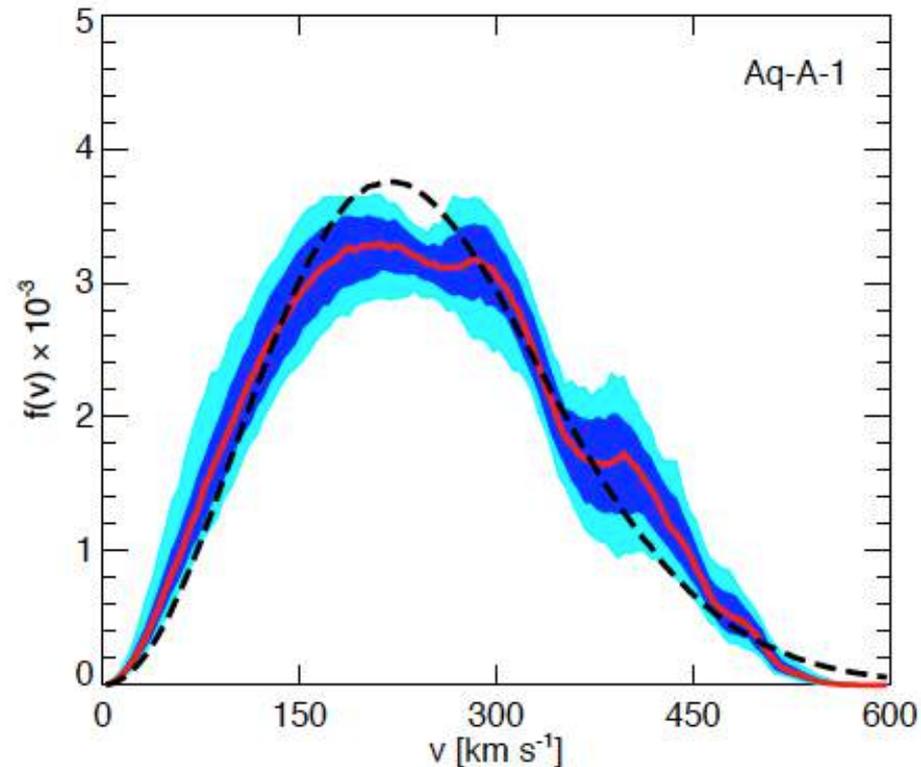
The ‘standard halo model’ has $v_c = 220$ km/s and is truncated at $v_{\text{esc}} = 544$ km/s (both numbers have large observational uncertainties)

High resolution numerical simulations however suggest significant deviations from the Maxwellian distribution, particularly at high velocities (\Rightarrow important implications for direct detection experiments)

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \frac{\partial f}{\partial \mathbf{v}} = 0$$

$$f(\mathbf{v}) = N \exp\left(-\frac{3|\mathbf{v}|^2}{2\sigma^2}\right)$$

$$\sigma = \sqrt{3/2} v_c$$

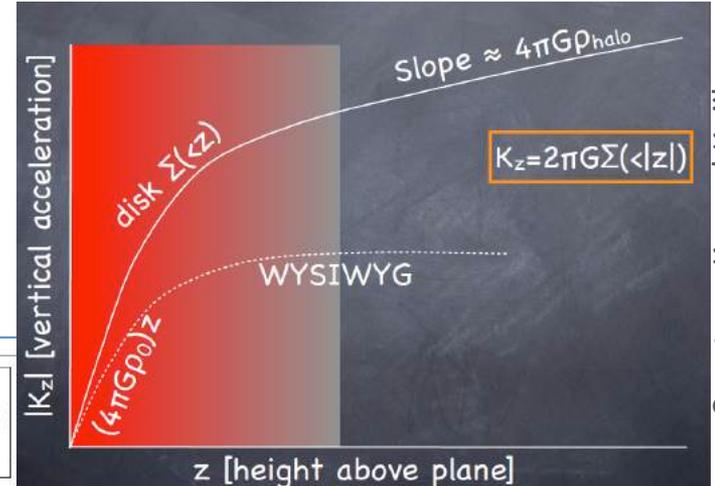
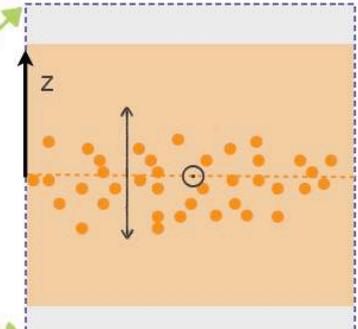
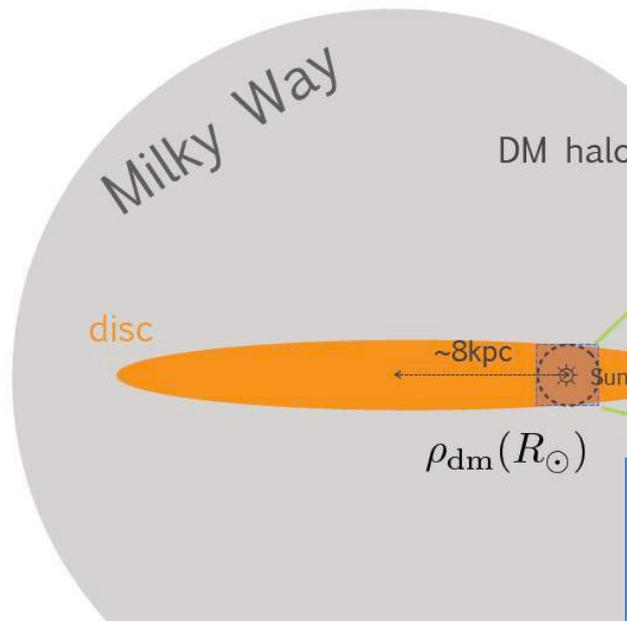


We can infer the *local* dark matter density by measuring vertical distribution of stars ... pioneered by Kapetyn (1922) and Oort (1932)

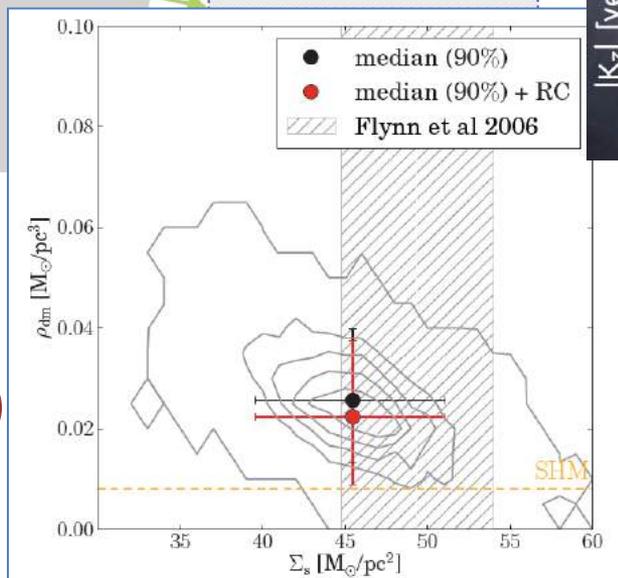
If galaxy is approximated as thin disk, then orthogonal to the Galactic plane:

$$\frac{d^2\psi(z)}{dz^2} = 4\pi G_N \rho_m \rightarrow \frac{d\psi(z)}{dz} = 2\pi G_N \Sigma_m$$

~100pc



Courtesy: Konrad Kuijken



Using Kuijken & Gilmore's data (MNRAS 239:605,1989) on K-dwarfs, Garbari *et al* get (MNRAS 425:1445,2012)

$\rho_{DM} = 0.85 \pm 0.6 \text{ GeV/cm}^3$

Using recent data on SDSS-SEGUE-G-dwarfs, Sivertsson *et al* (MNRAS 478:1667,2018) get: $\rho_{DM} = 0.46 \pm 0.07 \text{ GeV/cm}^3$... but there are *inconsistencies*

Expect update from GAIA

Bidin *et al* (ApJ 747:101,2012) claimed $\rho_{DM} < 0.04 \text{ GeV/cm}^3$, because of *incorrectly* assuming that the rotational velocity is independent of galactocentric radius at all z (Bovy & Tremaine, ApJ 756:89,2012)

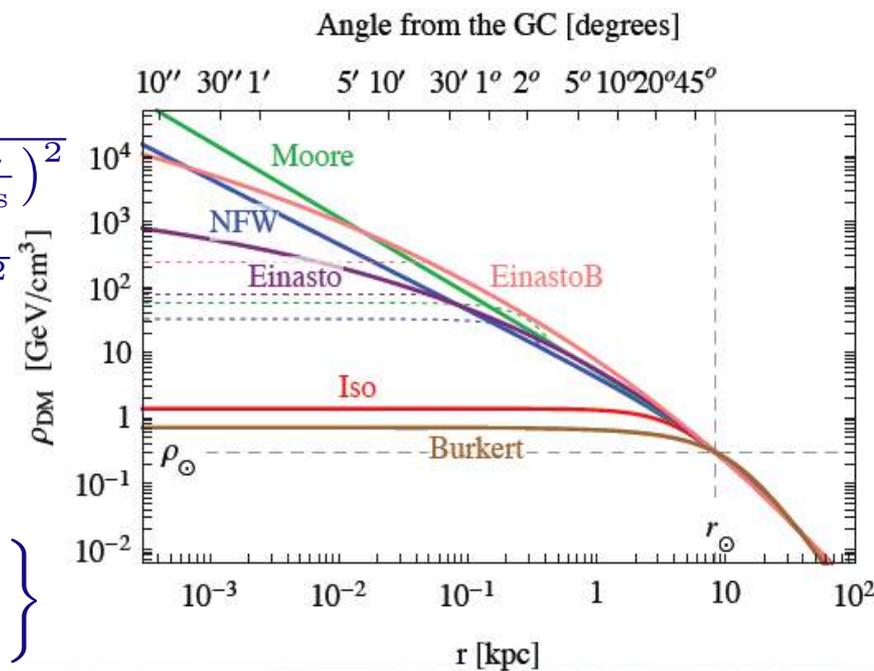
MODELLING DARK MATTER HALOS

Cored isothermal sphere: $\rho_{\text{isothermal}} = \frac{\rho_s}{\left(1 + \frac{r}{r_s}\right)^2}$

Navarro-Frenk-White profile: $\rho_{\text{NFW}} = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$
(indicated by CDM simulations)

Burkert profile: $\rho_{\text{Burkert}} = \frac{\rho_s}{\left(1 + \frac{r}{r_s}\right) \left[1 + \left(\frac{r}{r_s}\right)^2\right]}$
(fits observations better)

Einasto profile: $\rho_{\text{Einasto}} = \rho_s \exp \left\{ -d_n \left[\left(\frac{r}{r_s}\right)^{1/n} - 1 \right] \right\}$



where d_n is defined such that ρ_s is the density at the radius r_s enclosing half the total mass

... more generally define the **Hernquist profile:** $\rho_{\text{Hernquist}} = \rho_s \left(\frac{r}{r_s}\right)^{-\gamma} \left[1 + \left(\frac{r}{r_s}\right)^\alpha\right]^{\frac{\gamma-\beta}{\alpha}}$

Here r_s is a characteristic scale and α controls the sharpness of the transition from the inner slope $\lim_{r \rightarrow 0} d \ln(\rho) / d \ln(r) = -\gamma$ to the outer slope $\lim_{r \rightarrow \infty} d \ln(\rho) / d \ln(r) = -\beta$

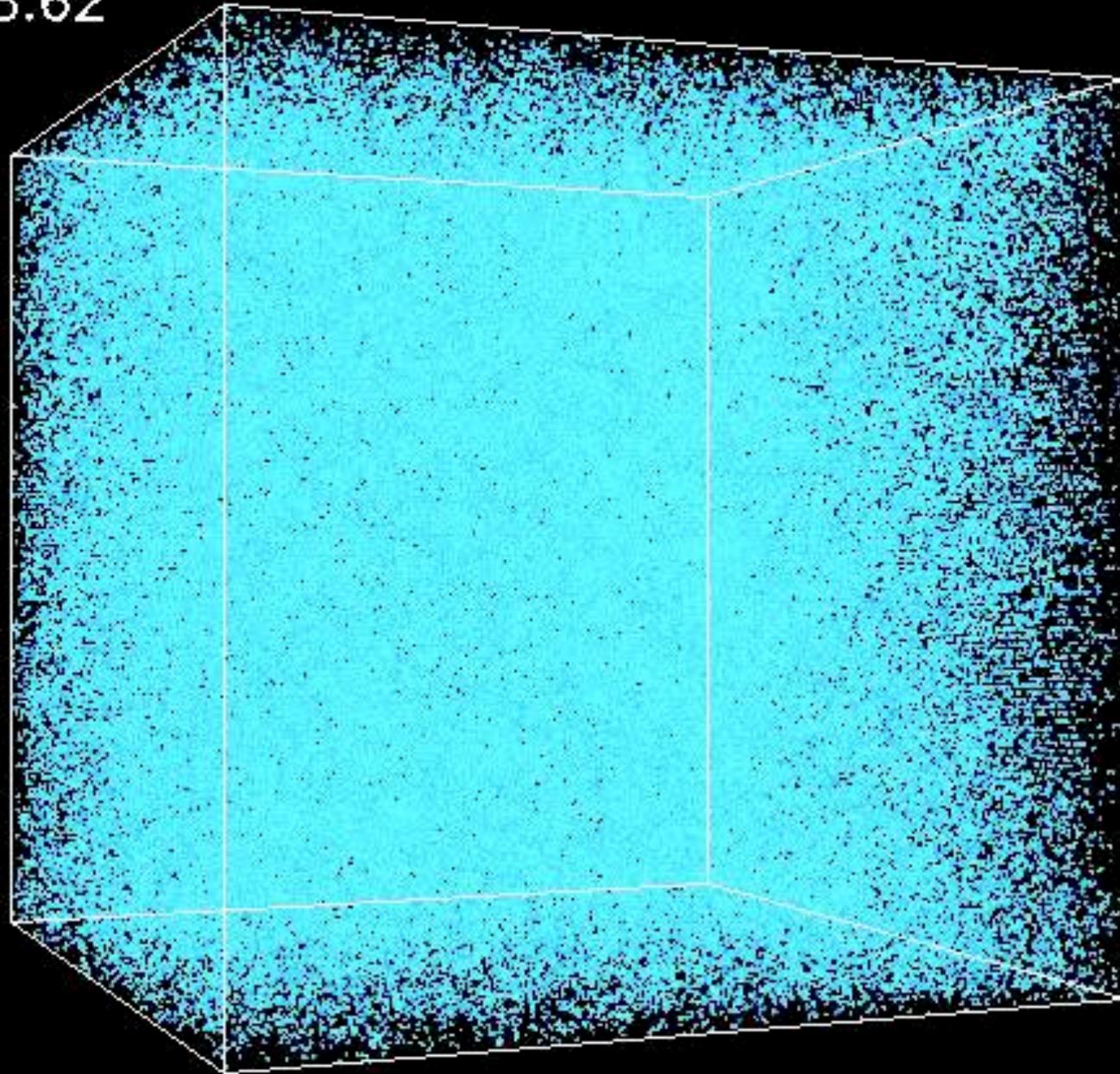
... e.g. the NFW profile corresponds to choosing $\alpha = 1, \beta = 3, \gamma = 1$, whereas a cored isothermal profile corresponds to choosing $\alpha = 1, \beta = 2, \gamma = 0$, and a Moore profile corresponds to $\alpha = 1.5, \beta = 2, \gamma = 1.5$ etc

For the Milky Way, the fit parameters are:

DM halo	α	r_s [kpc]	ρ_s [GeV/cm ³]
NFW	–	24.42	0.184
Einasto	0.17	28.44	0.033
EinastoB	0.11	35.24	0.021
Isothermal	–	4.38	1.387
Burkert	–	12.67	0.712
Moore	–	30.28	0.105

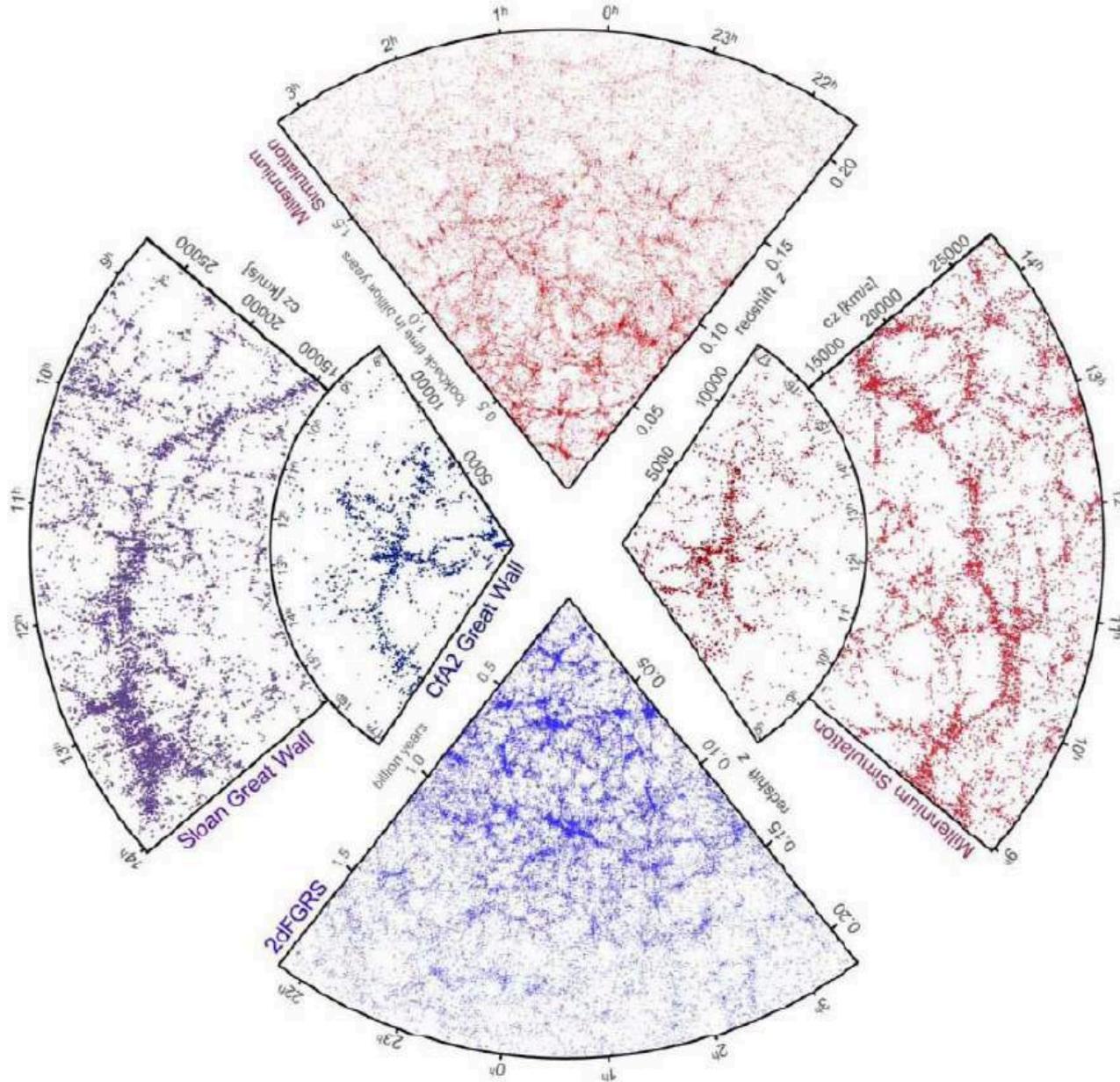
$Z=28.62$

SIMULATING THE UNIVERSE ON A COMPUTER

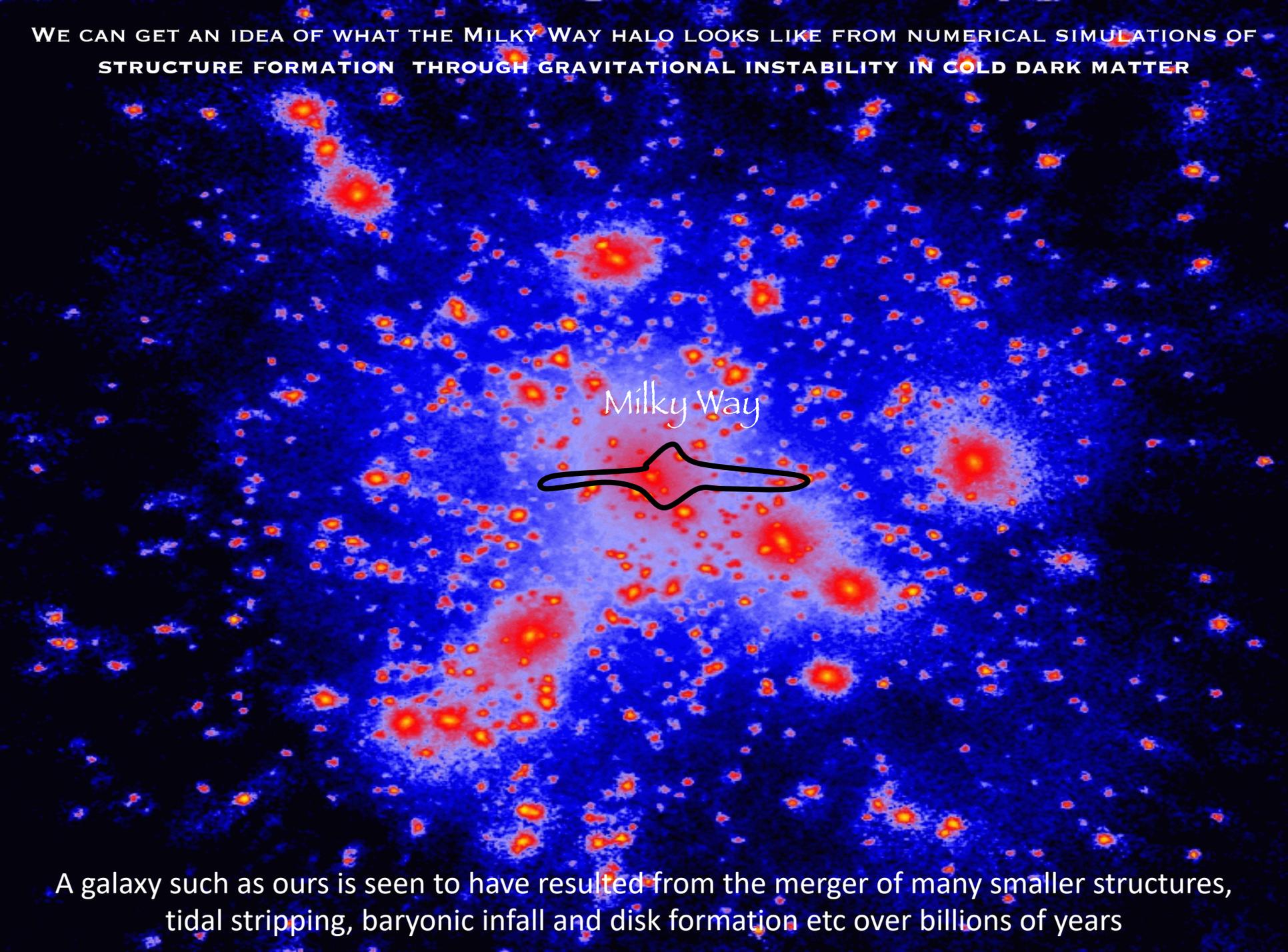


Λ CDM

SUCH NUMERICAL SIMULATIONS PROVIDE A PRETTY GOOD MATCH TO THE OBSERVED LARGE-SCALE STRUCTURE OF GALAXIES IN THE UNIVERSE



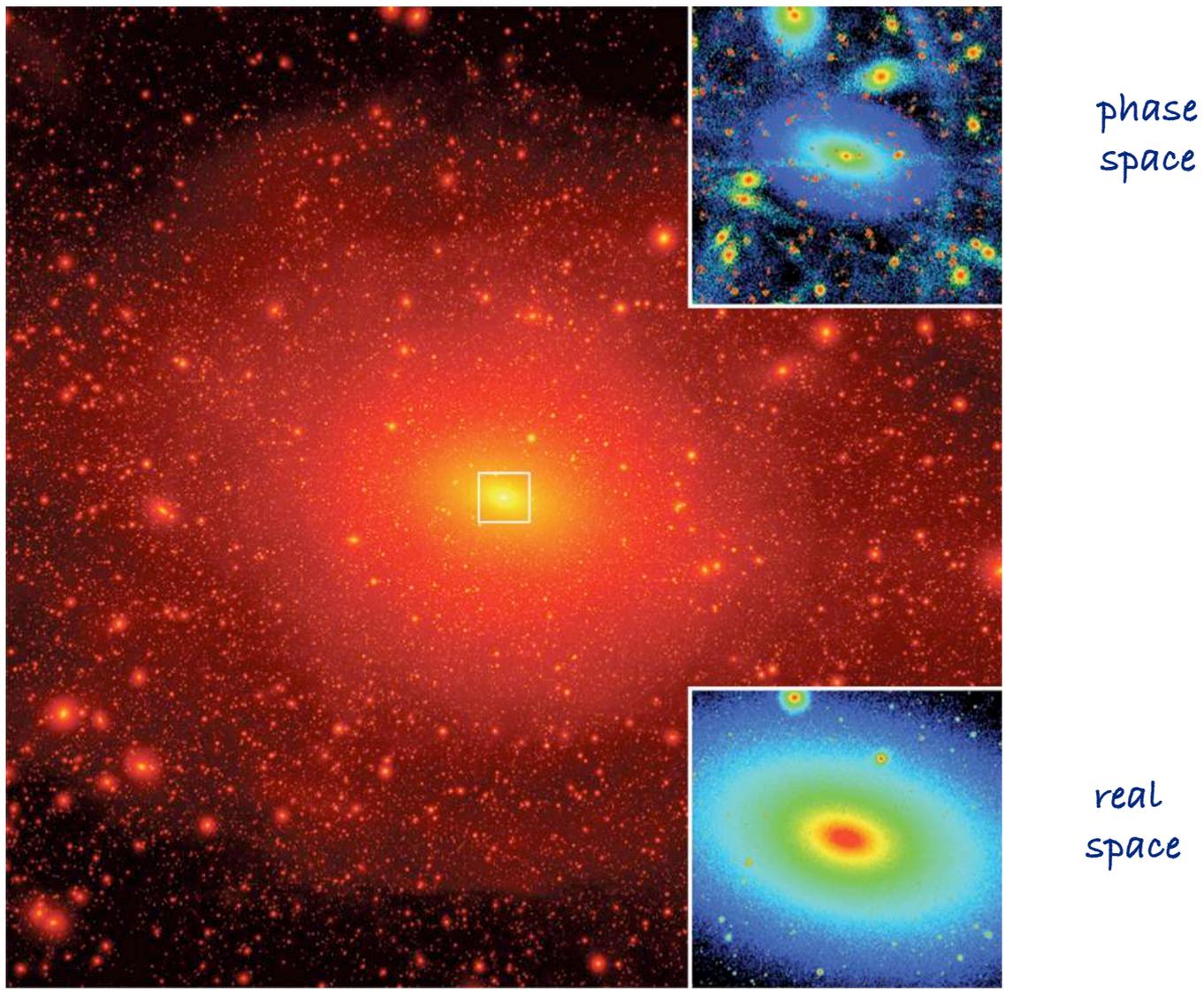
WE CAN GET AN IDEA OF WHAT THE MILKY WAY HALO LOOKS LIKE FROM NUMERICAL SIMULATIONS OF
STRUCTURE FORMATION THROUGH GRAVITATIONAL INSTABILITY IN COLD DARK MATTER



A galaxy such as ours is seen to have resulted from the merger of many smaller structures, tidal stripping, baryonic infall and disk formation etc over billions of years

SO THE PHASE SPACE STRUCTURE OF THE DARK HALO IS PRETTY COMPLICATED

Via Lactea II projected dark matter (squared-) density map



BUT REAL GALAXIES APPEAR SIMPLER THAN EXPECTED!

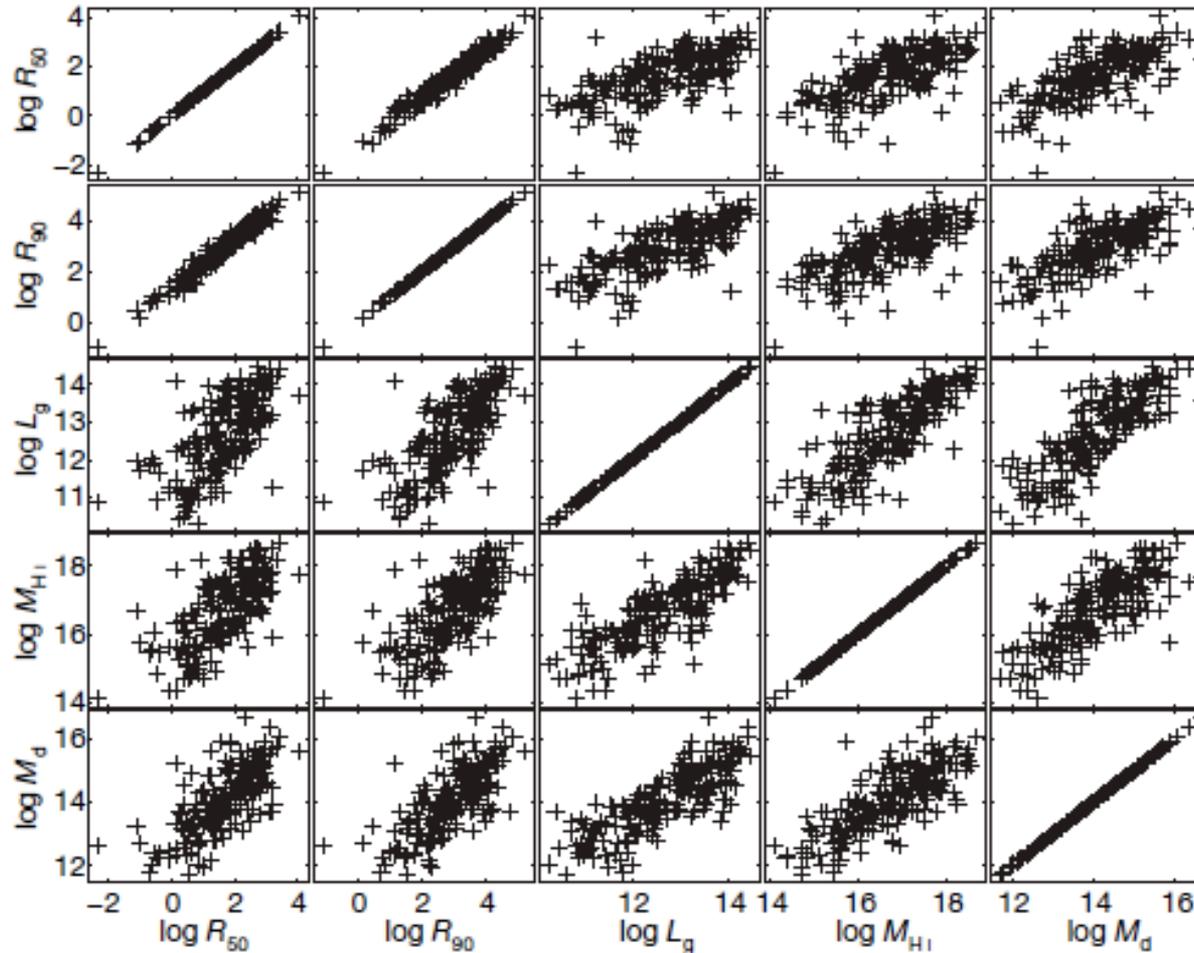
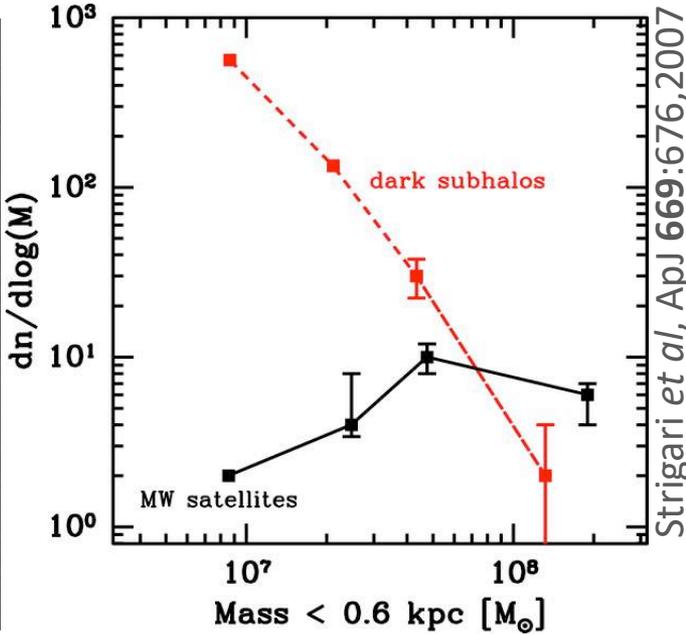
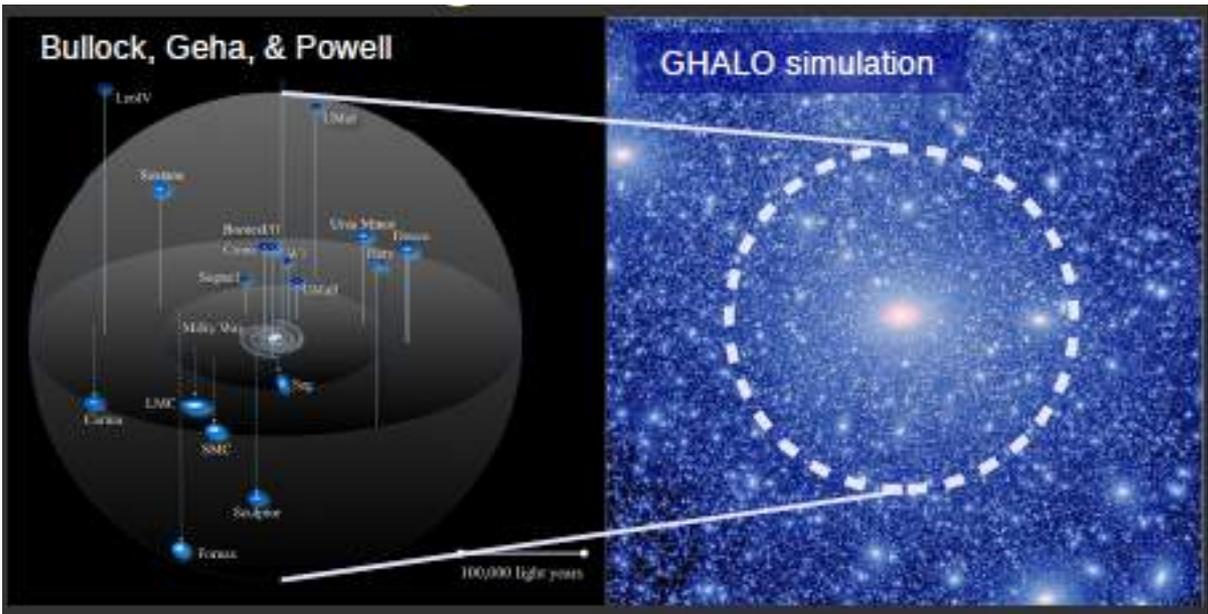


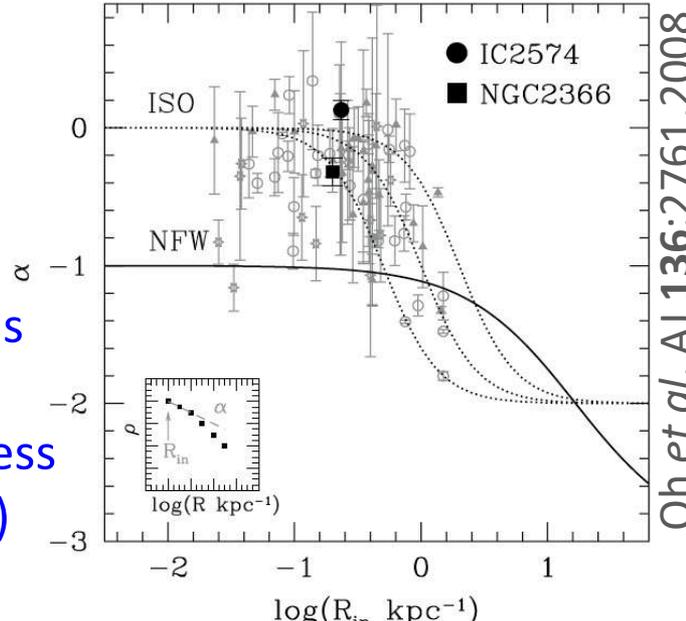
Figure 1 | Scatter plots showing correlations between five measured variables, not including colour. The variables are two optical radii, R_{50} and R_{90} (in parsecs), respectively containing 50 and 90% of the emitted light; and luminosity, L_g ; neutral hydrogen mass, M_{HI} ; and dynamical mass, M_d (inferred from the 21-cm linewidth, the radius and the inclination in the

MOREOVER WHEREAS THE MILKY WAY DOES HAVE SATELLITE GALAXIES AND SUBSTRUCTURE THERE IS A LOT LESS THAN IS EXPECTED FROM THE NUMERICAL SIMULATIONS



Also, the halo density profile for collisionless dark matter is predicted to be 'cuspy', whereas observations suggest 'cored' isothermal profiles

This *could* be because of the 'feedback effect' of baryons – computer simulations are just beginning to test this – or it could even be because dark matter is *not* collisionless but *self-interacting* (or perhaps 'warm' rather than cold)

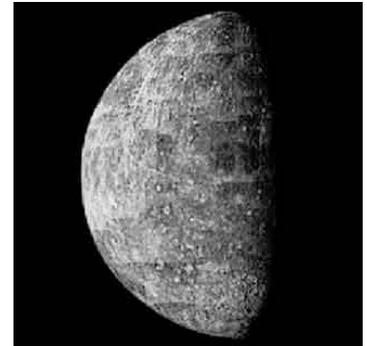


INFERENCES OF DARK MATTER ARE NOT ALWAYS RIGHT ... IT MAY INSTEAD BE A CHANGE IN THE DYNAMICS



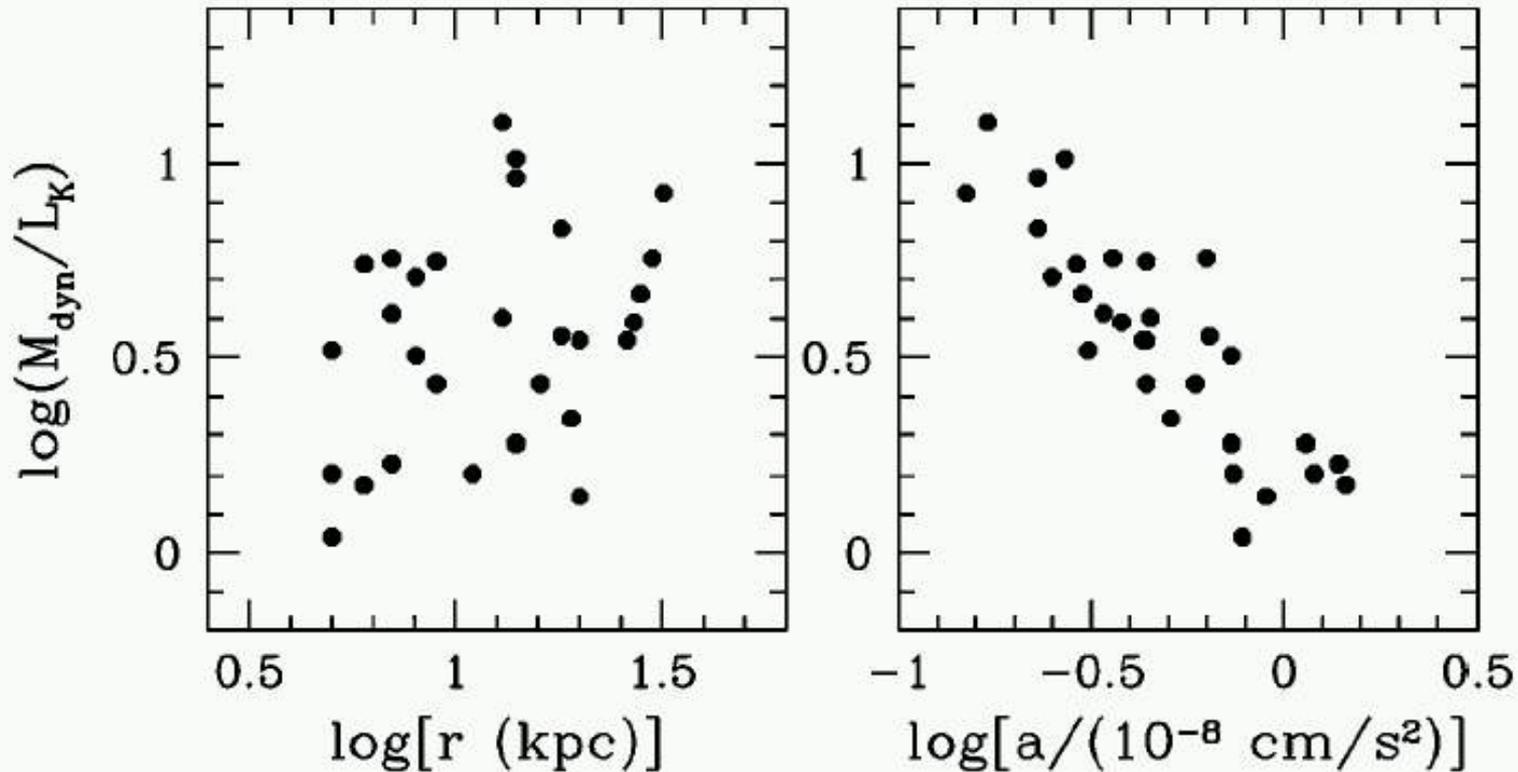
2nd January 1860: “Gentlemen, I Give You the Planet Vulcan” French mathematician Urbain Le Verrier announces the discovery of a new planet between Mercury and the Sun, to members of the Académie des Sciences in Paris (following up on his earlier prediction of Neptune in 1856).

Some astronomers even see Vulcan in the evening sky!



But the precession of Mercury is not due to a dark planet ... **but because Newton is superseded by Einstein**

DARK MATTER APPEARS TO BE REQUIRED ONLY WHERE THE TEST PARTICLE ACCELERATION IS LOW ($< a_0 \sim 10^{-8} \text{ cm/s}^2$) - IT IS *NOT* A SCALE-DEPENDENT EFFECT



What if Newton's law is modified in weak fields?

$$F_N \rightarrow \sqrt{\frac{GM}{r^2}} a_0$$

Milgrom, ApJ **270**:365,1983

BEKENSTEIN—MILGROM EQUATION

Suppose $\mathbf{F} = -\nabla\phi$ where

$$\nabla^2\phi_{\text{N}} = 4\pi G\rho \quad \rightarrow \quad \nabla \cdot [\mu(|\nabla\phi|/a_0)\nabla\phi] = 4\pi G\rho$$

where

$$\mu(x) \rightarrow \begin{cases} 1 & \text{for } x \gg 1 \\ x & \text{for } x \ll 1 \end{cases}$$

Then

$$0 = \nabla \cdot [\mu(|\nabla\phi|/a_0)\nabla\phi - \nabla\phi_{\text{N}}]$$

implies

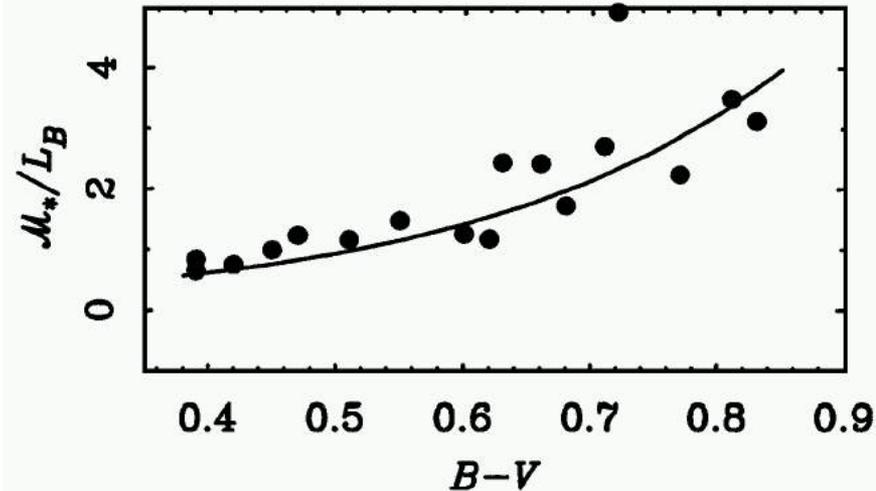
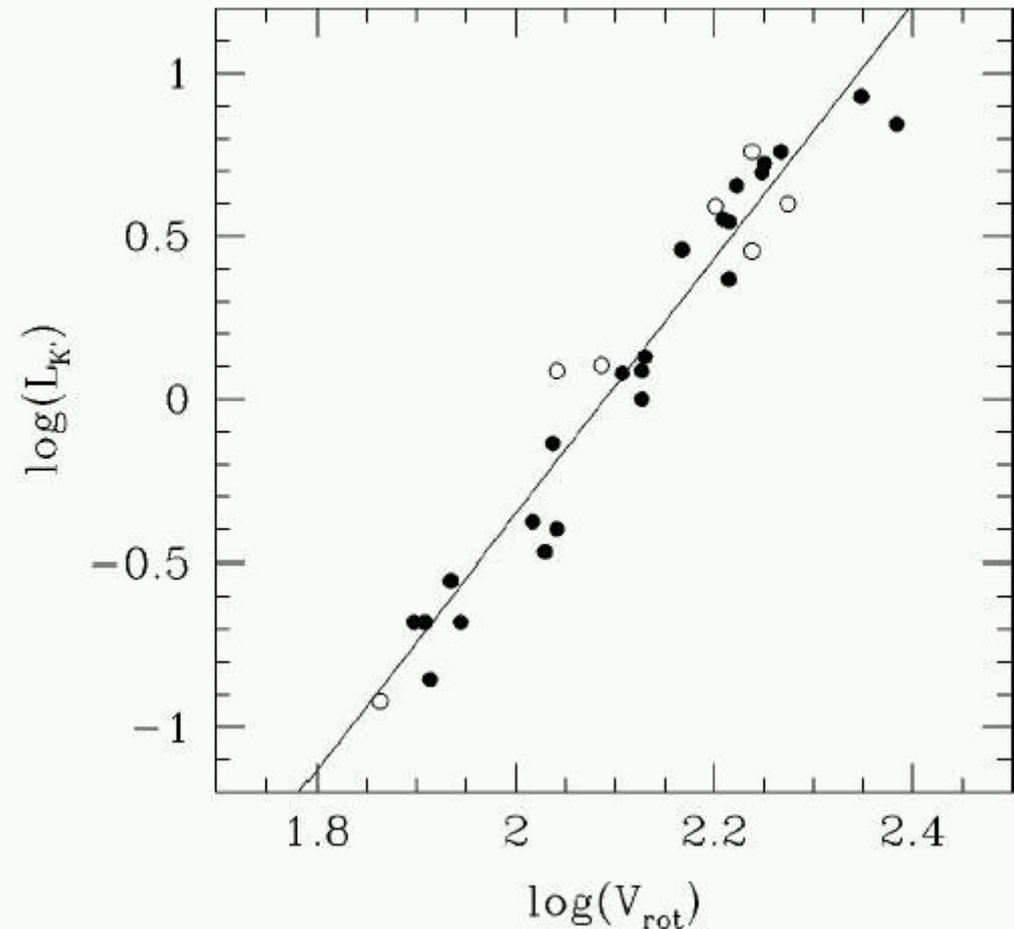
$$\mu(|\nabla\phi|/a_0)\nabla\phi = \nabla\phi_{\text{N}} + \nabla \times \mathbf{A}$$

so when $\mathbf{A} \simeq 0$ and $|\nabla\phi| \ll 1$

$$g_{r \rightarrow \infty} \rightarrow -\sqrt{M G a_0} \frac{\vec{r}}{r^2} + \mathcal{O}\left(\frac{1}{r^2}\right), \quad \frac{|\nabla\phi|^2}{a_0} = |\nabla\phi_{\text{N}}|$$

THE MOND HYPOTHESIS *DIRECTLY* IMPLIES:

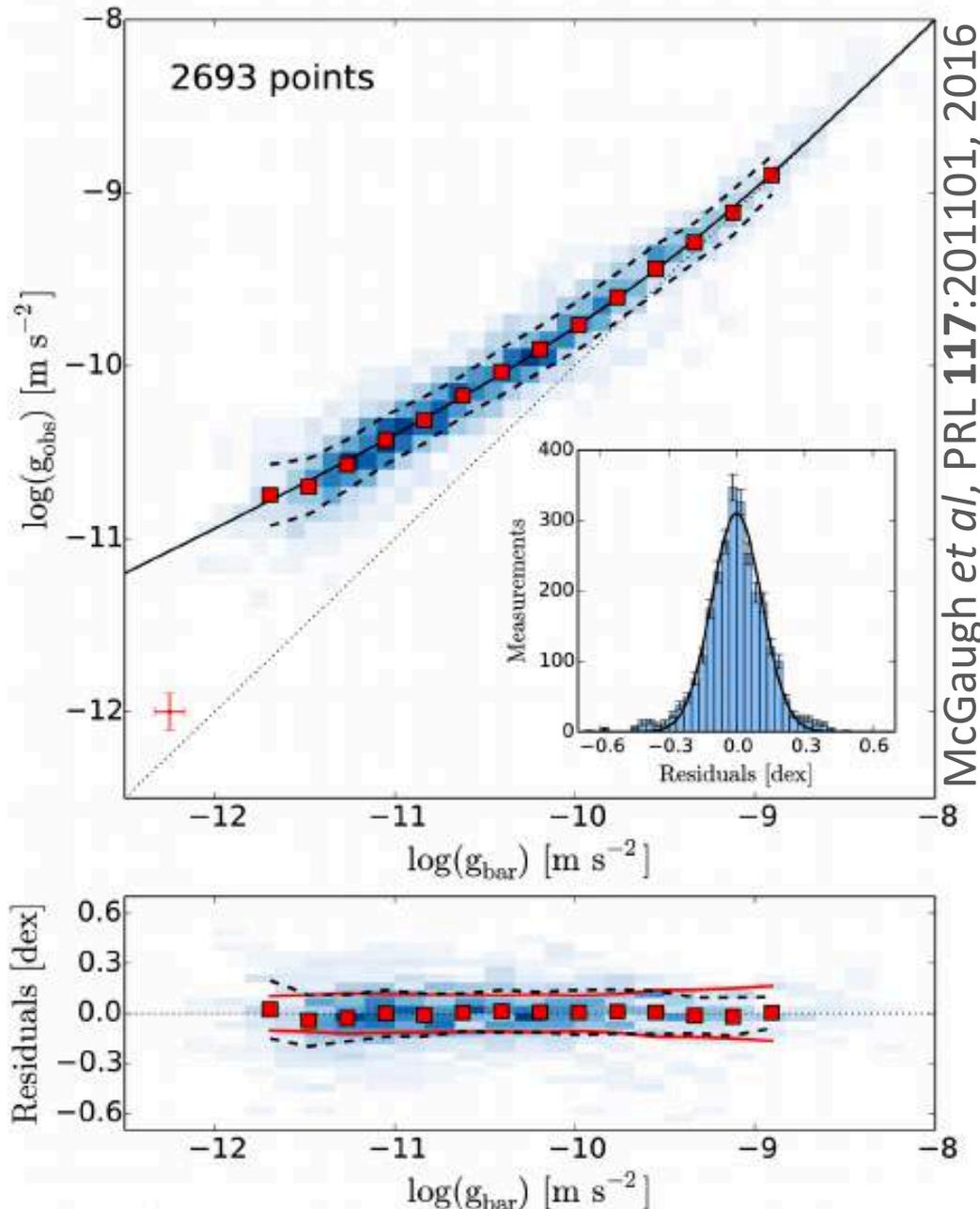
$$\frac{v^4}{r^2} = \frac{GM}{r^2} a_0 \quad \Rightarrow \quad M \propto v^4 \quad (\text{Tully-Fisher if } \frac{M}{L} = \text{const})$$



... the fitted value of the only free parameter (M/L) agrees very well with population synthesis models
Sanders & Verheijen, ApJ **503**:97,1998

This is an impressive correlation for which dark matter has no simple explanation

RECENTLY THIS HAS GAINED PROMINENCE AS THE 'MDAR RELATIONSHIP'



153 disk galaxies (SPARC set)
measured at multiple radii

$$g_{\text{obs}} = \mathcal{F}(g_{\text{bar}})$$

$$= \frac{g_{\text{bar}}}{1 - e^{-\sqrt{g_{\text{bar}}/g_{\dagger}}}}$$

$$g_{\dagger} = (1.2 \pm 0.24) \times 10^{-10} \text{m s}^{-2}$$

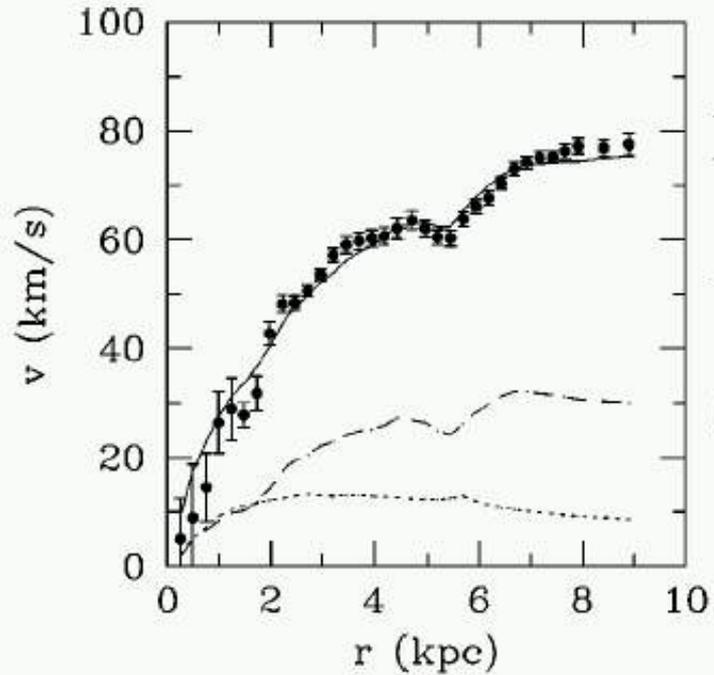
The functional form is *not* unique, e.g.

$$\mathcal{F}(y) = [1 + \sqrt{(1 + 4/y)}] / 2$$

which follows from $\mu(x) = x/(1+x)$, fits just as well (Milgrom, 1609.06642)

Can CDM simulations *including* baryon physics predict this curve?

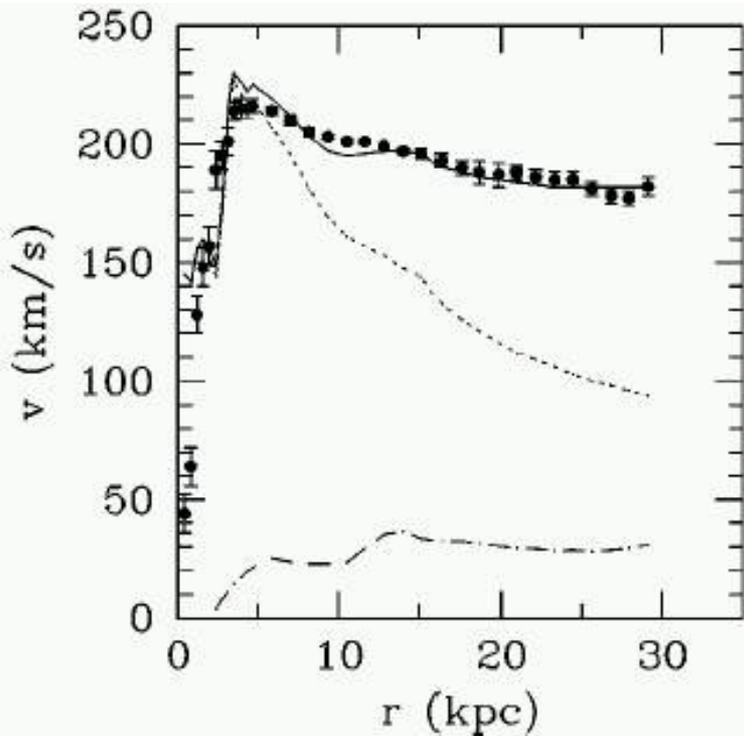
MOND fits galactic rotation curves with
 $a_0 = 1.2 \times 10^{-8} \text{ cm s}^{-2}$



NGC 1560

$\langle \mu_B \rangle = 23.2 \text{ mag/a}$

$(M/L_B)_{\text{disk}} = 0.4$



NGC 2903

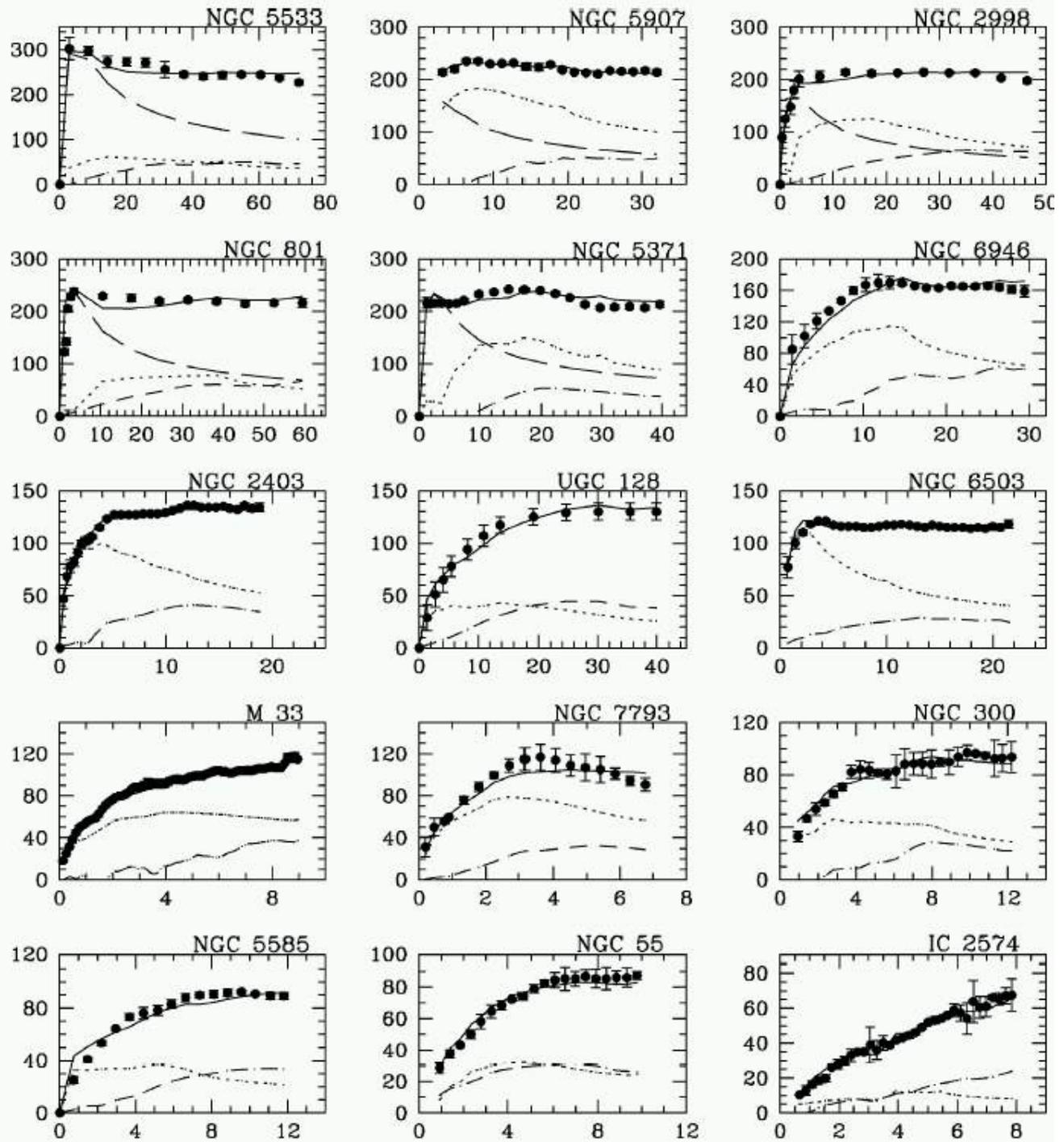
$\langle \mu_B \rangle = 20.5 \text{ mag/a}$

$(M/L_B)_{\text{disk}} = 1.9$

Features in the
baryonic disc have
counterparts in the
rotation curve

A huge variety of rotation curves is well fitted by MOND

... with fewer parameters than is required by the dark matter model



MOREOVER SOME GIANT ELLIPTICAL GALAXIES DO EXHIBIT KEPLERIAN FALL-OFF OF THE RANDOM VELOCITY DISPERSION, AS MOND PREDICTS

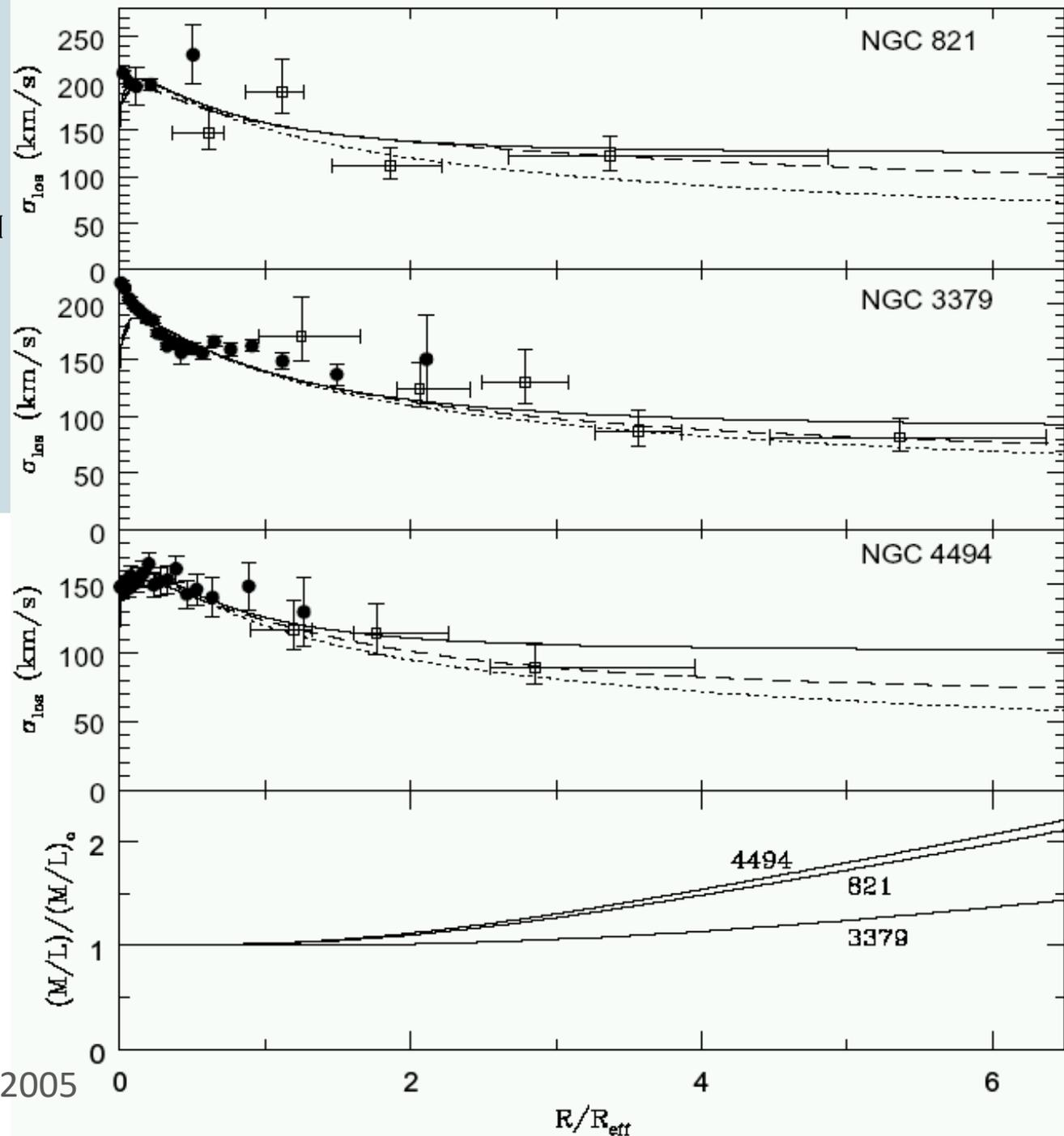
Romanowsky *et al*,
Science **301**:1696,2003

Models:

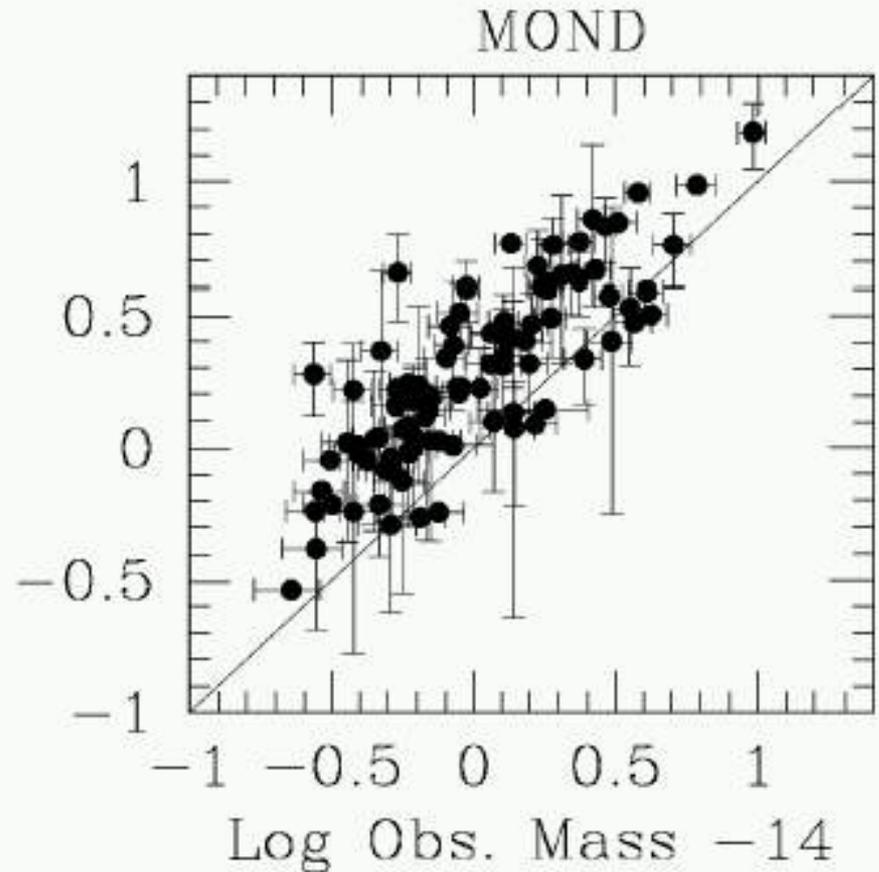
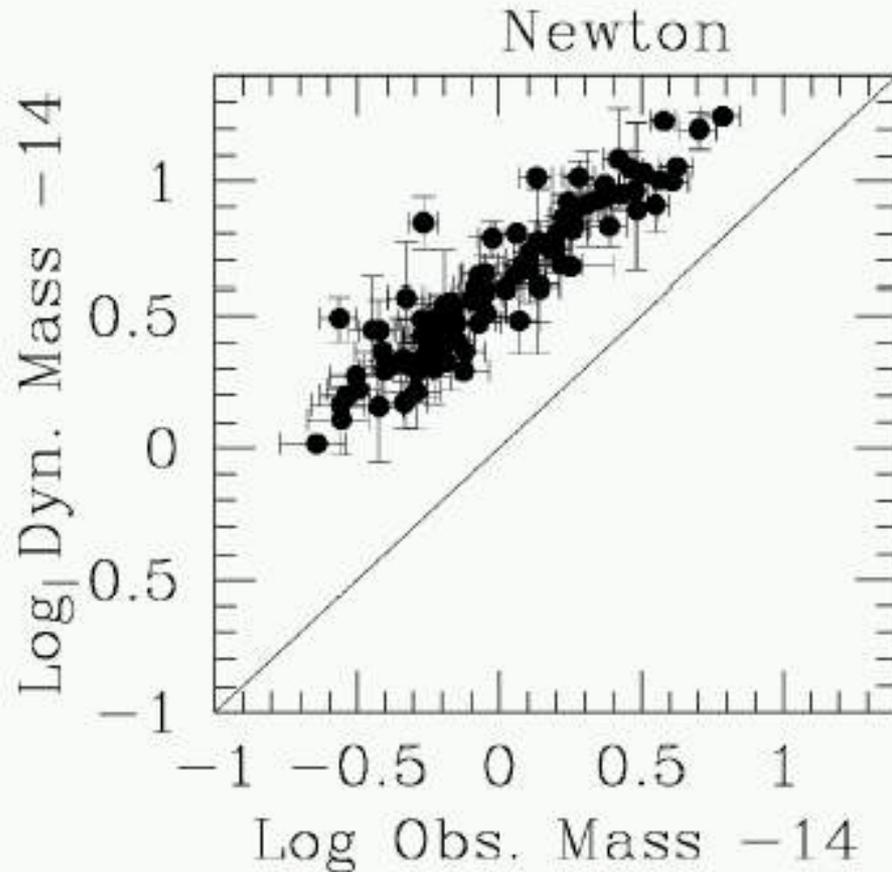
Milgrom & Sanders
ApJ **599**:L25,2003

This can be explained in a dark matter model only if stellar orbits are *very* elliptical

Dekel *et al*, Nature **437**:707,2005



HOWEVER MOND *FAILS* ON THE SCALE OF CLUSTERS OF GALAXIES



The “missing mass” cannot be accounted for entirely by invoking MOND ... **dark matter is required** (thus vindicating the original proposal of Zwicky)



Credit: Palomar Observatory/Caltech

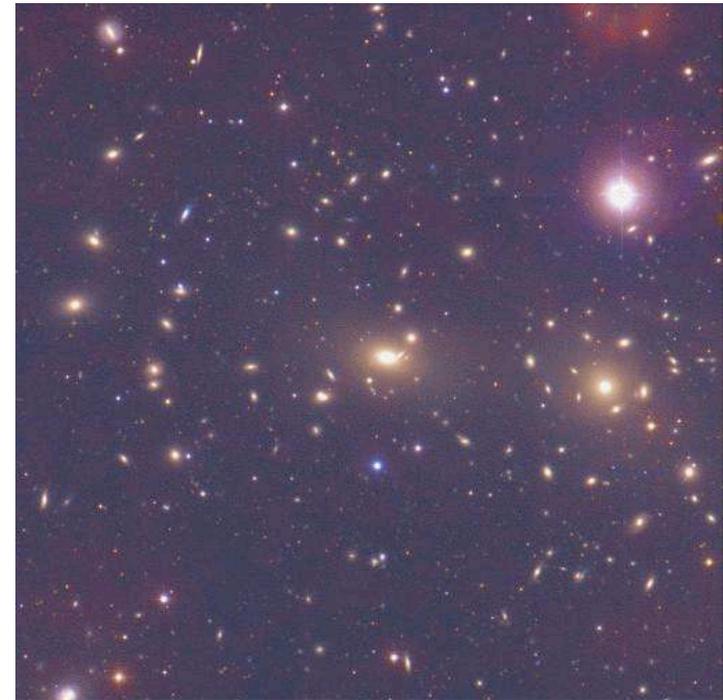
Fritz Zwicky (1933) measured the velocity dispersion in the Coma cluster to be as high as 1000 km/s $\Rightarrow M/L \sim O(100) M_{\odot}/L_{\odot}$

“... If this overdensity is confirmed we would arrive at the astonishing conclusion that **dark matter** is present (in Coma) with a much greater density than luminous matter”.

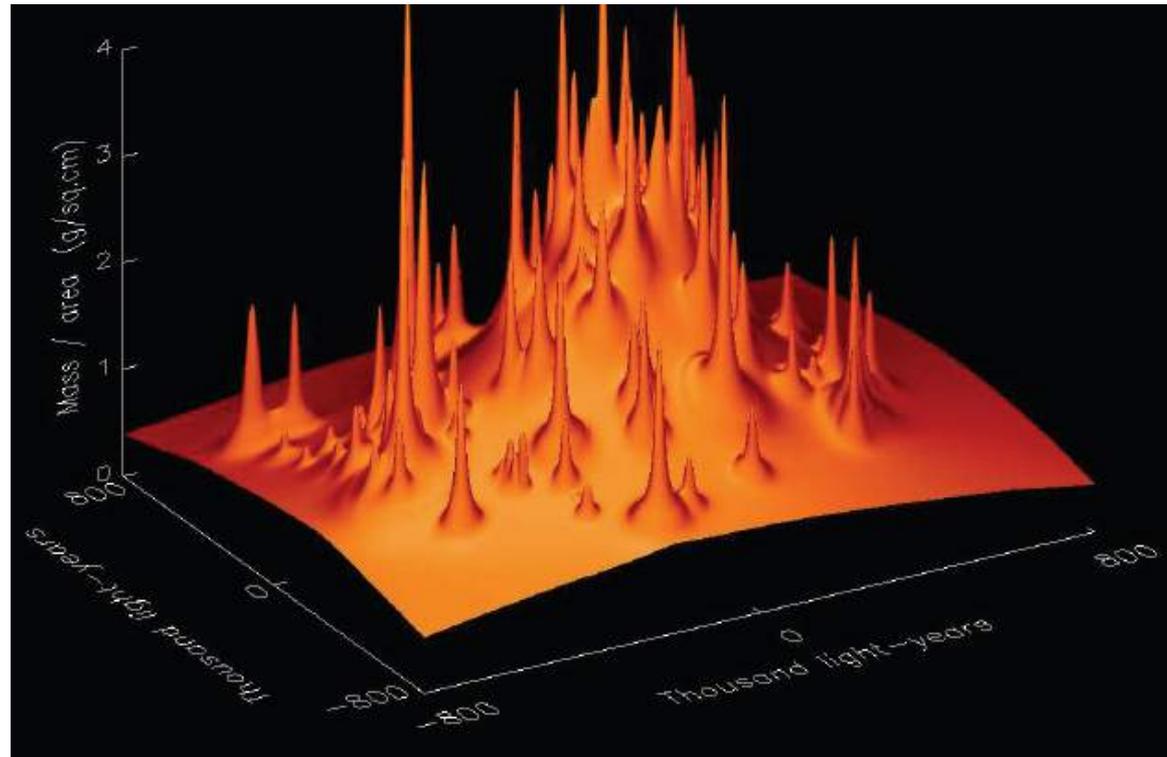
Virial Theorem: $\langle V \rangle + 2\langle K \rangle = 0$

$$V = -\frac{N^2}{2} G_N \frac{\langle m^2 \rangle}{\langle r \rangle}, \quad K = N \frac{\langle m v^2 \rangle}{2}$$

$$M = N \langle m \rangle \sim \frac{2 \langle r \rangle \langle v^2 \rangle}{G_N} \gg \sum m_{\text{galaxies}}$$

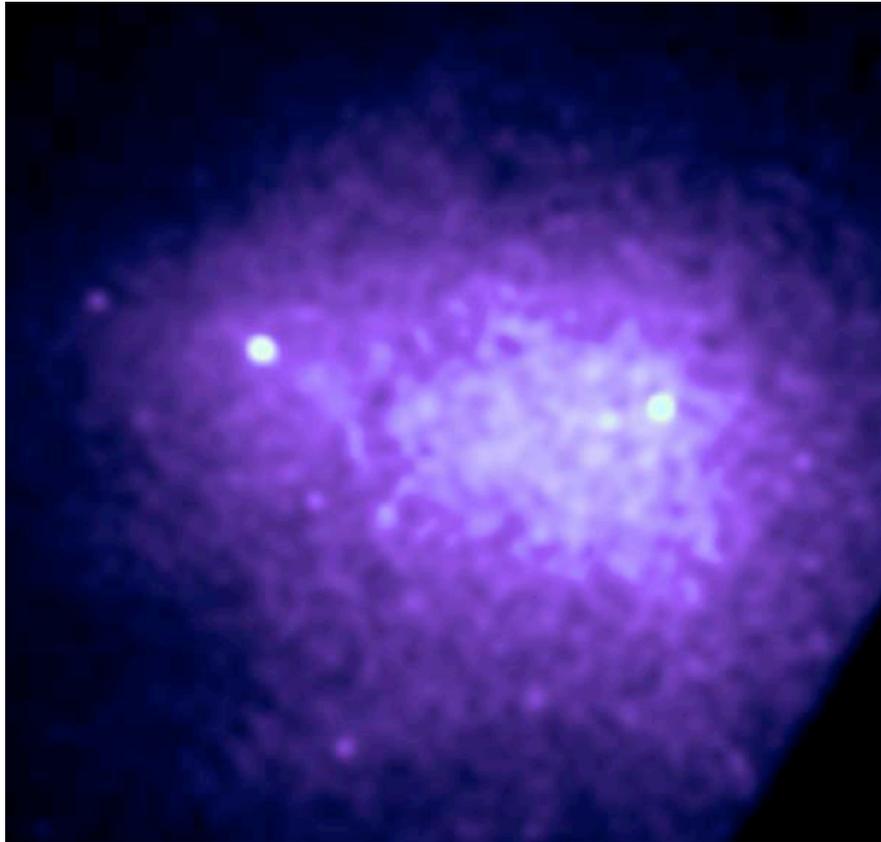


FURTHER EVIDENCE COMES FROM OBSERVATIONS OF GRAVITATIONAL
LENSING OF DISTANT SOURCES BY A FOREGROUND CLUSTER ...
ENABLING THE POTENTIAL TO BE RECONSTRUCTED



This reveals that the gravitational mass is dominated by an extended smooth distribution of dark matter

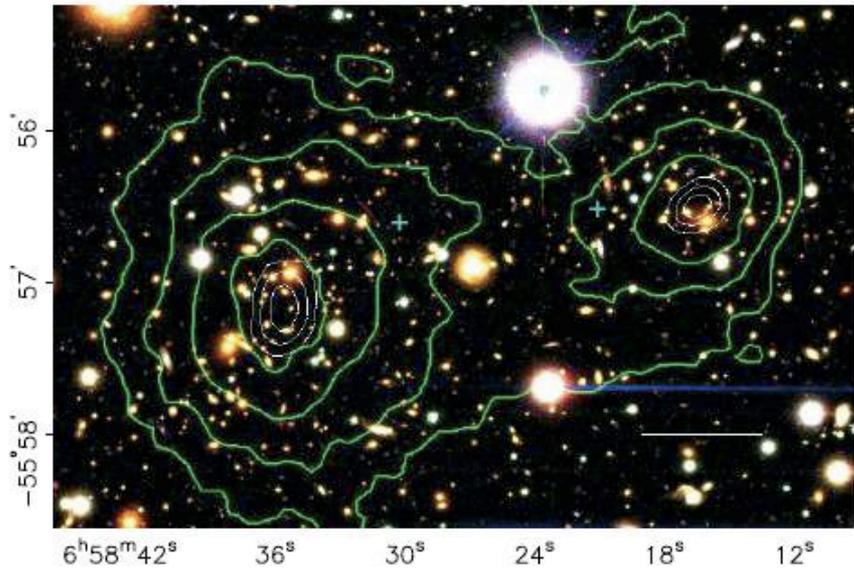
THE GRAVITATING MASS CAN ALSO BE OBTAINED FROM X-RAY OBSERVATIONS OF THE HOT GAS IN THE CLUSTER



... assuming it is in thermal equilibrium:

$$\frac{1}{\rho_{\text{gas}}} \frac{dP_{\text{gas}}}{dr} = \frac{G_N M(< r)}{r^2}$$

THE CHANDRA PICTURE OF THE *BULLET CLUSTER* SHOWS THAT THE X-RAY EMITTING BARYONIC MATTER IS DISPLACED FROM THE GALAXIES AND THE DARK MATTER (INFERRED THROUGH GRAVITATIONAL LENSING) ... FOR MANY THIS IS CONVINCING EVIDENCE OF DARK MATTER



Clowe et al, ApJ 648:L109,2006

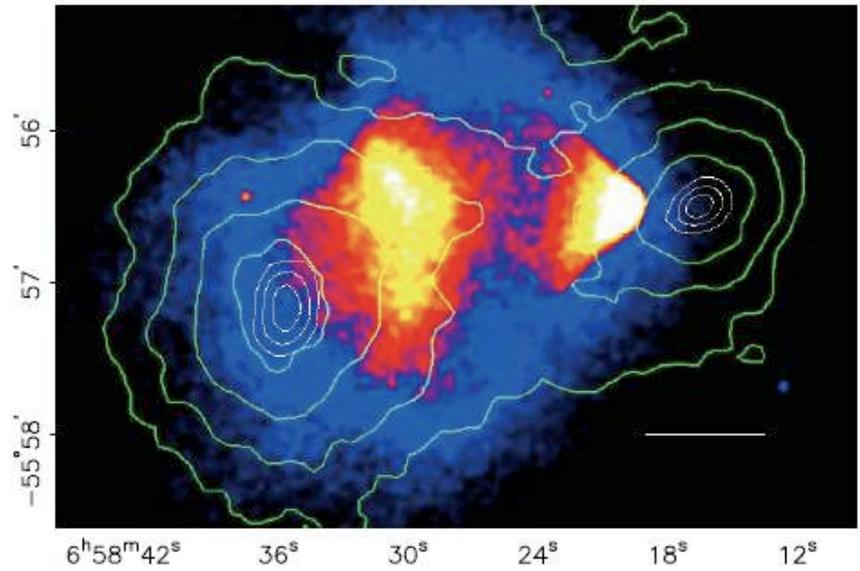


FIG. 1.—*Left panel*: Color image from the Magellan images of the merging cluster 1E 0657–558, with the white bar indicating 200 kpc at the distance of the cluster. *Right panel*: 500 ks *Chandra* image of the cluster. Shown in green contours in both panels are the weak-lensing κ reconstructions, with the outer contour levels at $\kappa = 0.16$ and increasing in steps of 0.07. The white contours show the errors on the positions of the κ peaks and correspond to 68.3%, 95.5%, and 99.7% confidence levels. The blue plus signs show the locations of the centers used to measure the masses of the plasma clouds in Table 2.

However this is nothing new ... it has been noted already that MOND fails (by a factor of ~ 2) to explain the ‘missing matter’ in galaxy clusters

Moreover the rather high relative velocity of the merging clusters is a puzzle in the LCDM cosmology as well – only ~ 0.1 such systems are expected (Kraljic & Sarkar, JCAP **04**:050,2015)
Problem even more pronounced for *El Gordo* (Asencio, Banik & Kroupa, MNRAS **500**:5249,2021)

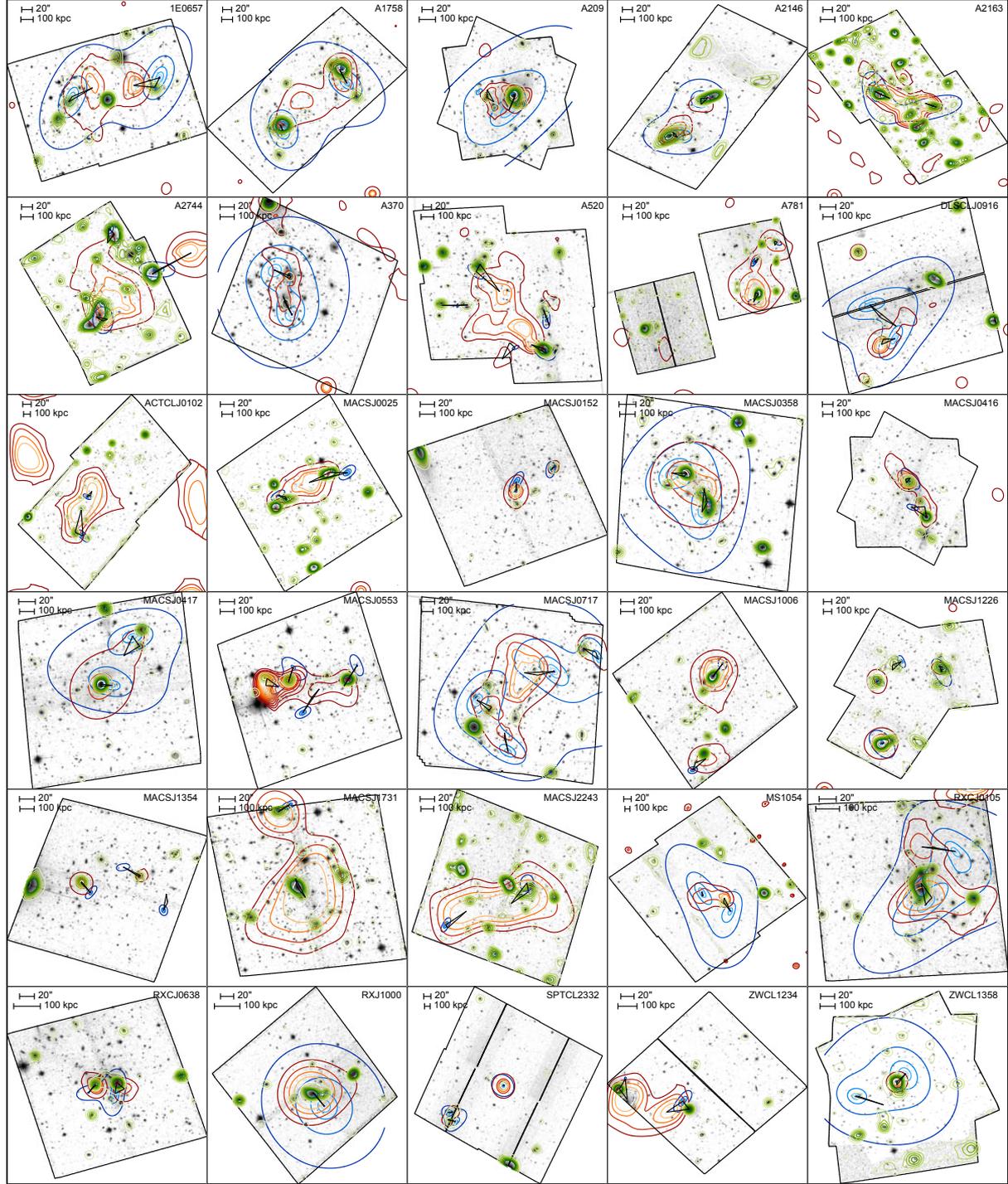
INFALLING SUBHALOS

There have been several studies on constraining DM self-interactions via the observation of DM sub-halos falling into galaxy clusters

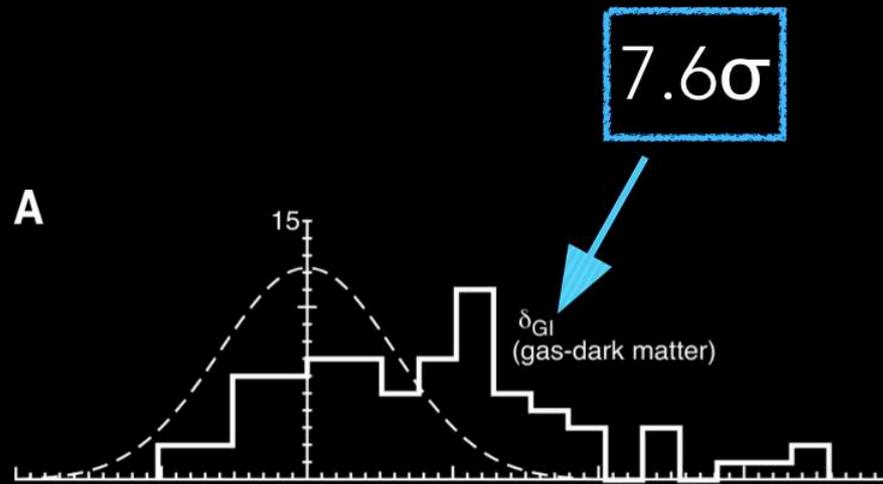
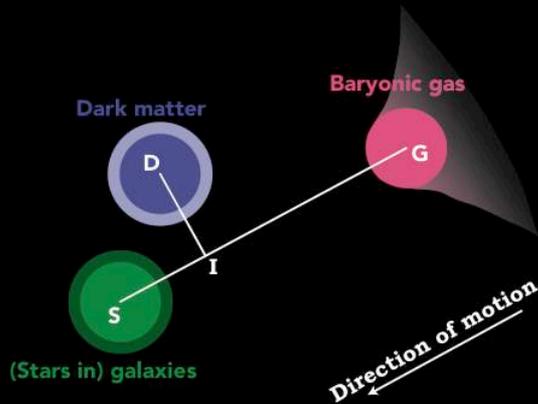
Through statistical analysis of a large number of gravitationally lensed clusters in the Chandra catalogue, the DM self-interaction is bounded as:

$$\sigma/m_\chi < 0.5 \text{ cm}^2/\text{g}$$

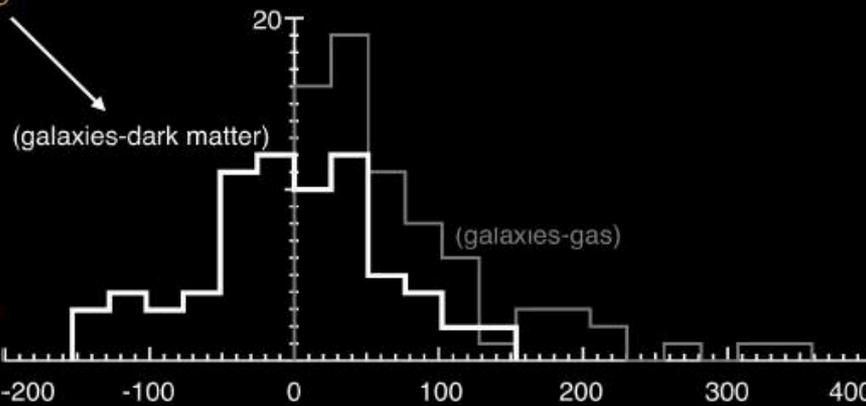
Massey *et al*, 1007.1924;
Harvey *et al*, 1305.2117,
1310.1731, 1503.07675



RESULTS FROM 72 MERGING SYSTEMS



$5.8 \pm 8.2 \text{ kpc}$



$25 \pm 29 \text{ kpc}$
(Bullet Cluster)

Observed offset between various components of substructure [kpc]

-500 -100 0 100 500 300 100

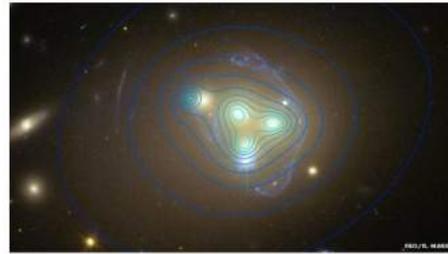
BUT IN A3827 AN OFFSET WAS OBSERVED BETWEEN A GALAXY AND ITS DM HALO!



Gravity from this cluster of four galaxies produces multiple distorted images (blue streams) of a more distant galaxy.
Subtly shifted star could force rethink of dark matter

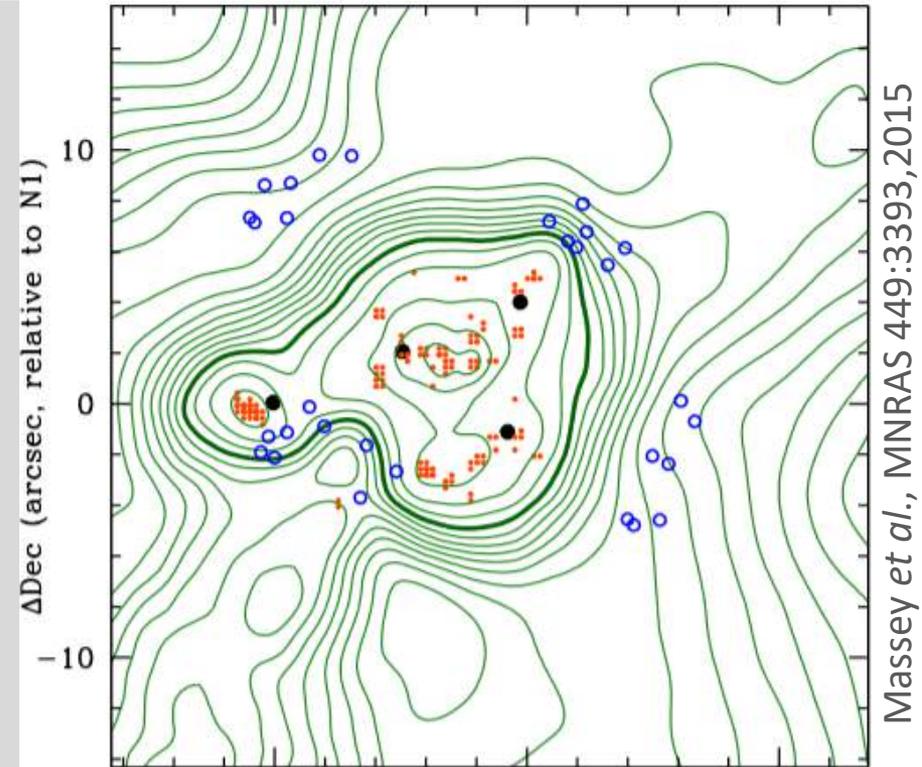
Dark matter becomes less 'ghostly'

By Paul Fincin
Science editor, BBC News website
15 April 2015 | Science & Environment



The behaviour of dark matter associated with 4 bright cluster galaxies in the 10 kpc core of Abell 3827

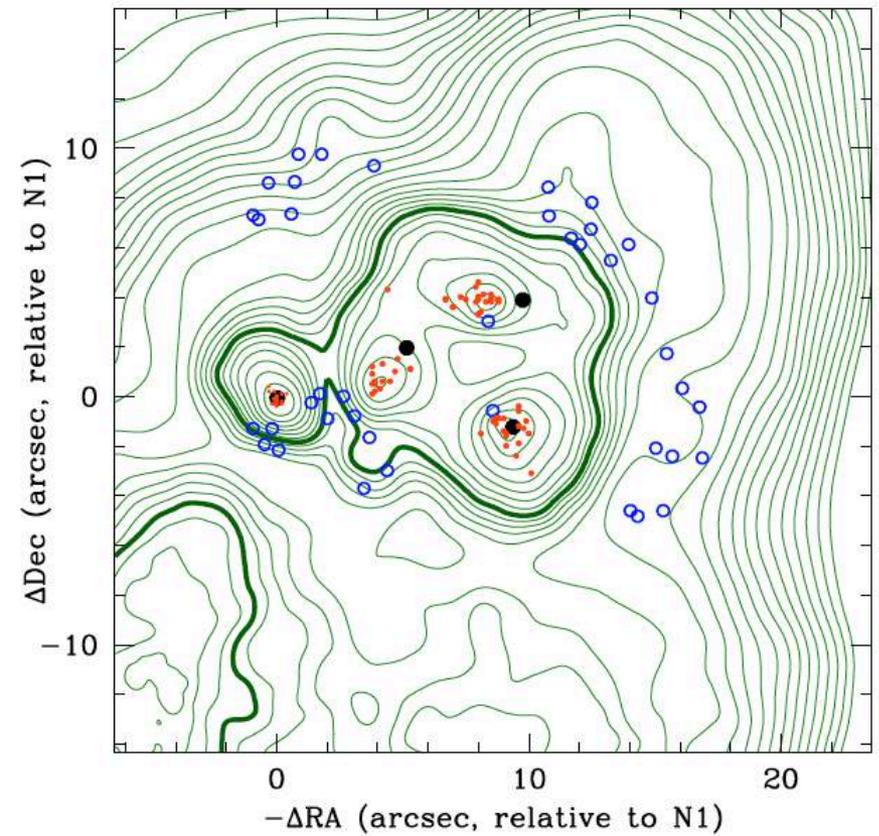
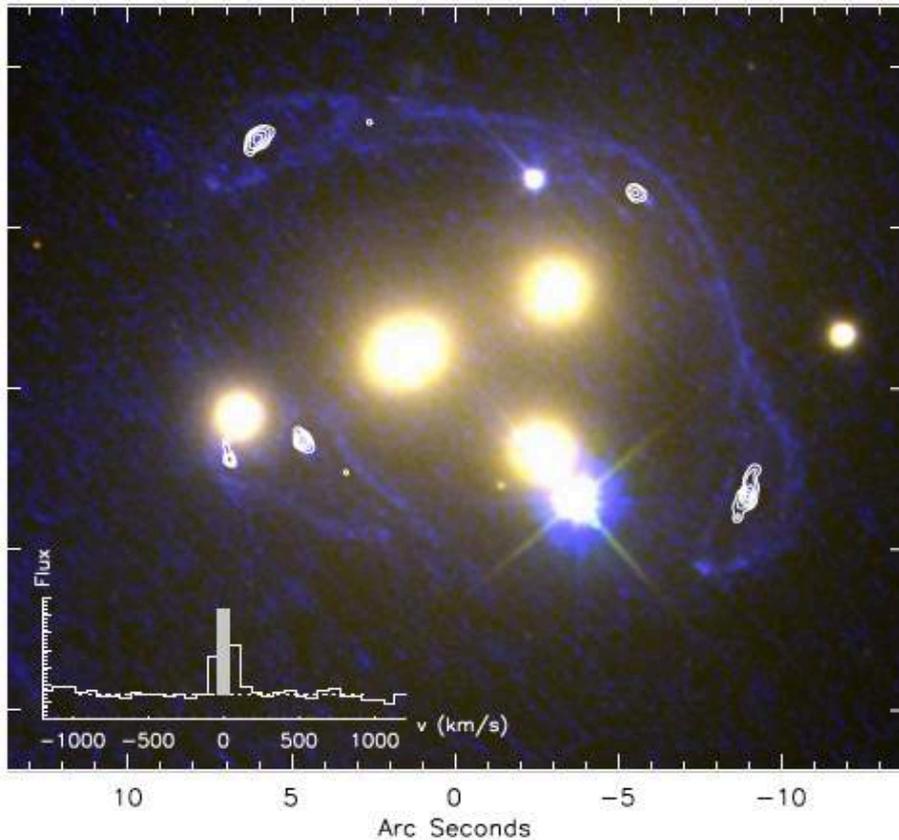
“The best-constrained offset is 1.62 ± 0.48 kpc, where the 68% confidence limit includes both statistical error and systematic biases in mass modelling. [...] With such a small physical separation, it is difficult to definitively rule out astrophysical effects operating exclusively in dense cluster core environments – but if interpreted solely as evidence for self-interacting dark matter, this offset implies a cross-section $\sigma/m = (1.7 \pm 0.7) \times 10^{-4} \text{ cm}^2/\text{g} (t/10^9 \text{ yr})^{-2}$ where t is the infall duration.”



... corrected to $\sigma/m \sim 1.5 \text{ cm}^2/\text{g}$, accounting for dynamics (Kahlhoefer et al, MNRAS 452:L54,2015)

BUT WITH NEW DATA FROM ALMA THE OFFSET IN ABELL 3827 HAS DISAPPEARED!

Now that the counterparts of the many multiply-imaged star-forming knots are better identified, the lensing reconstruction is more secure and uncertainties are reduced

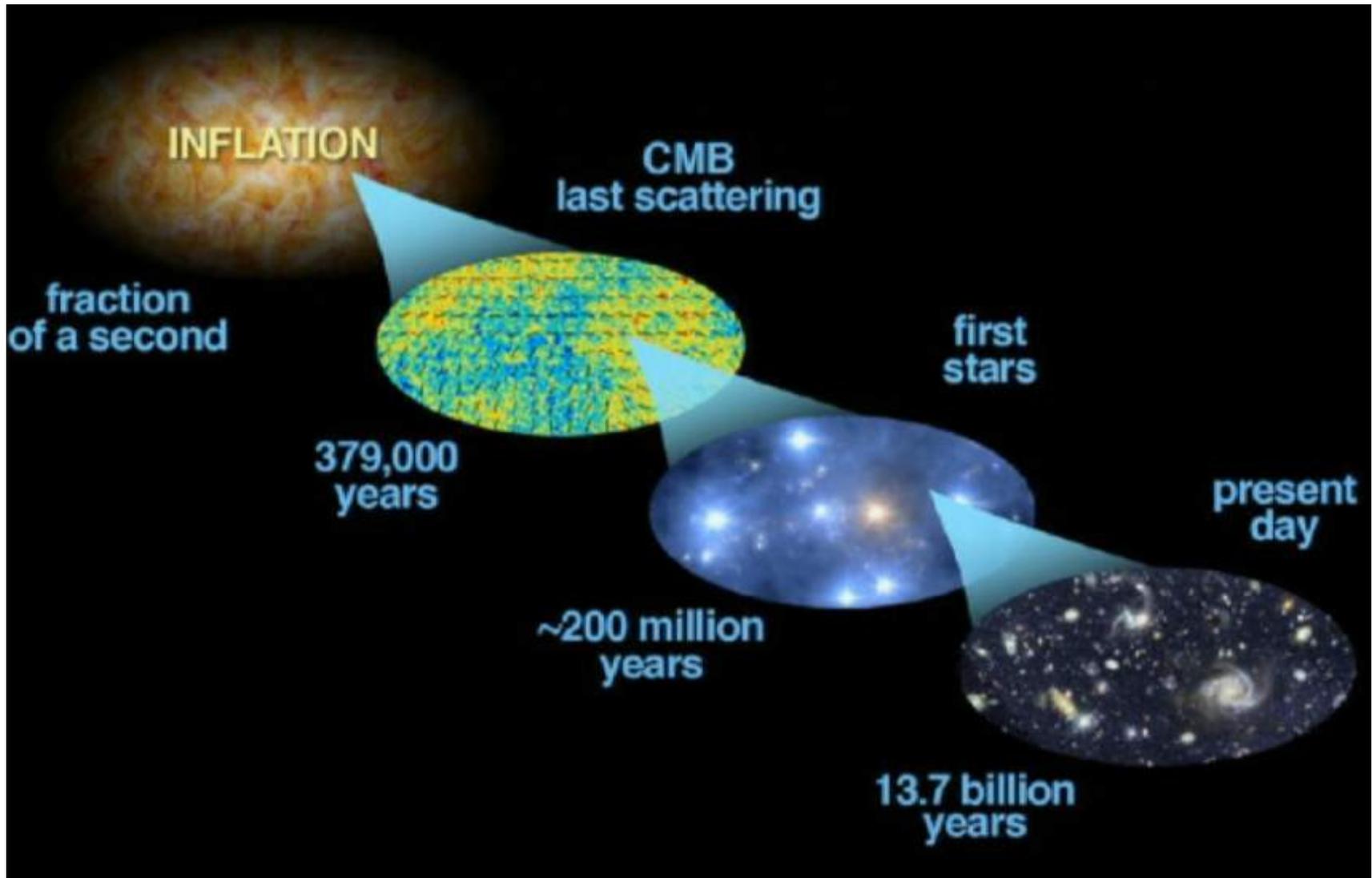


Massey et al., MNRAS 477:669,2017

Nevertheless arguments continue to be made in support of dark matter having self-interactions (which may be *velocity-dependent*) e.g. from observations of cores in dwarf spheroidal/low surface brightness galaxies, cluster mergers (e.g. A520), etc

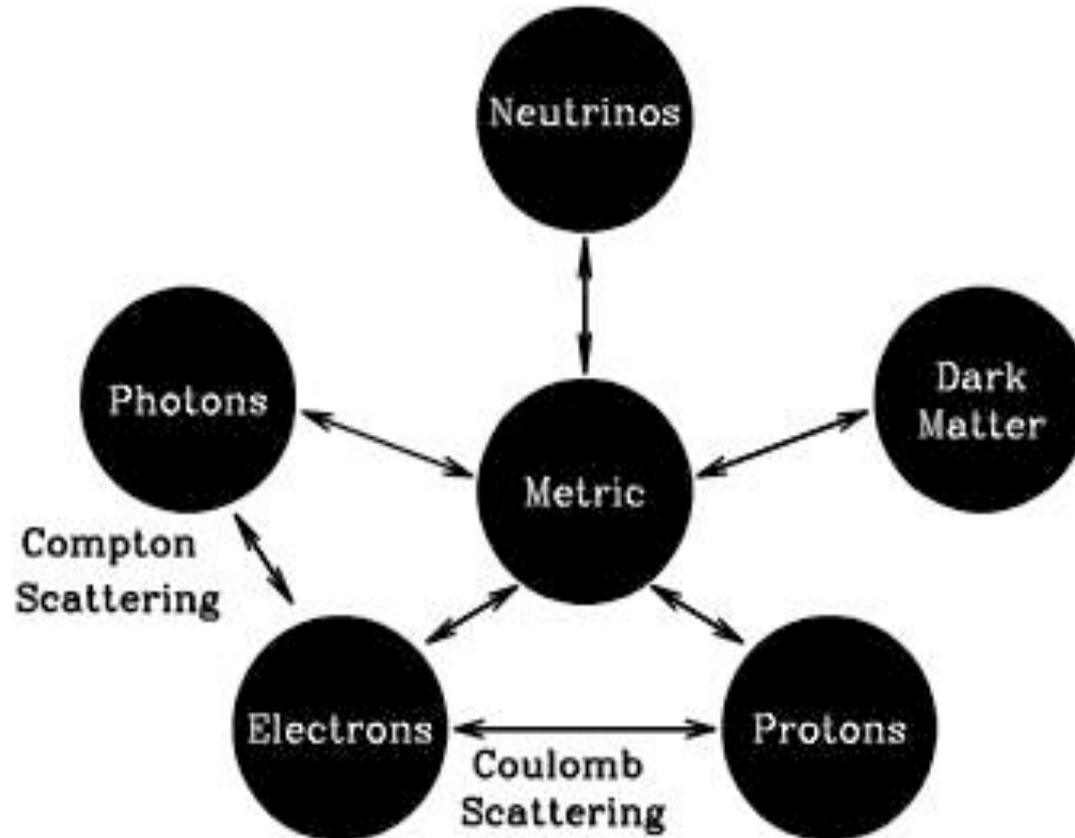
Can the predicted offset between the DM & galaxies in merging clusters be detected?

THE COMPELLING ARGUMENT FOR DM COMES FROM CONSIDERATIONS OF STRUCTURE FORMATION



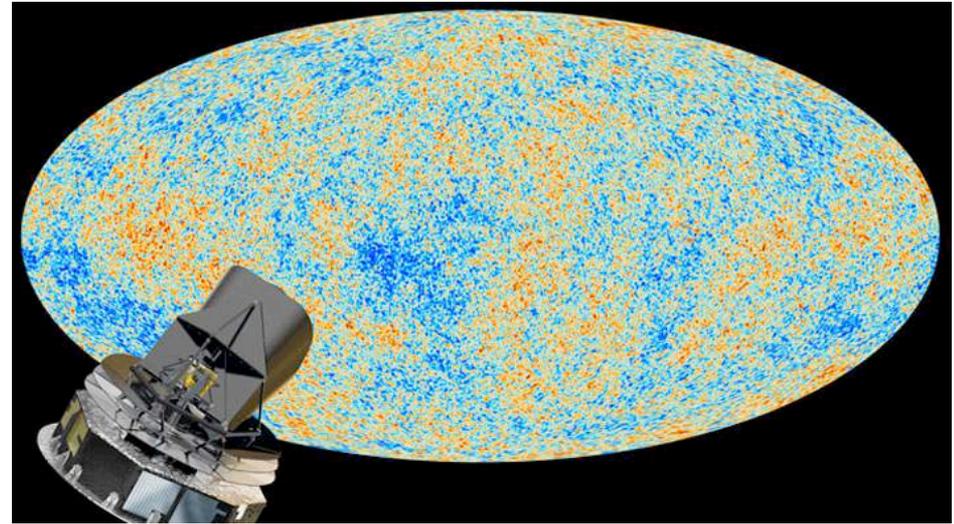
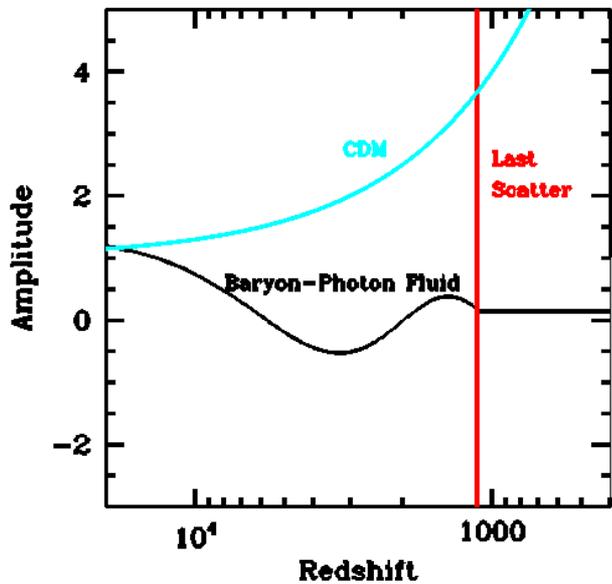
These temperature fluctuations are understood as due to **scalar density perturbations** with an \sim scale-invariant spectrum which were generated during an early phase of inflationary expansion ... these perturbations have subsequently grown into the **large-scale structure** of galaxies observed today through **gravitational instability** in a sea of **dark matter**

PERTURBATIONS IN METRIC (GENERATED DURING INFLATION) INDUCE PERTURBATIONS IN PHOTONS AND (DARK) MATTER



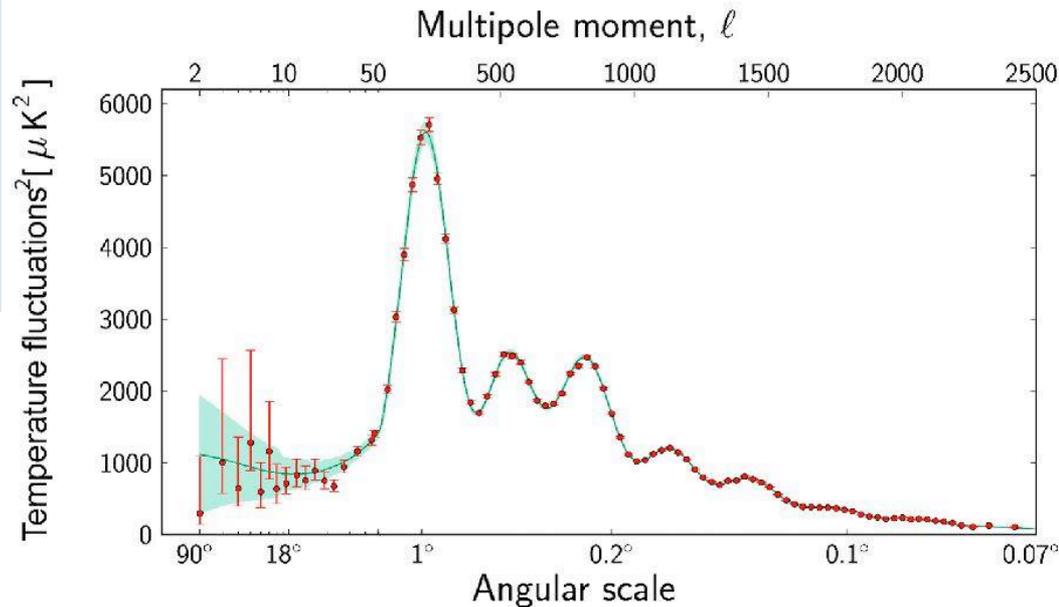
These perturbations begin to grow through gravitational instability after matter domination

BEFORE RECOMBINATION, THE PRIMORDIAL FLUCTUATIONS EXCITE SOUND WAVES IN THE BARYONIC PLASMA, BUT CAN BEGIN TO GROW IN THE NON-INTERACTING DARK MATTER ...

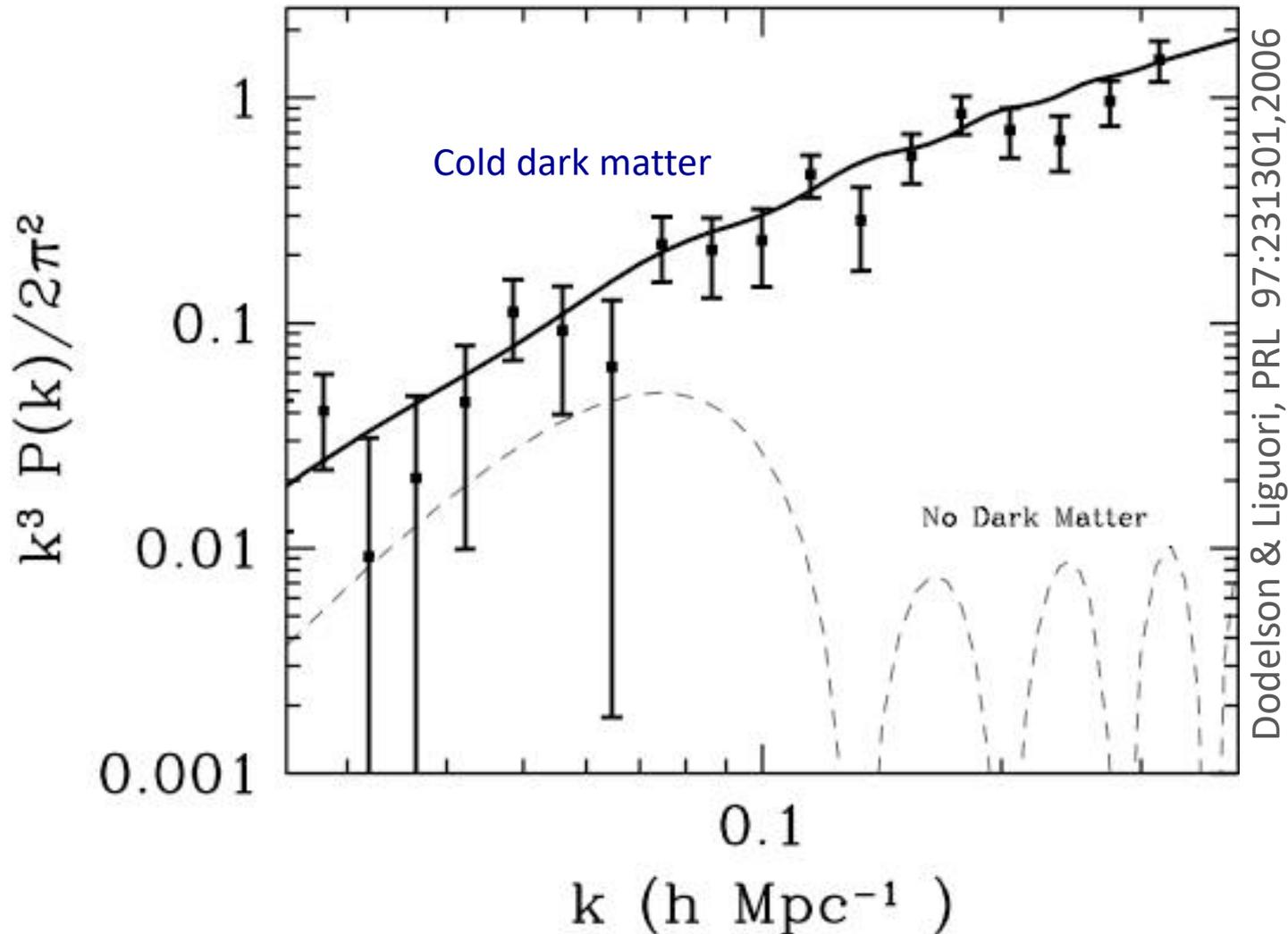


These sound waves leave an imprint on the last scattering surface as the universe turns neutral and transparent ... sensitive to the baryon/CDM densities

The angular power spectrum of the fluctuations can be well described only if dark matter dominates over baryonic matter ('Silk damping')

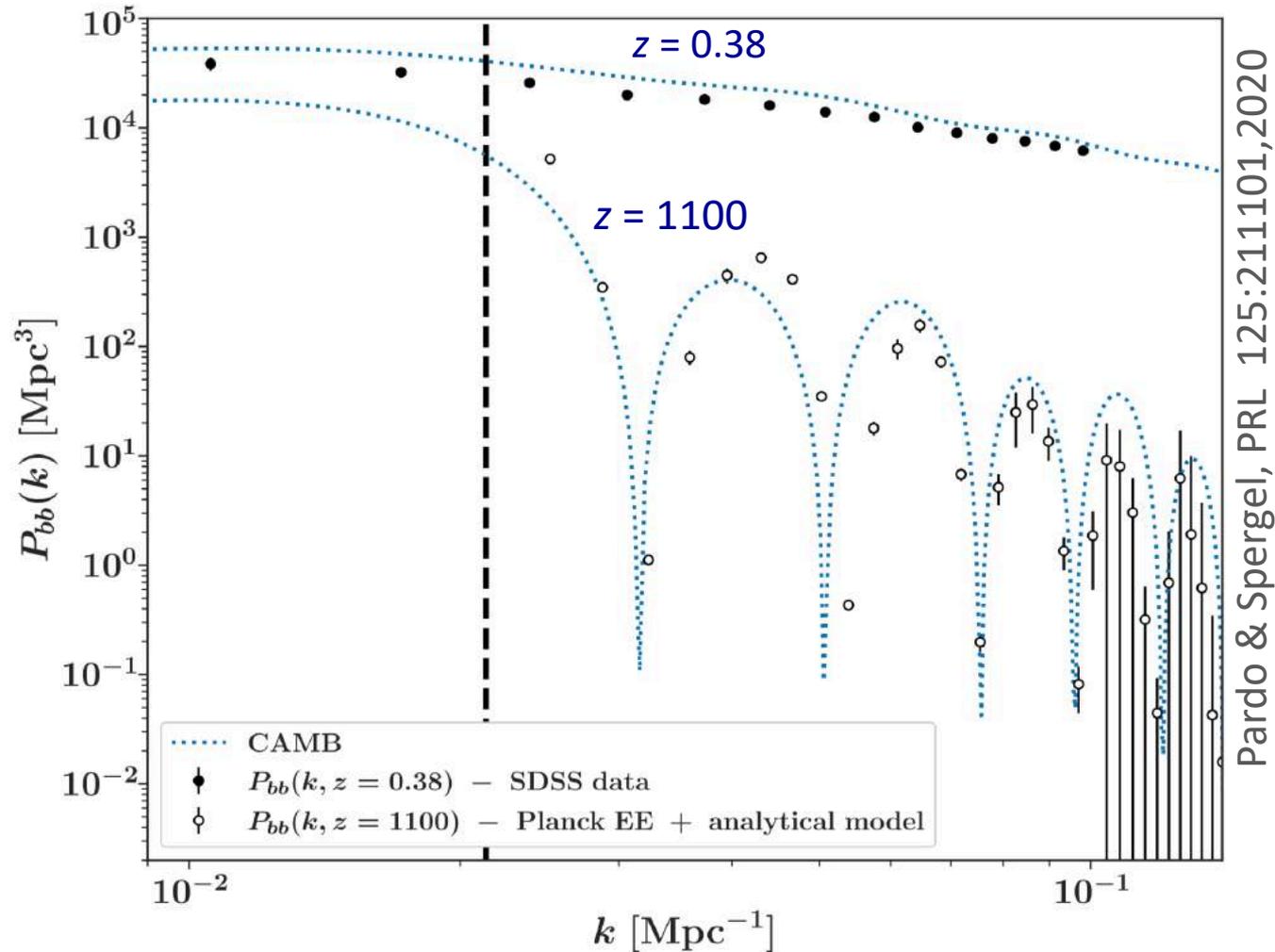


THE OBSERVED LARGE-SCALE STRUCTURE *REQUIRES* $\Omega_m \gg \Omega_B$ IF IT HAS RESULTED FROM THE GROWTH UNDER GRAVITY OF SMALL INITIAL DENSITY FLUCTUATIONS WHICH LEFT AN IMPRINT ON THE CMB AT LAST SCATTERING



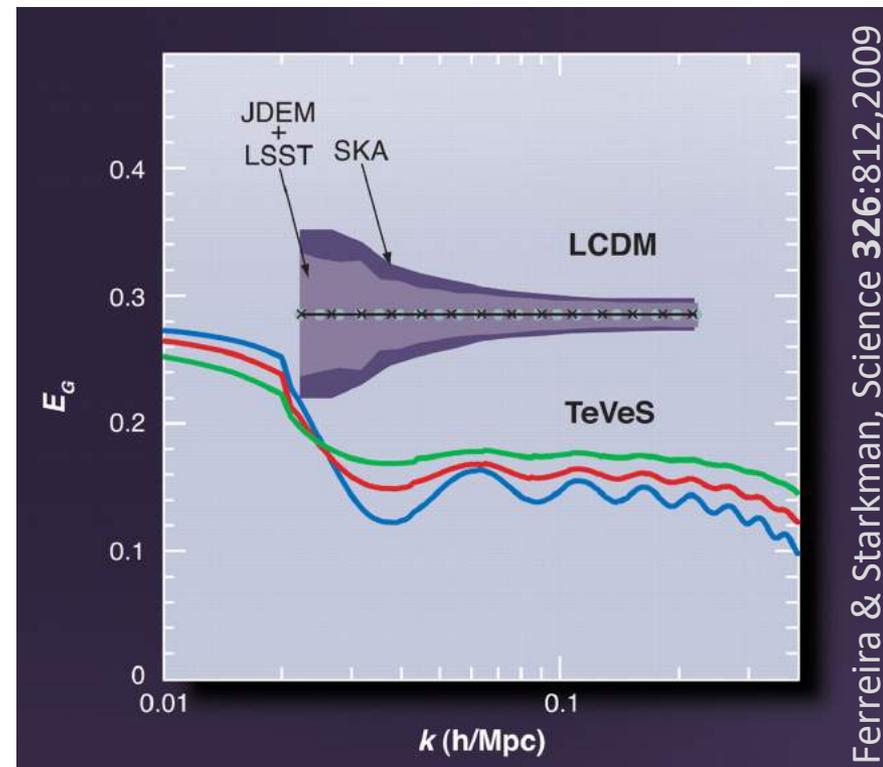
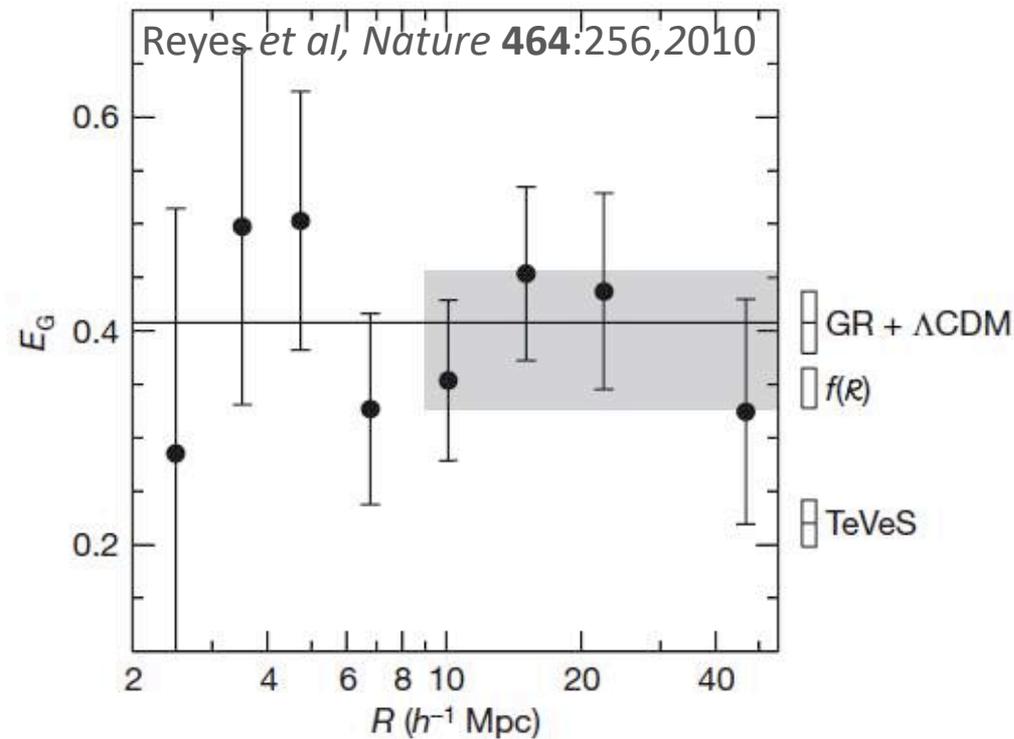
Detailed modelling of WMAP/Planck and 2dF/SDSS $\Rightarrow \Omega_m \sim 0.3, \Omega_B \sim 0.05$
... No MOND-like theory (e.g. TeVeS) can fit the data so well

THE BARYON POWER SPECTRUM AT $z = 0.38$ (INFERRED FROM BAO)
MATCHES THE GROWTH UNDER GRAVITY OF SMALL INITIAL DENSITY
FLUCTUATIONS IMPRINTED ON THE CMB AT LAST SCATTERING ($z = 1100$)



Detailed modelling of WMAP/Planck and 2dF/SDSS $\Rightarrow \Omega_m \sim 0.3, \Omega_B \sim 0.05$
Can a MOND-like theory (Skordis & Zlosnik, arXiv:2007.00082) fit as well?

ALTHOUGH *NEW* GRAVITATIONAL PHYSICS (UNDERLYING MOND) CAN IN PRINCIPLE PROVIDE ADEQUATE GROWTH OF COSMOLOGICAL STRUCTURE, THERE WILL ALWAYS BE AN OBSERVABLE DISTINCTION – THE ‘GRAVITATIONAL SLIP’ – BETWEEN GENERAL RELATIVITY AND THE NEW THEORY



Ferreira & Starkman, *Science* 326:812,2009

This can be tested through measurements of ‘weak lensing’ (shearing of galaxy shapes) and its cross-correlation with the galaxy density field

DOES DARK MATTER EXIST?

Modified Newtonian Dynamics (MOND) accounts *better* for galactic rotation curves than does dark matter - moreover it predicts the observed correlation between luminosity and rotation velocity: $L \sim v_{\text{rot}}^4$ (“Tully-Fisher relation”)

... however MOND *fails* on the scale of galaxy clusters and in particular it cannot explain the segregation of ‘bright’ and ‘dark’ matter seen in the merging ‘Bullet cluster’ (1E 0657-558)

Also MOND is not a *physical* theory – relativistic covariant theories that yield MOND exist (e.g. ‘TeVeS’ by Bekenstein, Phys.Rev.D**70**:083509,2004) they have *not* provided as satisfactory an understanding of CMB anisotropies and structure formation, as has the standard (cold) dark matter cosmology

... *Nevertheless you may like to keep an open mind until dark matter is directly detected and identified*