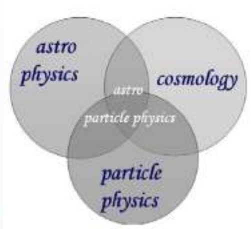
Astroparticle Physics

Hilary 2021



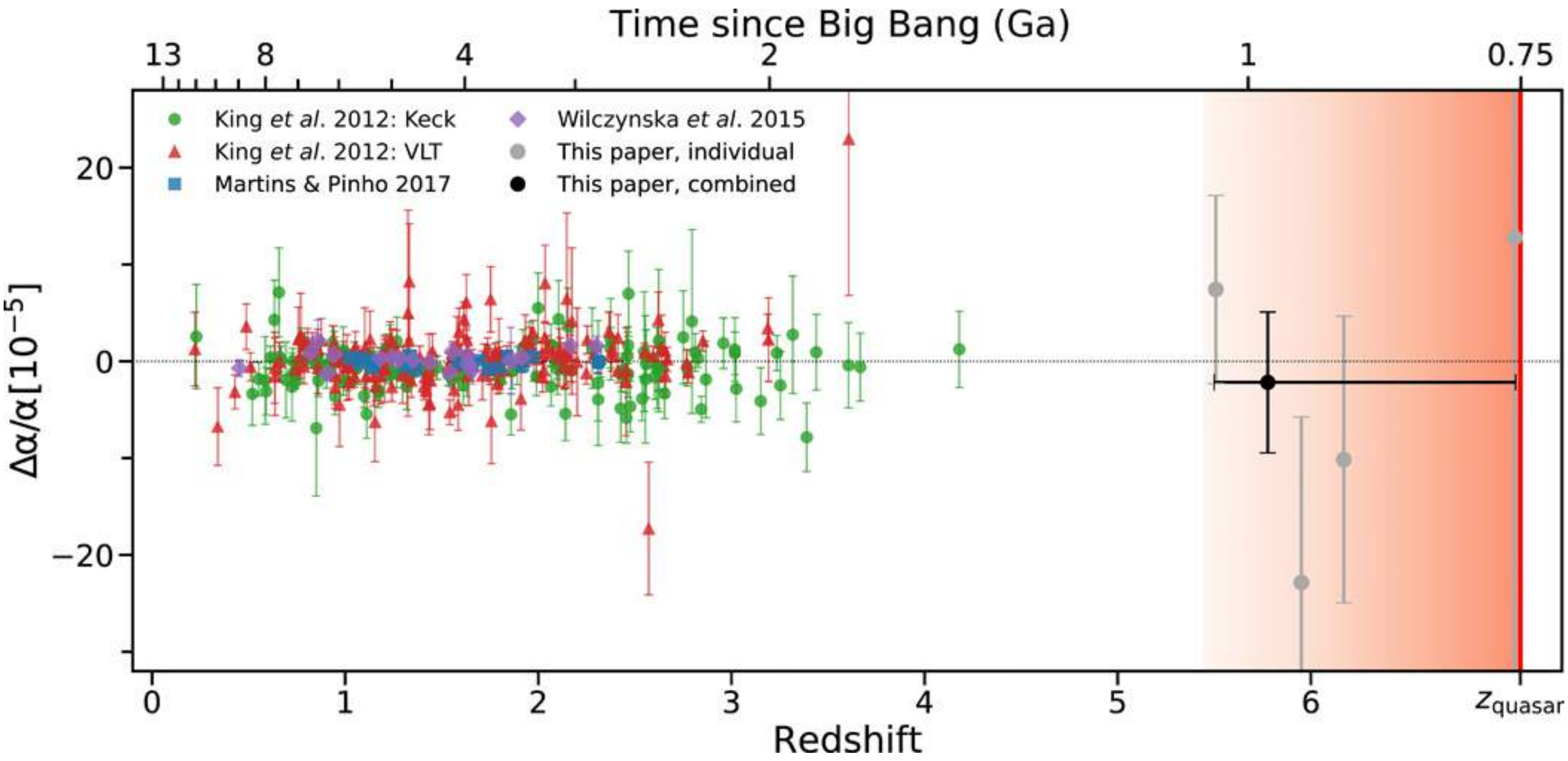
Oxford Physics

Oxford Master Course in Mathematical and Theoretical Physics

- The universe observed
- Relativistic world models
- Reconstructing the thermal history
- Big bang nucleosynthesis
- Dark matter: astrophysical observations
- Dark matter: relic particles
- Dark matter: direct detection
- Dark matter: indirect detection
- Cosmic rays in the Galaxy
- Antimatter in cosmic rays
- Ultrahigh energy cosmic rays
- High energy cosmic neutrinos
- The early universe: constraints on new physics
- The early universe: baryo/leptogenesis
- The early universe: inflation & the primordial density perturbation
- Cosmic microwave background & large-scale structure

 http://www-thphys.physics.ox.ac.uk/user/SubirSarkar/astropartphys.html

We can check *experimentally* that physical ‘constants’ such as α have been sensibly constant for the past ~12 billion years …



Wilczynska *et al*, Sci.Adv. 6: :eaay9672,2020

So we are entitled to extrapolate known physical laws back in time with confidence

(NB: In string theory all fundamental `constants’ are expectation values of moduli fields so may have been different in the *very* early universe … but we know they have been all fixed at least since the epoch of primordial nucleosyntheis at $t \sim 1$ s)

Knowing the **equation of state**, we can solve the Friedman equation …

For matter: $$\frac{d}{dt}(\rho a^3) = 0 \Rightarrow \quad \rho = \rho_0/a^3 = \rho_0(1+z)^3$$

Hence $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_0}{3a^3} \quad \Rightarrow \quad a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

For radiation: $$\frac{d}{dt}(\rho a^4) = 0 \Rightarrow \quad \rho = \rho_0/a^4 = \rho_0(1+z)^4$$

So radiation will dominate over other components as we go to early times

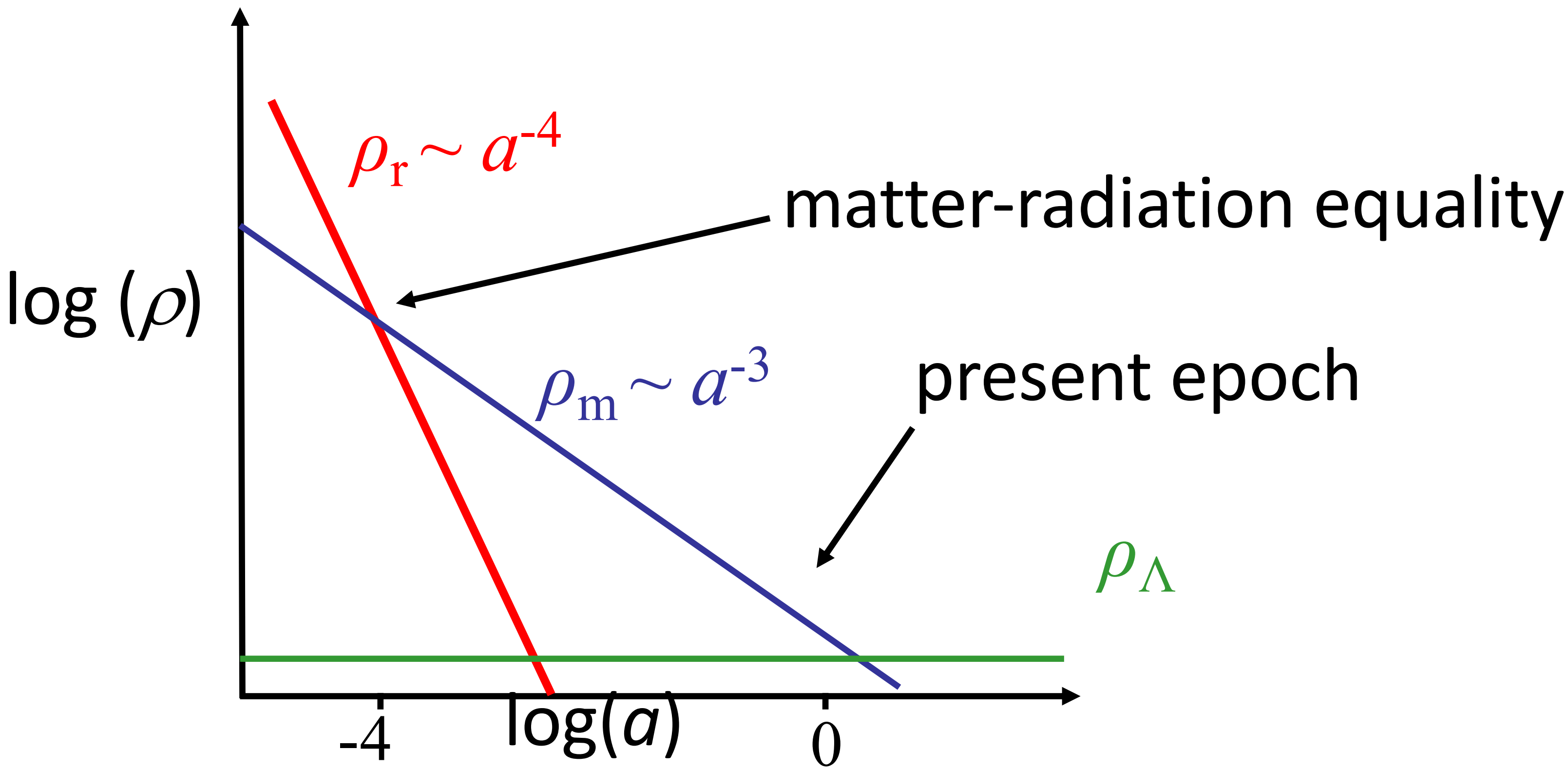
$$a(t) = \left(\frac{t}{t_0}\right)^{1/2} \Rightarrow \quad \rho_{\mathrm{r}} \propto t^{-2}$$

Radiation-dominated era

But at $a_{\mathrm{eq}} = \rho_{\mathrm{r},0}/\rho_{\mathrm{m},0}$ the matter density will come to dominate

Note that $\rho_{\mathrm{m}} \propto t^{-2}$ during the **Matter-dominated era** too

Evolution of energy components



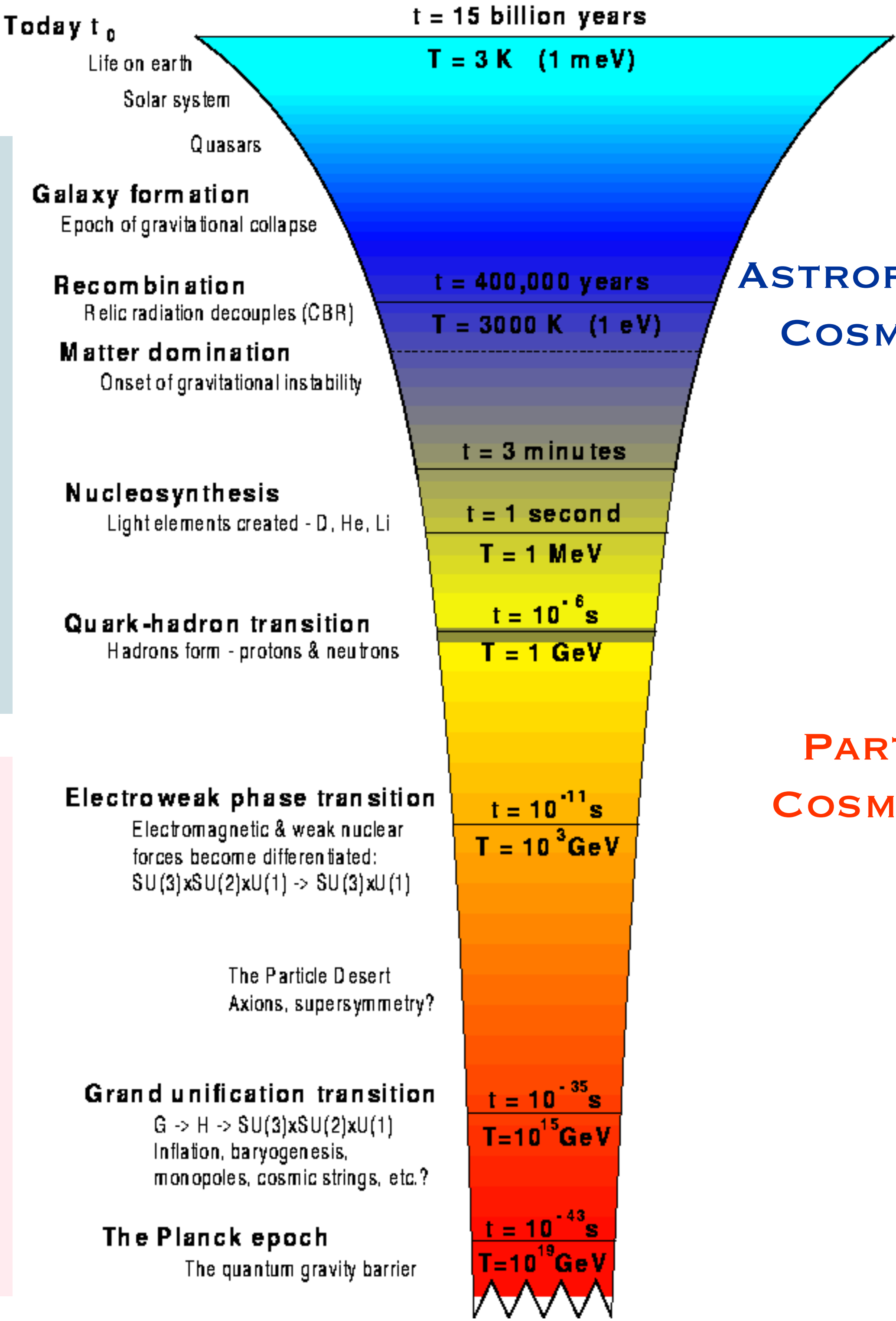
Very recently (at z~1) the expansion has supposedly become dominated by a 'cosmological constant': $\Lambda \sim 2\, H_0^2 \Rightarrow \rho_\Lambda \sim 2\, H_0^2\, \mathrm{M_P}^2$

This creates a severe 'why now?' problem as $\rho_\Lambda << \rho_{m,\,r}$ at earlier epochs

But this also means that the high redshift universe was matter and/or radiation dominated so well-described by an Einstein-de Sitter model

On the basis of known physics, the evolution of the universe can be extrapolated into our past, quite reliably up to the nucleosyntheis era and (with some caveats) back through the QCD phase transition up to the electroweak unification scale

New physics is required to account for the observed asymmetry between matter and antimatter, to explain dark matter, and also generate the density fluctuations which seeded the formation of structure



ASTROPHYSICAL COSMOLOGY

PARTICLE COSMOLOGY

Does the universe have any net quantum numbers?

The chemical potential is additively conserved in all reactions

hence zero for photons and Z^0 bosons which can be emitted or absorbed in any number (at high enough temperatures) – and consequently *equal and opposite* for a particle and its antiparticle, which can annihilate into such gauge bosons

A finite chemical potential corresponds to a *particle-antiparticle asymmetry*, i.e. a non-zero value for any associated conserved quantum number

The net electric charge of the universe is consistent with being zero

e.g. $q_{e-p} < 10^{-26} e$ from the isotropy of the CMB (Caprini & Ferreira, JCAP **02**:006,2005)

The net baryon number is very small relative to the number of photons:

$$\boxed{\frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} \sim \frac{n_B}{n_\gamma} \simeq 5 \times 10^{-10}}$$

… and presumably so is any net lepton number

There can be a large lepton asymmetry in *neutrinos* (if $B - L$ is non-zero) but this is constrained to be small due to ν oscillations (Dolgov *et al*, Nucl.Phys. **B632**:363,2002)

(NB: The dark matter may be a particle with a relic *asymmetry* … similar to that of baryons)

Thermodynamics of ultra-relativistic plasma in equilibrium

$$\boxed{f_i^{\mathrm{eq}}(q,T) = \left[\exp\left(\frac{E_i-\mu_i}{T}\right) \mp 1\right]^{-1}}$$

For negligible chemical potential, this integrates to:

Number density: $$n_i^{\mathrm{eq}}(T) = g_i \int f_i^{\mathrm{eq}}(q,T)\,\frac{\mathrm{d}^3q}{(2\pi)^3} = \frac{g_i}{2\pi^2}T^3 I_i^{11}(\mp)\ ,$$

Energy density: $$\rho_i^{\mathrm{eq}}(T) = g_i \int E_i(q)\, f_i^{\mathrm{eq}}(q,T)\,\frac{\mathrm{d}^3q}{(2\pi)^3} = \frac{g_i}{2\pi^2}T^4 I_i^{21}(\mp)\ ,$$

Pressure density: $$p_i^{\mathrm{eq}}(T) = g_i \int \frac{q^2}{3E_i(q)}\, f_i^{\mathrm{eq}}(q,T)\,\frac{\mathrm{d}^3q}{(2\pi)^3} = \frac{g_i}{6\pi^2}T^4 I_i^{03}(\mp)$$

where: $$I_i^{mn}(\mp) \equiv \int_{x_i}^{\infty} y^m (y^2-x_i^2)^{n/2}\,(\mathrm{e}^y \mp 1)^{-1} dy\ , \qquad x_i \equiv \frac{m_i}{T}$$

bosons :	$I_{\mathrm{R}}^{11}(-) = 2\zeta(3)\ ,$	$I_{\mathrm{R}}^{21}(-) = I_{\mathrm{R}}^{03}(-) = \frac{\pi^4}{15}$
fermions :	$I_{\mathrm{R}}^{11}(+) = \frac{3\zeta(3)}{2}\ ,$	$I_{\mathrm{R}}^{21}(+) = I_{\mathrm{R}}^{03}(+) = \frac{7\pi^4}{120}$

Non-relativistic particles (x >>1) have the Boltzmann distribution:

$$n_{\mathrm{NR}}^{\mathrm{eq}}(T) = \frac{\rho_{\mathrm{NR}}^{\mathrm{eq}}(T)}{m} = \frac{g}{(2\pi)^{3/2}}T^3 x^{3/2}\mathrm{e}^{-x}, \qquad p_{\mathrm{NR}} \simeq 0$$

The particle *i* will stay in *kinetic* equilibrium with the plasma (i.e. $T_i = T$) as long as the **scattering rate** $\Gamma_s = n\langle\sigma v\rangle$ **exceeds the Hubble rate** $H = (8\pi G\rho/3)^{1/2} \sim 1.66\sqrt{g}\, T^2/M_P$

It will decouple at $T_i = T_D$ when $\Gamma_s(T_D) = H(T_D)$

If it is *relativistic* at this time (i.e. $m_i << T_d$) then it would also have been in *chemical* equilibrium $(\mu_i + \mu_{\bar{i}} = \mu_{l^+} + \mu_{l^-} = \mu_\gamma = 0)$ and its abundance will just be:

$$\boxed{n_i^{\rm eq}(T_{\rm D}) = \frac{g_i}{2} n_\gamma(T_{\rm D}) f_{\rm B,\,F} \quad (f_{\rm B} = 1, f_{\rm F} = 3/4)}$$

The decoupled *i* particles expand freely without interactions so that their **number in a comoving volume is conserved** and their pressure and energy density are functions of the scale-factor a alone. Although non-interacting, their phase space distribution will retain the equilibrium form, with T substituted by T_i, as long as the particles remain *relativistic*, which ensures that both E_i and T_i will scale as a^{-1}

Subsequently T_i will continue to track the photon temperature T but as the universe cools below various mass thresholds, the corresponding particles will become non-relativistic and annihilate – this will heat the photons (and any other *interacting* particles), but *not* the decoupled *i* particles, so T_i will now *drop below* T and therefore n_i/n_γ will *decrease* below its value at decoupling

To calculate this write (Alpher, Follin & Herman, Phys.Rev.**92**:1347,1953):

$$p = p_{\mathrm{I}}(T) + p_{\mathrm{D}}(a), \; \rho = \rho_{\mathrm{I}}(T) + \rho_{\mathrm{D}}(a)$$

The energy conservation equation: $a^3 \frac{\mathrm{d}p}{\mathrm{d}T} = \frac{\mathrm{d}}{\mathrm{d}T}\left[a^3(\rho + p)\right]$

then reduces to: $\frac{\mathrm{d}\ln a}{\mathrm{d}\ln T} = -\frac{1}{3}\frac{(\mathrm{d}\rho_{\mathrm{I}}/\mathrm{d}\ln T)}{(\rho_{\mathrm{I}} + p_{\mathrm{I}})}$ (using $n_{\mathrm{D}}a^3 = \mathrm{const}$)

Combining with the 2nd law of thermodynamics, this yields:

$$\frac{\mathrm{d}\ln a}{\mathrm{d}\ln T} = -1 - \frac{1}{3}\frac{\mathrm{d}\ln\left(\frac{\rho_{\mathrm{I}} + p_{\mathrm{I}}}{T^4}\right)}{\mathrm{d}\ln T}$$

which integrates to: $\ln a = -\ln T - \frac{1}{3}\ln\left(\frac{\rho_{\mathrm{I}} + p_{\mathrm{I}}}{T^4}\right) + \mathrm{constant}$

Hence if $(\rho_{\mathrm{I}} + p_{\mathrm{I}})/T^4$ is constant (as for a gas of blackbody photons), this yields the *adiabatic invariant*: $aT = \mathrm{constant}$

Epochs where the number of interacting species is *different* can now be related through the conservation of specific **entropy** in a comoving volume, i.e. $\mathrm{d}(s_\mathrm{I}\, a^3)/\mathrm{d}T = 0$, where:

$$s_\mathrm{I} \equiv \frac{\rho_\mathrm{I} + p_\mathrm{I}}{T} = \sum_{\mathrm{int}} s_i \ , \ s_i(T) = g_i \int \frac{3m_i^2 + 4q^2}{3E_i(q)\, T} f_i^{\mathrm{eq}}(q, T) \frac{\mathrm{d}^3 q}{(2\pi)^3}$$

Here s_i can be parameterised in terms of the value for photons:

$$s_i(T) \equiv \left(\frac{g_{s_i}}{2}\right) \left(\frac{4}{3}\frac{\rho_\gamma}{T}\right) \ , \ g_{s_i} = \frac{45}{4\pi^4} g_i \left[I_i^{21}(\mp) + \frac{1}{3} I_i^{03}(\mp) \right]$$

So the number of *interacting* degrees of freedom is:

$$g_{s_\mathrm{I}} \equiv \frac{45}{2\pi^2} \frac{s_\mathrm{I}}{T^3} = \sum_{\mathrm{int}} g_{s_i}$$

Analogous to the *total* number of degrees of freedom:

$$\rho_i^{\mathrm{eq}}(T) \equiv \left(\frac{g_{\rho_i}}{2}\right) \rho_\gamma \ , \qquad g_{\rho_i} = \frac{15}{\pi^4} g_i\, I_i^{21}(\mp) \ = \sum_{\mathrm{B}} g_i + \frac{7}{8} \sum_{\mathrm{F}} g_i$$

We can now calculate how the temperature of a particle *i* which decoupled at $T_{\rm D}$ relates to the photon temperature T at a later epoch

For $T < T_{\rm D}$, the entropy in the decoupled i particles and the entropy in the still interacting j particles are *separately conserved*:

$$S - S_{\rm I} = s_i\, a^3 \quad = \frac{2\pi^2}{45} g_{s_i}(T)\,(a\,T)_i^3,$$

$$S_{\rm I} = \sum_{j \neq i} s_j(T)\, a^3 \quad = \frac{2\pi^2}{45} g_{s_{\rm I}}(T)\,(a\,T)^3$$

Since $T_i = T$ at decoupling, this yields for the subsequent ratio of temperatures

Srednicki *et al,* Nucl.Phys.B**310**:693,1988, Gondolo & Gelmini, *ibid* B**360**:145,1991

$$\frac{T_i}{T} = \left[\frac{g_{s_i}(T_{\rm D})}{g_{s_i}(T)}\,\frac{g_{s_{\rm I}}(T)}{g_{s_{\rm I}}(T_{\rm D})}\right]^{1/3}$$

After decoupling, the degrees of freedom specifying the conserved total entropy is:

$$g_s(T) \equiv \frac{45}{2\pi^2}\frac{S}{T^3 a^3} = g_{s_{\rm I}}(T)\left[1 + \frac{g_{s_i}(T_{\rm D})}{g_{s_{\rm I}}(T_{\rm D})}\right]$$

We now have an useful fiducial in the total entropy density, which *always* scales as a^{-3}:

$$\boxed{s(T) \equiv \frac{2\pi^2}{45} g_s(T) T^3}$$

Therefore the ratio of the decoupled particle density to the blackbody photon density is subsequently related to its value at decoupling as:

$$\frac{(n_i/n_\gamma)_T}{(n_i^{\mathrm{eq}}/n_\gamma)_{T_\mathrm{D}}} = \frac{g_s(T)}{g_s(T_\mathrm{D})} = \frac{N_\gamma(T_\mathrm{D})}{N_\gamma(T)}$$

where $N_\gamma = a^3 n_\gamma$ is the total number of blackbody photons in a comoving volume

The total energy density may similarly be parameterised as:

$$\rho(T) = \sum \rho_i^{\mathrm{eq}} \equiv \left(\frac{g_\rho}{2}\right) \rho_\gamma = \frac{\pi^2}{30} g_\rho T^4 \ , \quad g_\rho \simeq \sum_\mathrm{B} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_\mathrm{F} g_i \left(\frac{T_i}{T}\right)^4$$

So the relationship between *a* and *T* writes: $\frac{\mathrm{d}a}{a} = -\frac{\mathrm{d}T}{T} - \frac{1}{3} \frac{\mathrm{d}g_{s_\mathrm{I}}}{g_{s_\mathrm{I}}}$

During the radiation-dominated era, the expansion rate is:

$$H \equiv \frac{\dot{a}}{a} \simeq \sqrt{\frac{8\pi G_{\rm N}\rho}{3}}$$

Integrating this yields the time-temperature relationship:

$$t = -\int \left(\frac{45\, M_{\rm P}^2}{4\pi^3}\right)^{1/2} g_\rho^{-1/2} \left(1 + \frac{1}{3}\frac{{\rm d}\ln g_{s{\rm I}}}{{\rm d}\ln T}\right) \frac{{\rm d}T}{T^3}$$

During the periods when ${\rm d}g_{s{\rm I}}/{\rm d}T \simeq 0$, i.e. away from mass thresholds and phase transitions, this yields the useful commonly used approximation:

$$(t/{\rm s}) = 2.42\, g_\rho^{-1/2}\, (T/{\rm MeV})^{-2}$$

So we can work out when events of physical significance occurred (according to the Standard $SU(3)_c\mathrm{x}SU(2)_L\mathrm{x}U(1)_Y$ Model … and beyond)

E.g. consider the decoupling of massless neutrinos in the Standard Model

The thermally-averaged #-section is: $<\sigma v> \sim G_F^2\, E^2 \sim G_F^2\, T^2$ ($m_n << T$)
so the interaction rate is: $G = n<\sigma v> \sim G_F^2 T^5$ (since $n \approx T^3$)

This equals the expansion rate $H \sim T^2/M_P$ at the decoupling temperature

$$T_{\rm D}(\nu) \sim (G_{\rm F}^2 M_{\rm P})^{-1/3} \sim 1 \text{ MeV}$$

At this time $n_\nu^{eq} = (3/4)n_\gamma$ since $T_\nu = T$ and $g_\nu = 2$. Subsequently as T drops below m_e, the electrons and positrons annihilate (almost) totally, heating the photons but *not* the decoupled neutrinos. While g_ν does not change, the number of other interacting degrees of freedom decreases from $11/2$ (γ, $e^\pm$) to 2 (γ only), hence the comoving number of blackbody photons *increases* by:

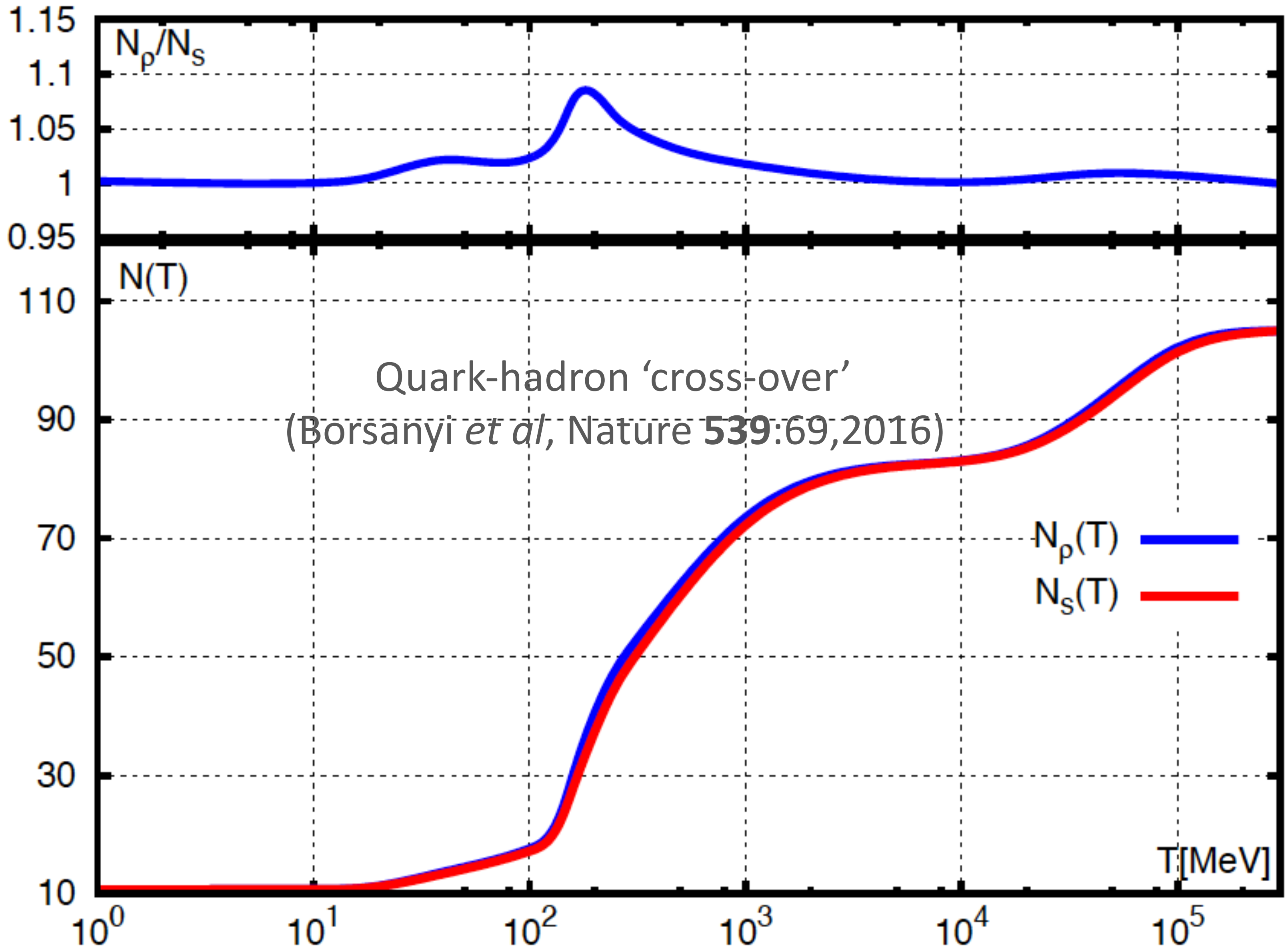
$$\frac{N_\gamma\,(T \ll m_{\rm e})}{N_\gamma\,(T = T_{\rm D}(\nu))} = \left[\frac{(aT)_{T \ll m_{\rm e}}}{(aT)_{T = T_{\rm D}(\nu)}}\right]^3 = \frac{11}{4} \text{ so } \left(\frac{n_\nu}{n_\gamma}\right)_{T \ll m_{\rm e}} = \frac{4}{11}\left(\frac{n_\nu^{\rm eq}}{n_\gamma}\right)_{T = T_{\rm D}(\nu)} = \frac{3}{11}$$

Hence the d.o.f. characterising the entropy & energy densities today are:

$$g_s\,(T \ll m_{\rm e}) = g_\gamma + \tfrac{7}{8}\,N_\nu\,g_\nu \left(\tfrac{T_\nu}{T}\right)^3 = \tfrac{43}{11}\,,$$

$$g_\rho\,(T \ll m_{\rm e}) = g_\gamma + \tfrac{7}{8}\,N_\nu\,g_\nu \left(\tfrac{T_\nu}{T}\right)^4 = 3.36$$

To construct our thermal history we must then count all boson and fermion species contributing to the number of relativistic degrees of freedom … and take into account our understanding of (possible) phase transitions

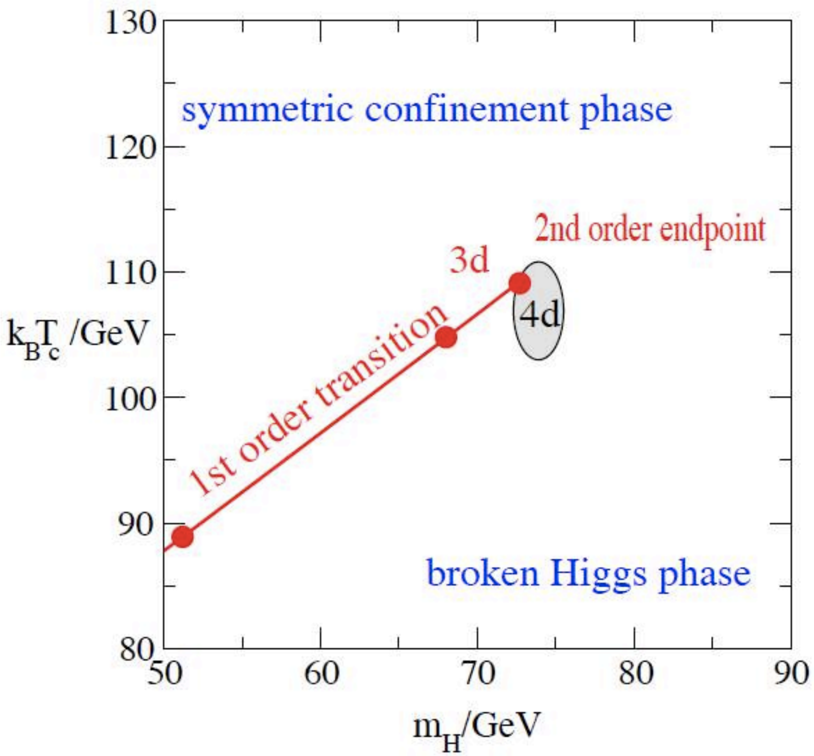


The Standard Model of the Early Universe

History of $g(T)$

$T \sim 200$ GeV	all present	106.75	
$T \sim 100$ GeV	EW transition	(no effect)	
$T < 170$ GeV	top-annihilation	96.25	
$T < 80$ GeV	$W^{\pm}, Z^0, H^0$	86.25	
$T < 4$ GeV	bottom	75.75	
$T < 1$ GeV	charm, τ^-	61.75	
$T \sim 150$ MeV	QCD transition	17.25	(u,d,g$\to \pi^{\pm,0}$, $37 \to 3$)
$T < 100$ MeV	$\pi^{\pm}, \pi^0, \mu^-$	10.75	$e^{\pm}, \nu, \bar{\nu}, \gamma$ left
$T < 500$ keV	e^- annihilation	(7.25)	$2 + 5.25(4/11)^{4/3} = 3.36$

The phase diagram of the Standard Model (based on a dimensionally reduced $SU(2)_{\mathrm{L}}$ theory with quarks and leptons, with the Abelian hypercharge symmetry $U(1)_{\mathrm{Y}}$ neglected). The 1st-order transition line ends at the 2nd-order endpoint:
$m_H \simeq 72 \pm 2\ \mathrm{GeV/c^2}$, $k_{\mathrm{B}}T_{\mathrm{E}} \simeq 110$ GeV;
for higher Higgs mass it is a 'crossover'
Rummukainen *et al,* Nucl.Phys.B**532**:283,1998



What is the highest temperature the Universe reached?

On dimensional grounds, the 2 ➛2 scattering/annihilation cross-section (at temperatures higher than the masses of particles) must go as $\sim\alpha^2/T^2$, i.e. the rate will go as: $\Gamma \sim n\langle\sigma v\rangle \sim \alpha^2 T$

Comparing this to the Hubble expansion rate, $H \sim (g_* T^4/10\, M_\mathrm{P}^4)^{1/2}$, we see that the thermalisation temperature *cannot* exceed:

$$T_\mathrm{therm} \sim \alpha^2 M_\mathrm{P}/3\sqrt{g_*} \sim 10^{-4}\, M_\mathrm{P} \text{ (taking: } \alpha = 1/24,\ g_* \sim 200)$$

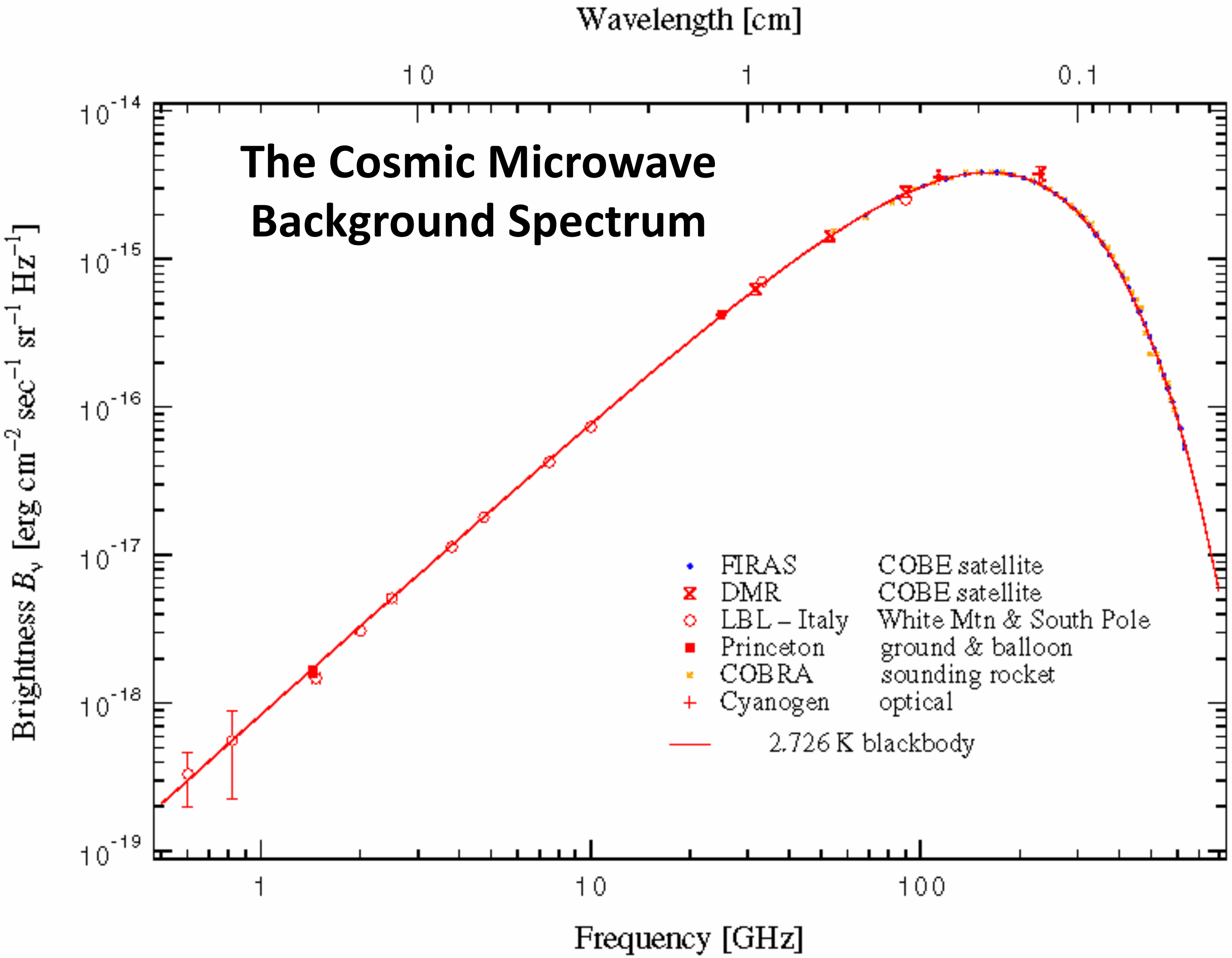
So the universe could never have got as hot as even the GUT scale!

A careful calculation (incl. the temperature dependence of α_QCD) gives:

$$T_\mathrm{therm} \sim 3\mathrm{x}10^{14}\,\mathrm{GeV}$$ (Enqvist & Sirkaa, Phys. Lett. B**314**:298,1993)

Ought to revisit earlier discussions of GUT-scale baryogenesis, monopole problem *etc*

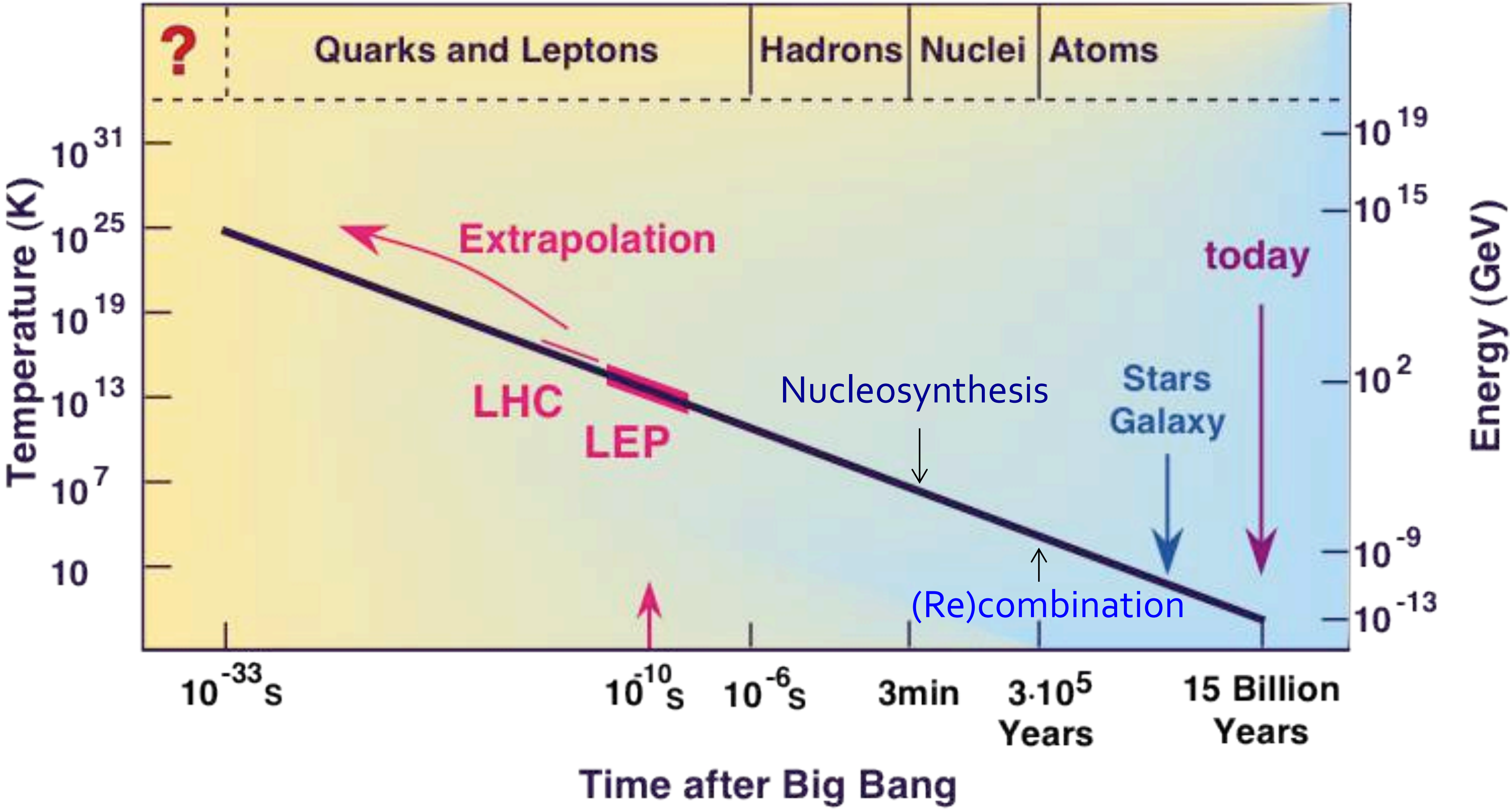
EVIDENCE THAT THE EARLY UNIVERSE WAS IN THERMAL EQUILIBRIUM



This *perfect* **blackbody** is testimony to our hot, dense past and directly demonstrates that the expansion was **adiabatic** (with negligible energy release) back at least to $t \sim 1$ day

By studying nucleosynthesis we can show this holds further back to ~1 s

The blackbody temperature can be used as a clock (assuming adiabatic expansion: $aT = \mathrm{constant}$), so our thermal history can be reconstructed



The furthest we 'see' directly is back to $t \sim 1$ s (when light elements were synthesised) but the small variations in CMB temperature must have been generated *much* earlier

Thomson scattering on electrons: $\gamma + e \rightarrow \gamma + e$

Photon interaction rate ($x = n_p/n_B$): $\Gamma_{\text{Thomson}} = n_e \langle \sigma_T |v| \rangle \propto x_e T^3 \sigma_T$

cf. expansion rate of the universe (MD era): $H \propto T^{3/2}$

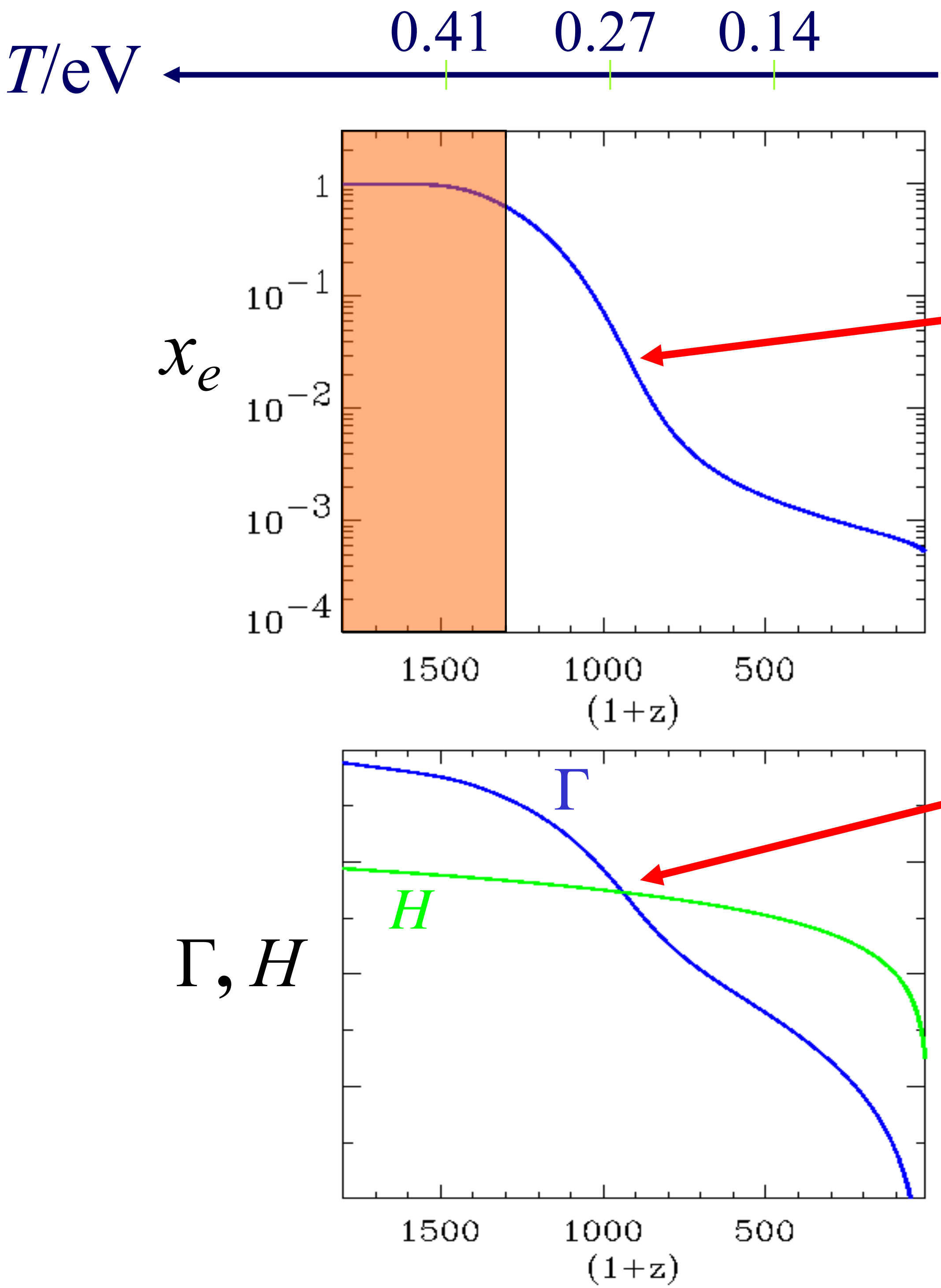
$\Gamma_{\text{Thomson}} > H \Rightarrow$ Photons/matter in equilibrium

$\Gamma_{\text{thomson}} < H \Rightarrow$ Photons/matter decouple

The ionisation fraction x_e drops rapidly at (re)combination so the Thomson scattering rate also decreases sharply below the Hubble expansion rate – this defines a ***last scattering surface*** for the relic photons … which we see today as the **cosmic microwave background**

While $p + e^- \longrightarrow \mathrm{H} + \gamma$ is in **chemical equilibrium**, $\mu_p + \mu_e = \mu_\mathrm{H}$ (since $\mu_\gamma = 0$) so,
$n_\mathrm{H} = (g_\mathrm{H}/g_p g_e) n_p n_e (m_e T/2\pi)^{3/2}\, \mathrm{e}^{B/T}$ (where $B = m_p + m_e - m_\mathrm{H} = 13.6$ eV)

In terms of the ionisation fraction x_e and the baryon-to-photon ratio, $\eta = n_B/n_\gamma$, this is the Saha ionisation equation: $\dfrac{1 - x_e}{x_e^2} = \dfrac{4\sqrt{2}\zeta(3)}{\sqrt{\pi}} \eta \left(\dfrac{T}{m_e}\right)^{3/2} \mathrm{e}^{-B/T}$



Recombination

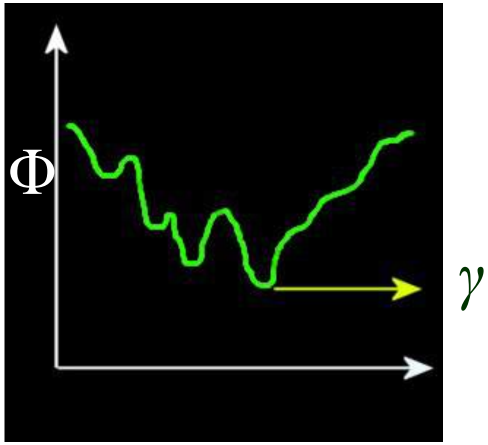
(according to the Saha ionisation eq.)

$T_{\mathrm{rec}} \sim 0.35$ eV, $z_{\mathrm{rec}} \sim 1300$

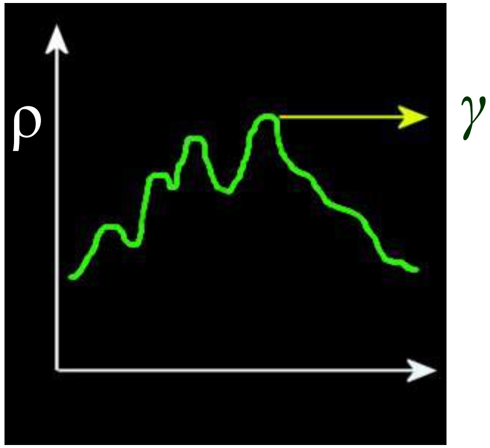
Decoupling of photons and baryons

$T_{\mathrm{rec}} \sim 0.29$ eV, $z_{\mathrm{dec}} \sim 1100$

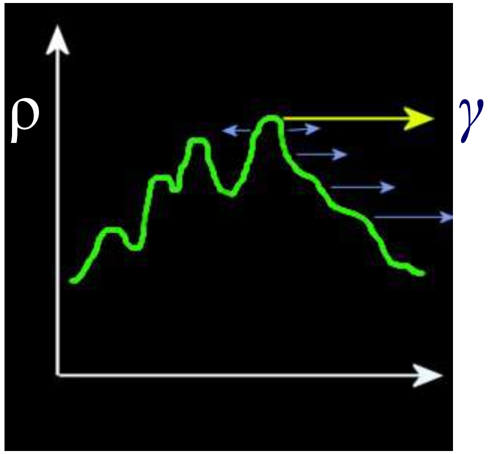
More precise calculation by
Seager *et al*, ApJ **523**:L1,1999
(Codes: *CosmoRec*, *HyRec*)



Photons are redshifted as they move out of gravitational potential wells



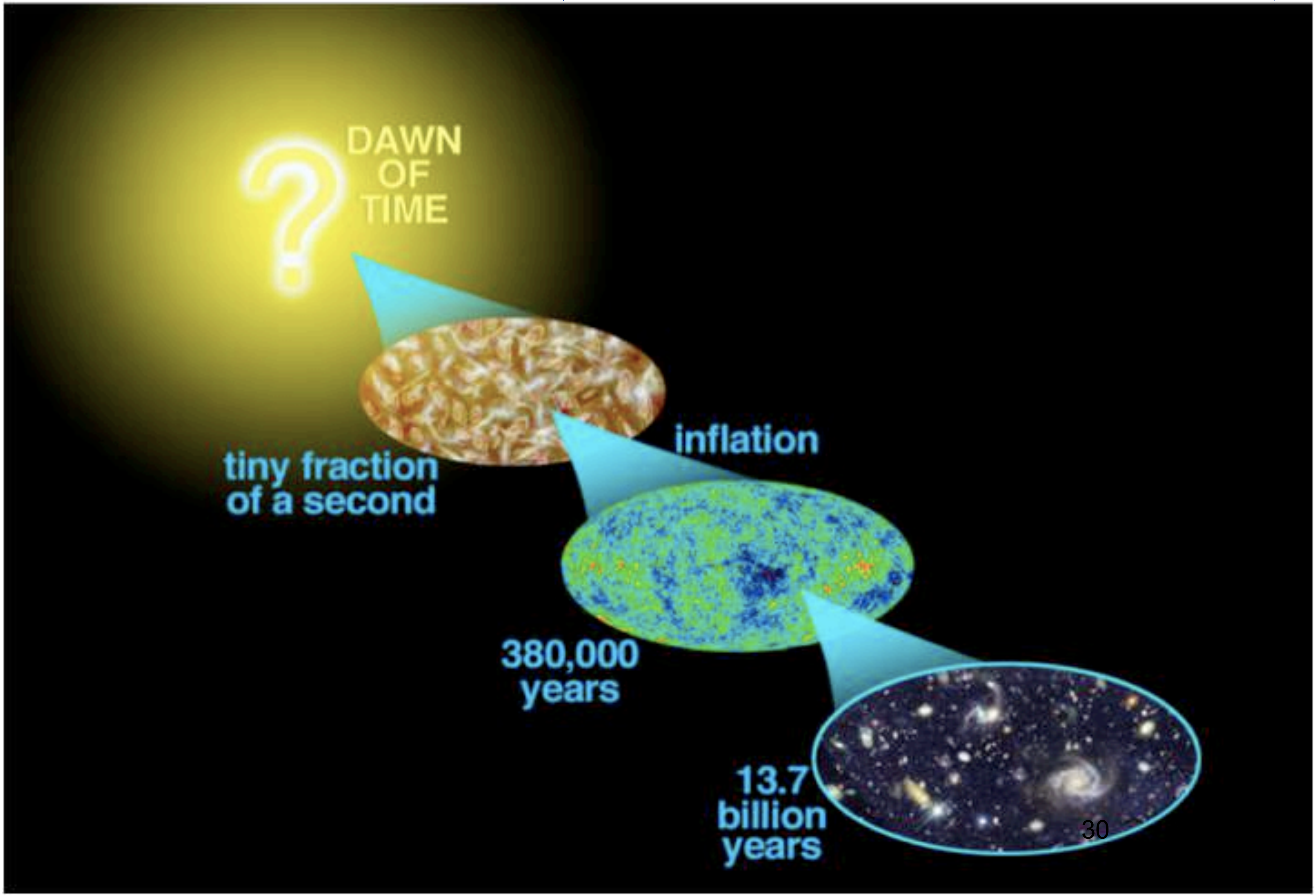
Dense regions have higher temperature ⇒ photons have higher energy



Photons emitted from a moving surface are red/blue-shifted

Fortunately the effects do not *quite* cancel so the CMB carries a memory of the past

The CMB carries the imprint of primordial fluctuations which grow into Large-Scale structure (in the potential wells of dark matter)



Formation of structure in the universe

$t = 10^{-34}$ s

Quantum mechanical fluctuations during **inflation**

$V(\phi)$

$t \sim 400000$ yr

Perturbation Growth: pressure vs. gravity

$\Omega_m, \Omega_r, \Omega_b, f_\nu$

Photons freestream Inhomogeneities turn into anisotropies

$z_{reion}, \Omega_{de}, w$

Matter perturbations grow into non-linear structures observed today

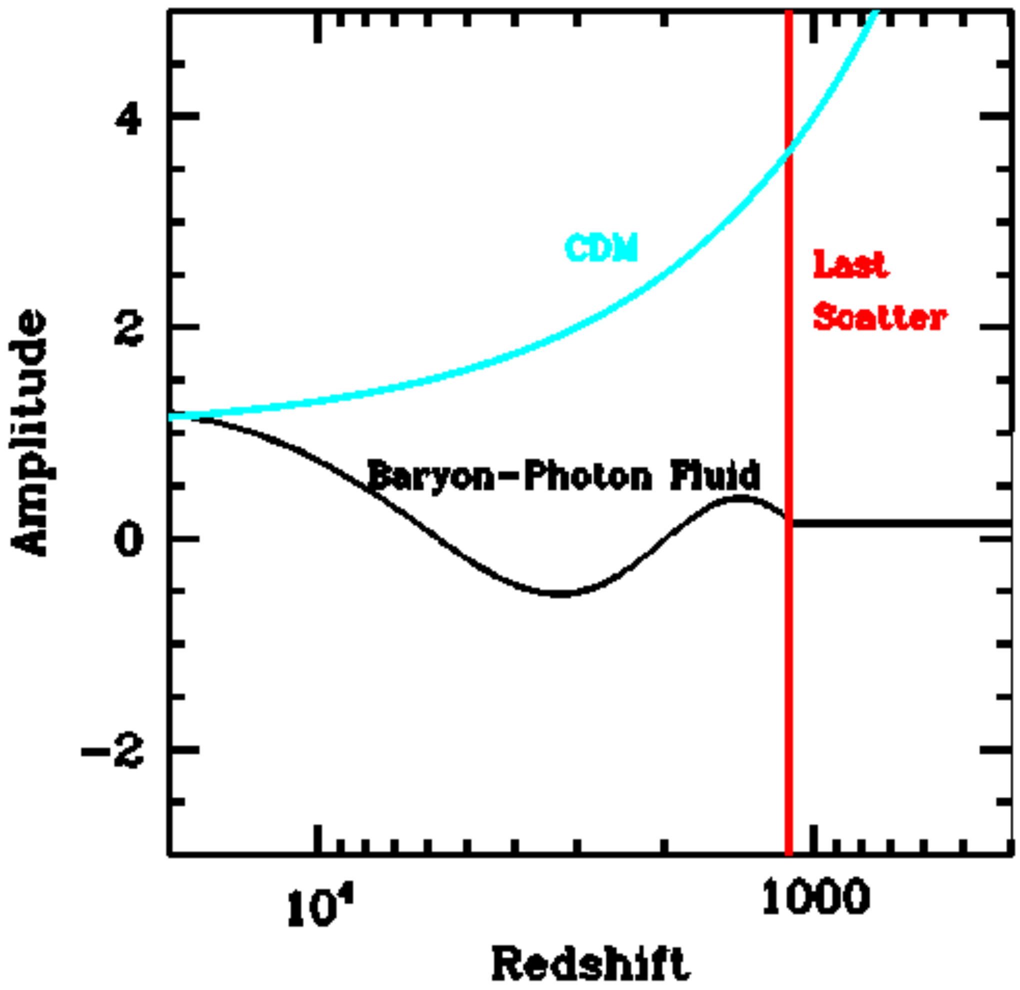
Growth of Fluctuations

- Linear theory

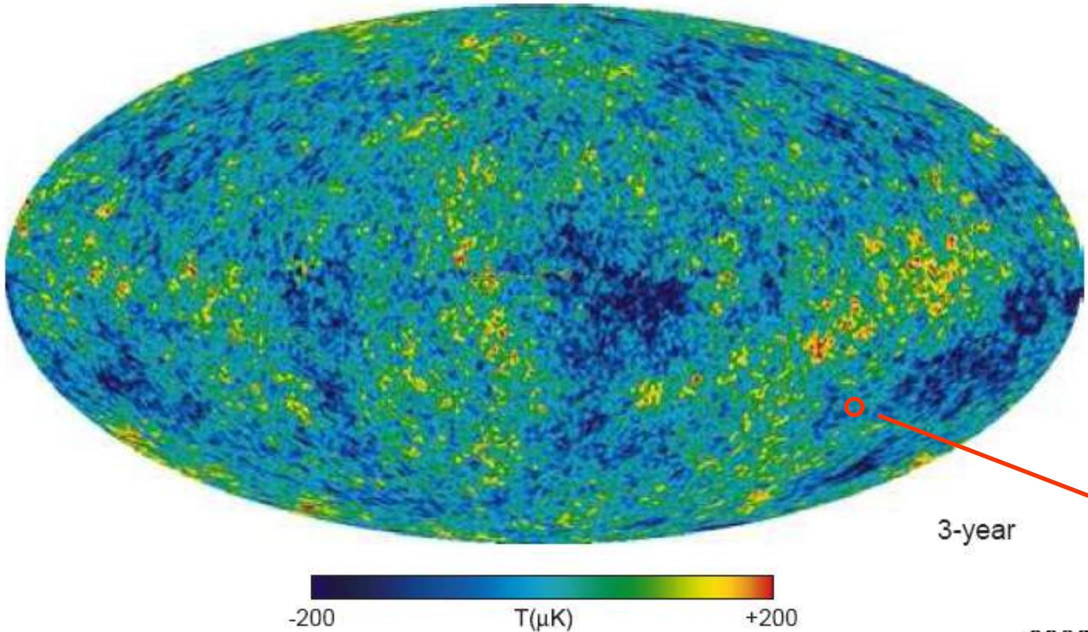
- Basic elements have been understood for (Peebles 1970, Sunyaev and Zeldovich 1970)

- Numerical codes agree at better than 0.1%

(e.g. CMBFAST, CAMB … Seljak *et al.* 2003)

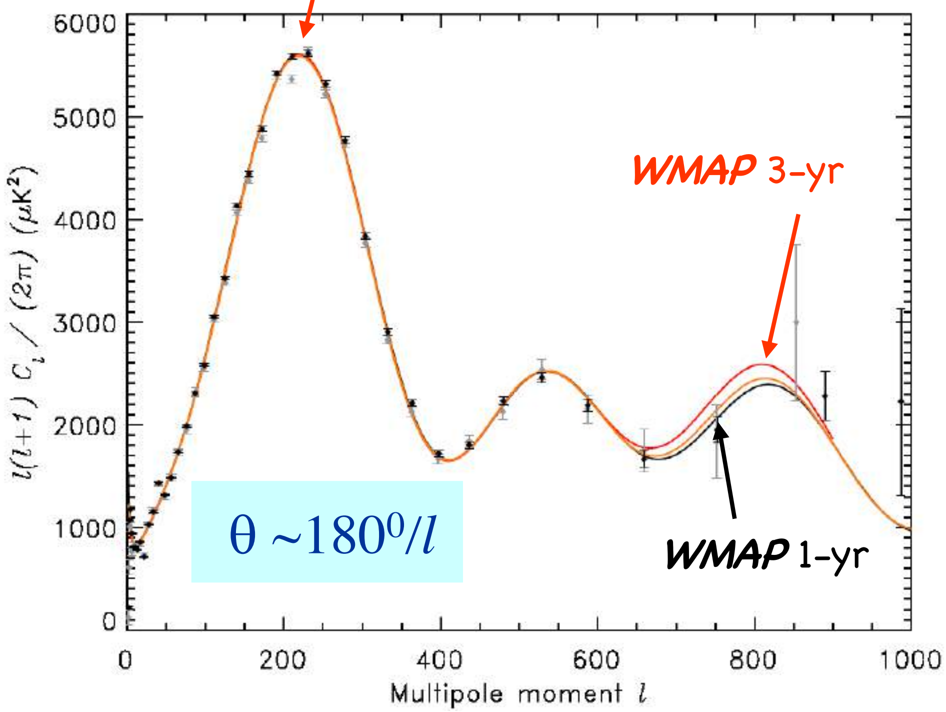


‘Internal Linear Combination’ map



Coherent oscillations in photon-baryon plasma, excited by primordial density perturbations on *super*-horizon scale

(Hubble radius at t_{rec})



$$\Delta T(\mathbf{n}) = \sum a_{lm} Y_{lm}(\mathbf{n})$$

$$C_l \equiv \frac{1}{2l+1} \sum |a_{lm}|^2$$

C_l’ s mildly *correlated* since (due to Galactic foreground) only ~85% of sky can be used

The CMB angular power spectrum is sensitive to cosmological parameters

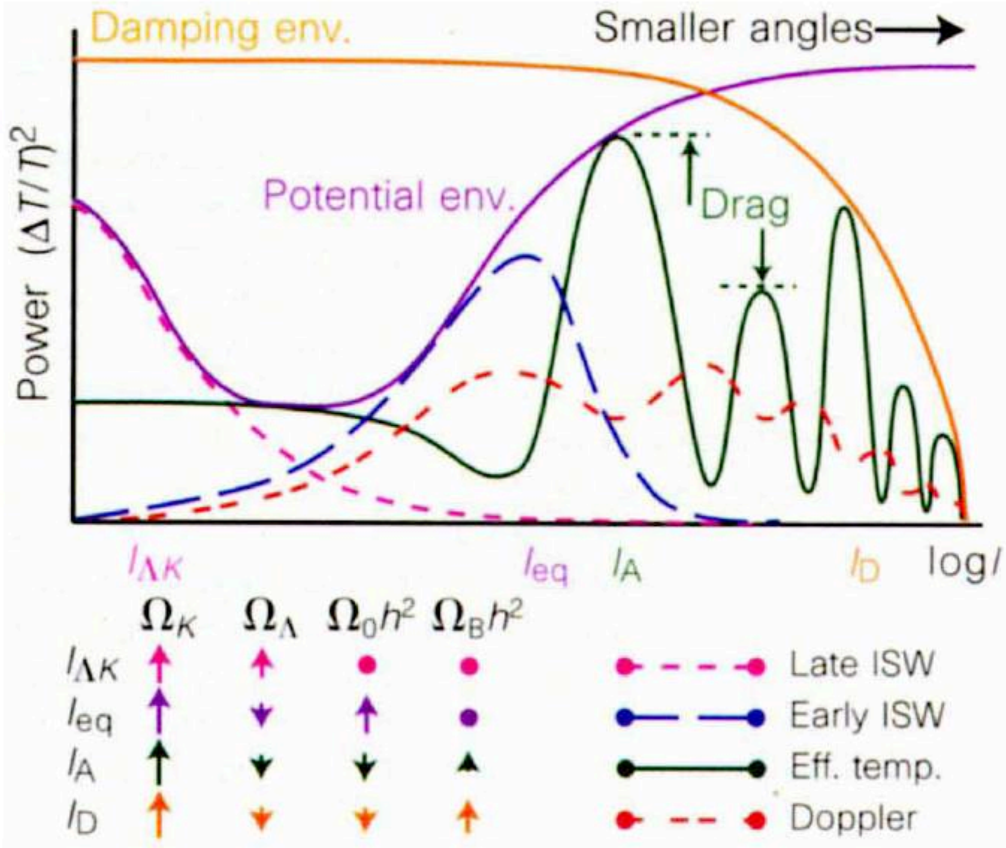


Figure 1 Schematic decomposition of the anisotropy spectrum and its dependence on cosmological parameters, in an adiabatic model. Four fundamental angular scales characterized by the angular wavenumber $l \propto \theta^{-1}$ enter the spectrum: $l_{\Lambda K}$ and l_{eq} which enclose the Sachs-Wolfe plateau in the potential envelope, l_A the acoustic spacing, and l_D the diffusion damping scale. The inset table shows the dependence of these angular scales on four fundamental cosmological parameters: $\Omega_K (\equiv 1 - \Omega_\Lambda - \Omega_0)$, Ω_Λ, $\Omega_0 h^2$ and $\Omega_B h^2$ (see Box 1 for definitions). Baryon drag enhances all compressional (here, odd) maxima of the acoustic oscillation, and can probe the spectrum of fluctuations at last scattering and/or $\Omega_B h^2$. Projection effects smooth Doppler more than effective-temperature features.

Hu, Sugiyama, Silk, Nature **40**:171,2002

SO THIS IS HOW PRETTY PICTURES SUCH AS THIS ONE ARE ACTUALLY CONSTRUCTED