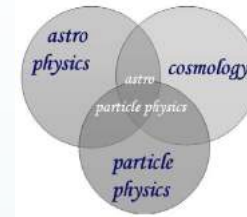




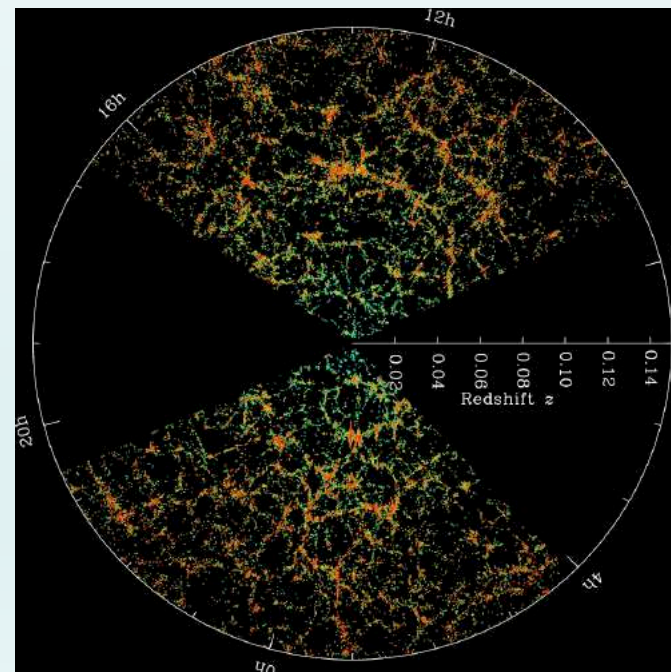
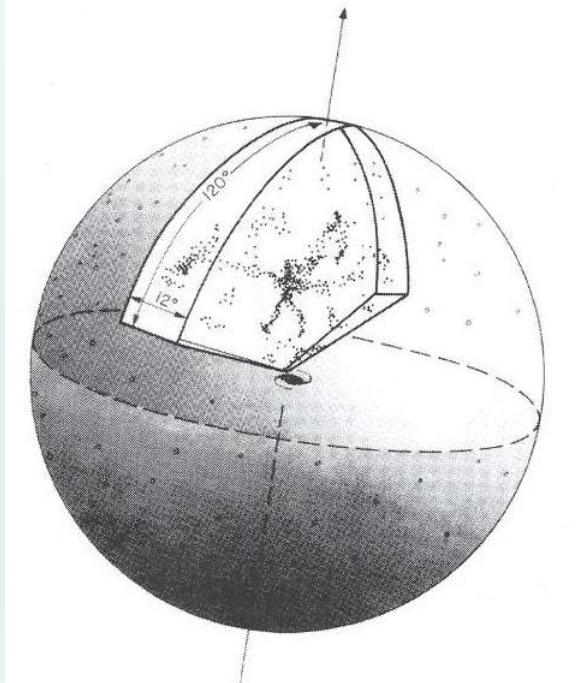
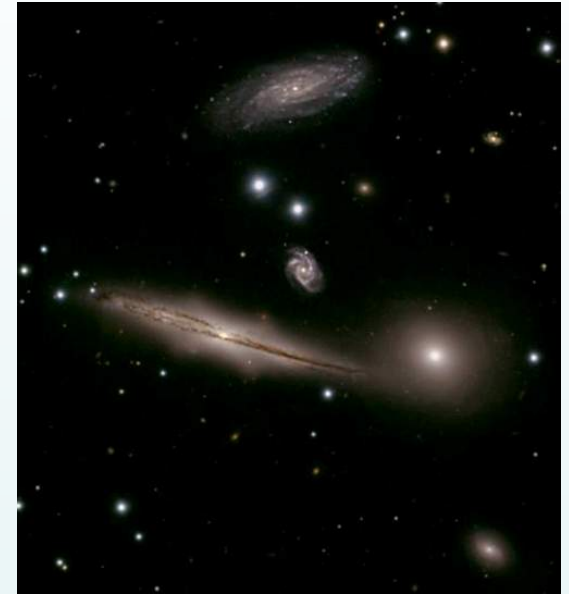
Hilary 2021



Oxford Master Course in Mathematical and Theoretical Physics

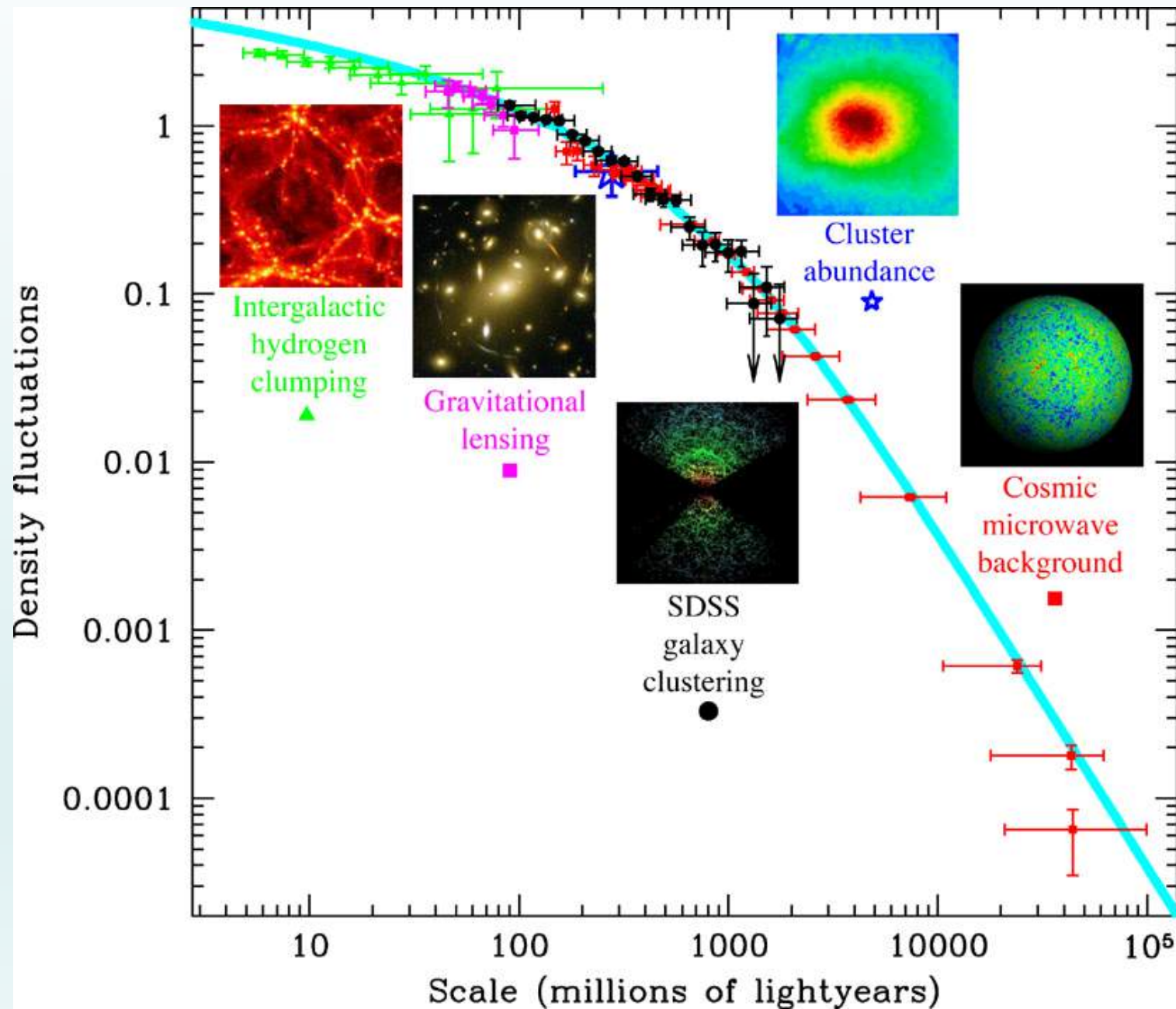
- ✧ **The universe observed**
- ✧ Relativistic world models
- ✧ Reconstructing the thermal history
- ✧ Big bang nucleosynthesis
- ✧ Dark matter: astrophysical observations
  - ✧ Dark matter: relic particles
  - ✧ Dark matter: direct detection
  - ✧ Dark matter: indirect detection
  - ✧ Cosmic rays in the Galaxy
  - ✧ Antimatter in cosmic rays
  - ✧ Ultrahigh energy cosmic rays
  - ✧ High energy cosmic neutrinos
- ✧ The early universe: constraints on new physics
  - ✧ The early universe: baryo/leptogenesis
- ✧ The early universe: inflation & the primordial density perturbation
  - ✧ Cosmic microwave background & large-scale structure

# THE UNIVERSE APPEARS COMPLEX & STRUCTURED ON MANY SCALES ...



How can we possibly describe it by a simple mathematical model?

ALTHOUGH THE UNIVERSE IS LUMPY, IT SEEMS TO BECOME SMOOTHER AND SMOOTHER WHEN AVERAGED OVER LARGER AND LARGER SCALES



Courtesy: Max Tegmark

## THE UNIVERSE SEEMS TO BE *ISOTROPIC* AROUND US

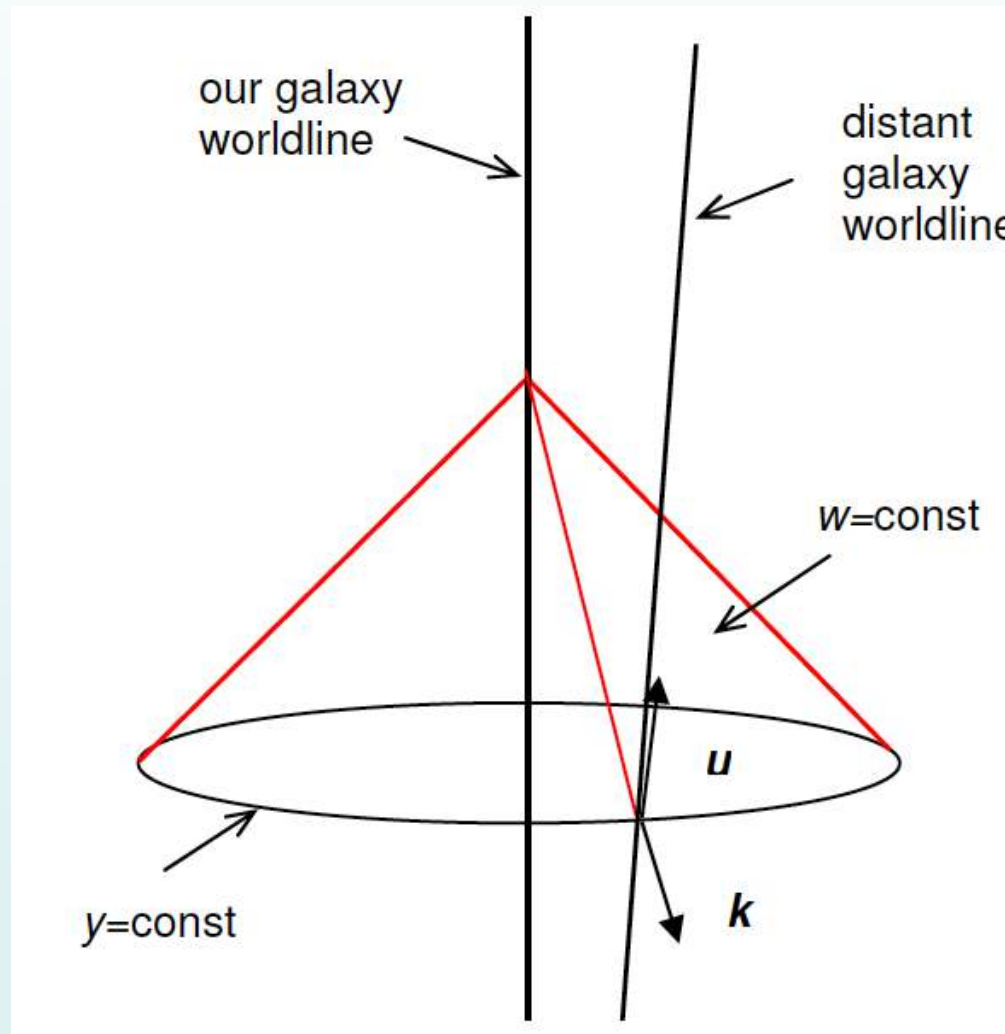
e.g. this is the distribution of the 3100 brightest radio sources at  $\lambda \sim 1-6$  cm



Peebles, *Principles of Physical Cosmology*, 1993

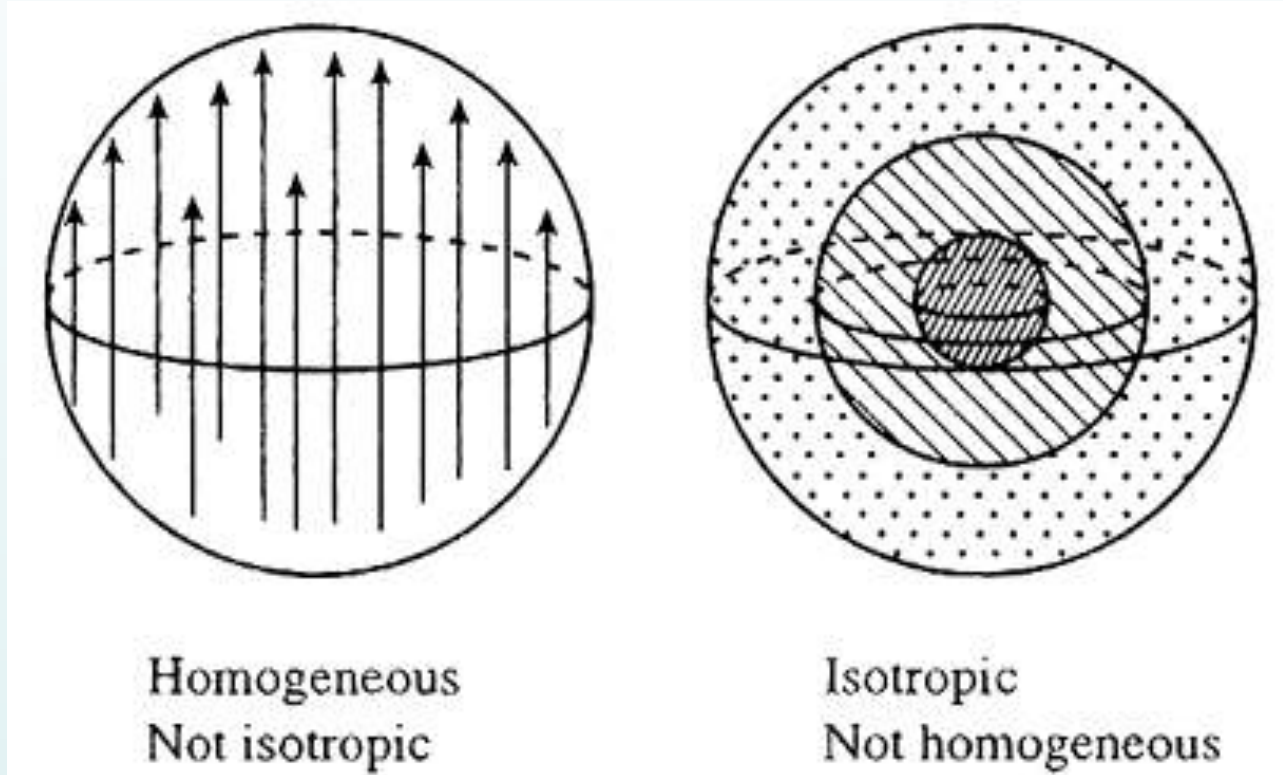
But is the universe *homogeneous*?

# ALL WE CAN *EVER* LEARN ABOUT THE UNIVERSE IS CONTAINED WITHIN OUR PAST LIGHT CONE



We *cannot* move over cosmological distances and check that the universe looks the same from 'over there' as it does from here ... so there are **limits to what we can know about the universe** ("cosmic variance")

# ISOTROPY DOES NOT IMPLY HOMOGENEITY

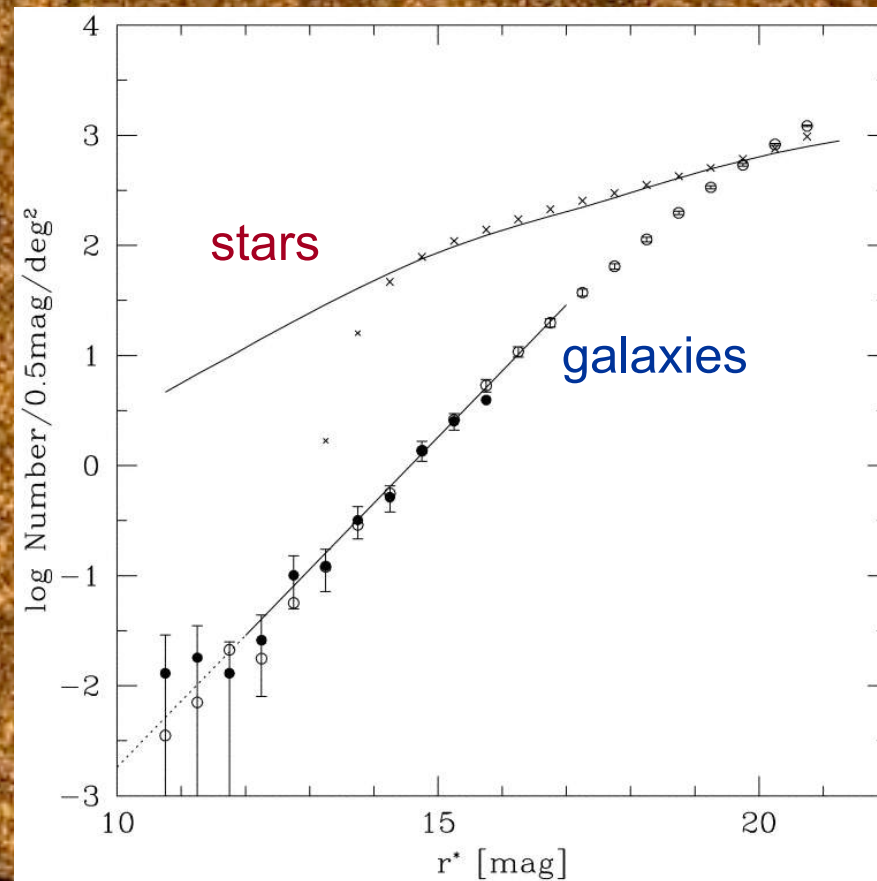


... unless it is so about *every* point in space

We cannot move (very far) in space so must *assume* that our position is typical - “The Cosmological Principle” (Milne 1935)

HUBBLE SHOWED THAT THE DISTRIBUTION OF GALAXIES IS HOMOGENEOUS

i.e.  $N(>S) \propto S^{-3/2} \Rightarrow N(<m) \propto 10^{0.6m}$ , where  $m \equiv -2.5 \log(S/S_0)$



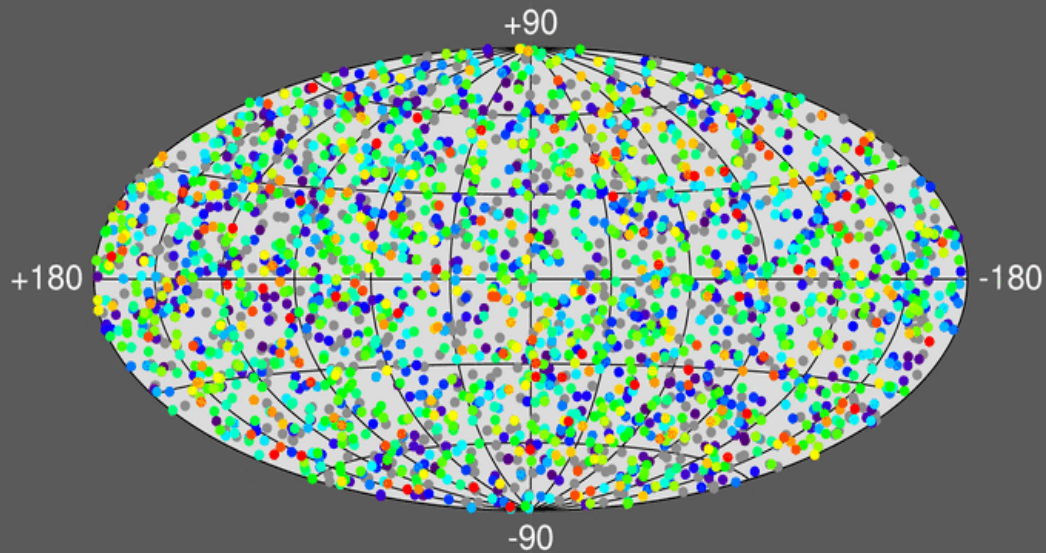
Here is the test done on galaxies in the **Sloan Digital Sky Survey**

Note that for stars,  $N(<m) \propto 10^{0.4m}$ , reflecting their 2D distribution

THIS IS A TEST ROUTINELY CARRIED OUT FOR ALL NEW CLASSES OF SOURCES

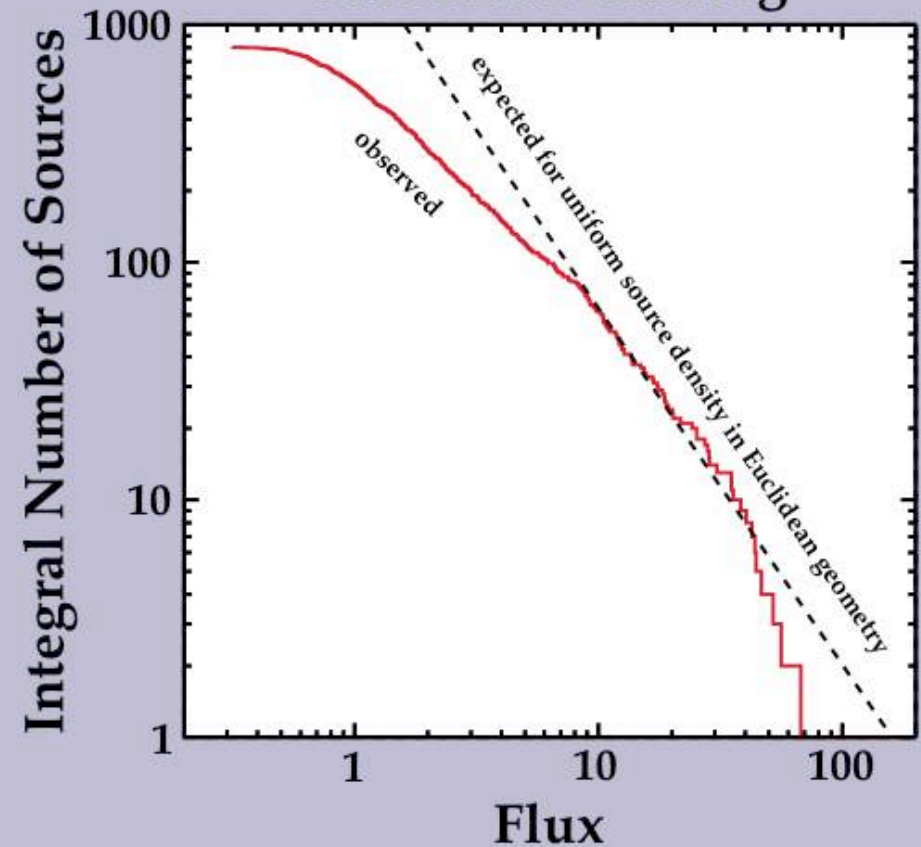
e.g. it shows that  $\gamma$ -ray bursts are homogeneously distributed  
... therefore presumably at cosmological distances

### 2704 BATSE Gamma-Ray Bursts



Note deviation from the  $S^{-3/2}$  expectation at the faint end  
- are we seeing the 'edge' of the  $\gamma$ -ray burst distribution?

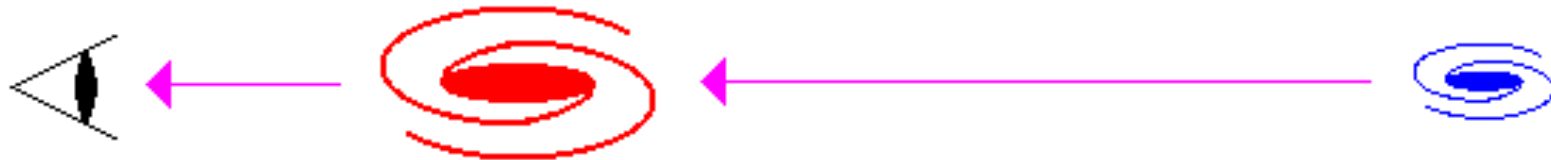
### BATSE 3B Catalog





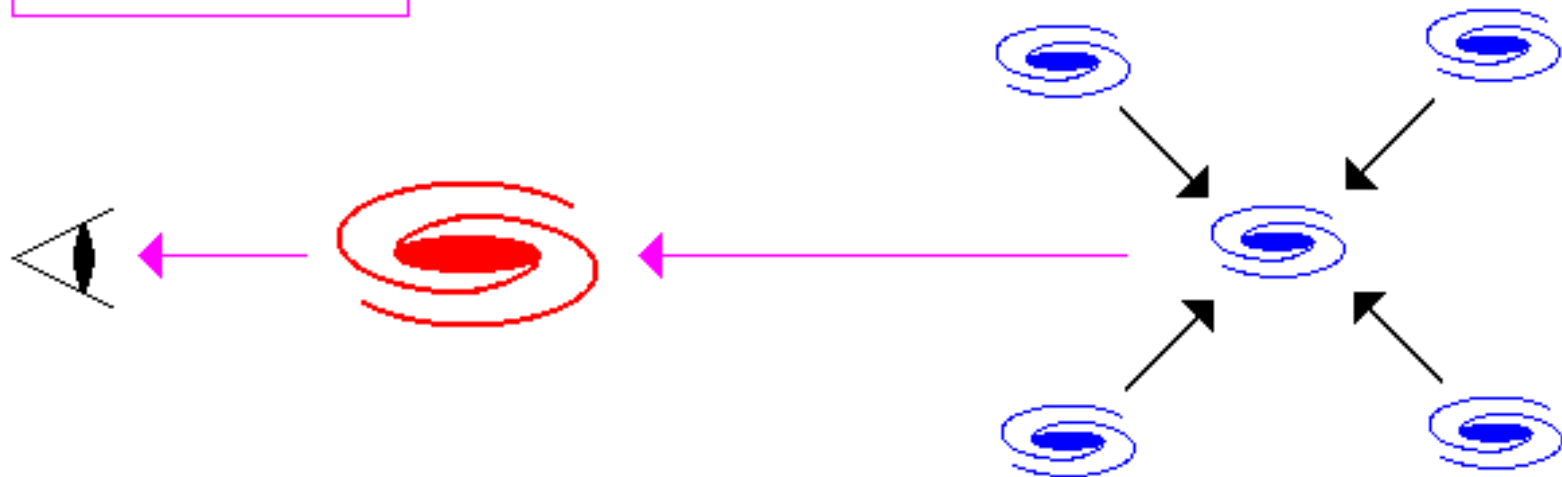
# HOWEVER SUCH TESTS ARE COMPLICATED BY *EVOLUTION EFFECTS*

## Colour Evolution



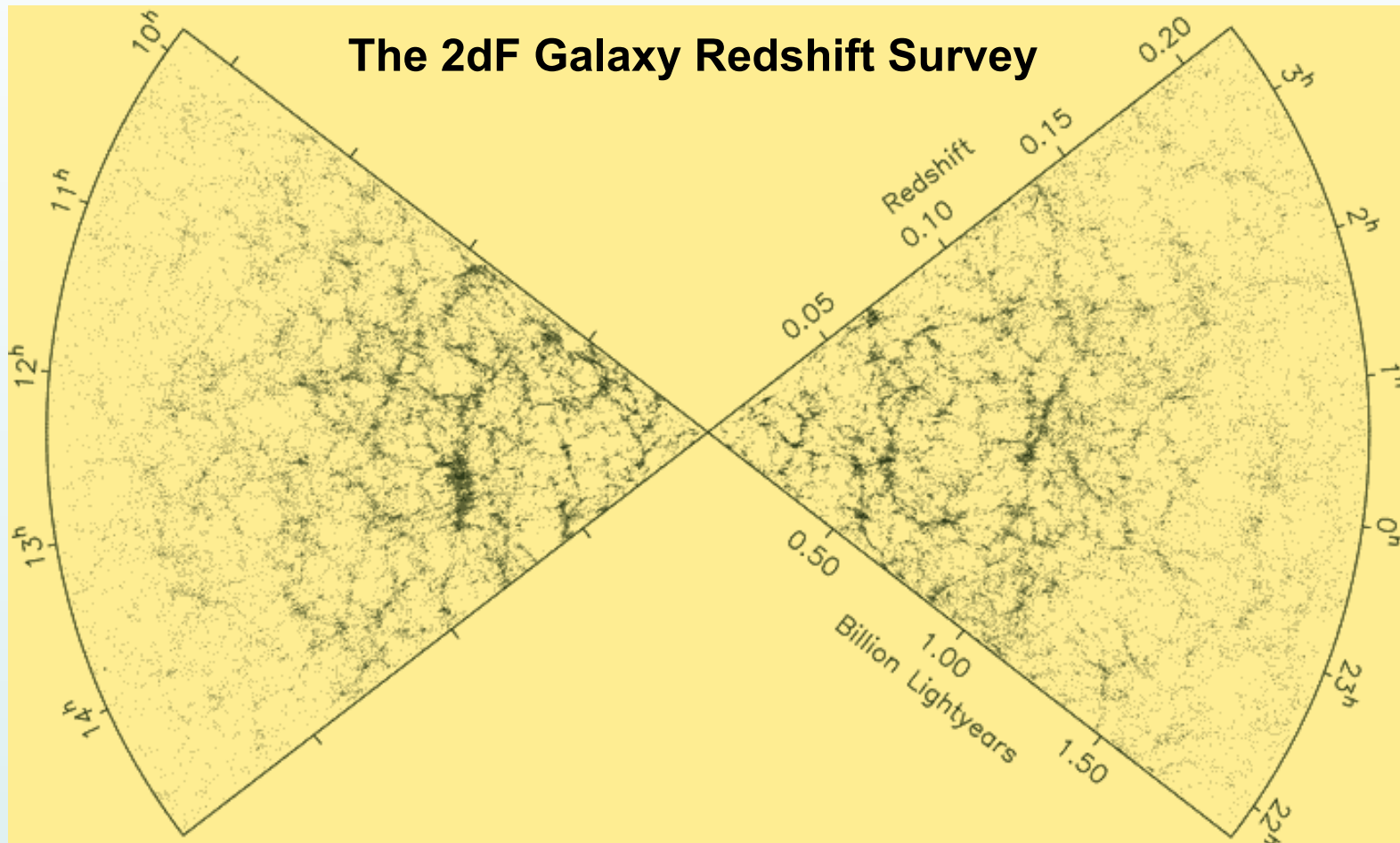
distant galaxies are bluer since we are looking back in time, and are seeing them at a younger age, younger stars = hotter stars = bluer stars

## Number Evolution



small galaxies merger at early epochs to form present-day galaxies. More galaxies are seen as we look back into the past.

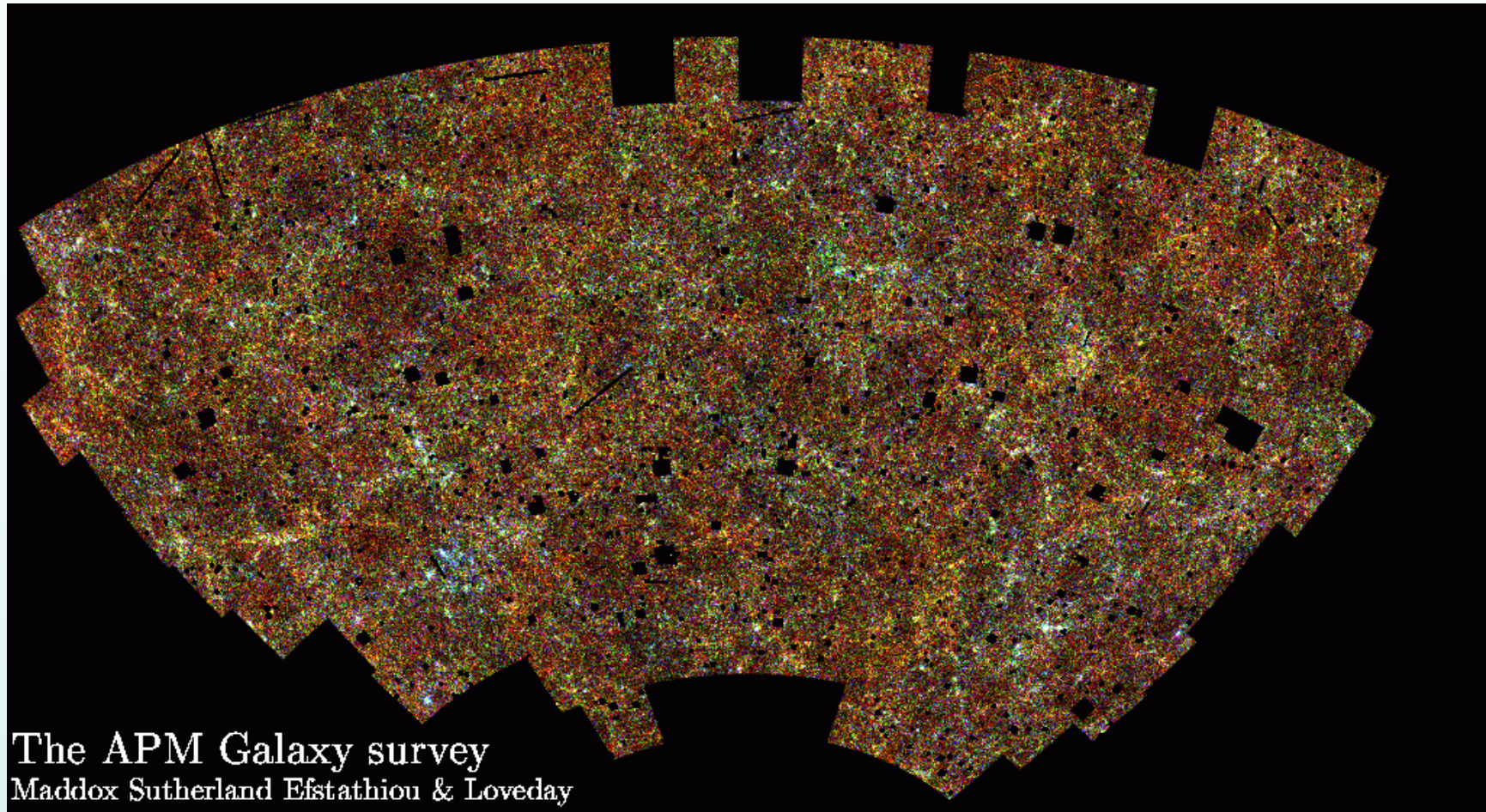
EINSTEIN “ANTICIPATED” (WITHOUT ANY DATA!) THAT THE UNIVERSE IS **HOMOGENEOUS AND ISOTROPIC** WHEN AVERAGED OVER LARGE SCALES



The galaxy distribution is in fact *fractal* on small scales ... but averaged on very large scales ( $\gtrsim 100$  Mpc) it supposedly becomes homogeneous

However there is *still* structure ('walls', 'voids') on the largest scales probed ...  
**so what is the scale of transition to homogeneity?**

**A CONSISTENCY TEST IS THE SCALING OF THE GALAXY ANGULAR CORRELATION FUNCTION WITH THE SURVEY DEPTH**



If the distribution is homogeneous on large scales (with fractional over-densities on small scales), then the characteristic **angular scale of clustering** should be *smaller* for fainter galaxies - which are on average further away - than for the (nearby) brighter ones ...

THIS IS INDEED FOUND TO BE THE CASE IN THE APM SURVEY WHICH MEASURED THE POSITIONS OF 2 MILLION GALAXIES EXTENDING TO  $\sim 600$  MPC

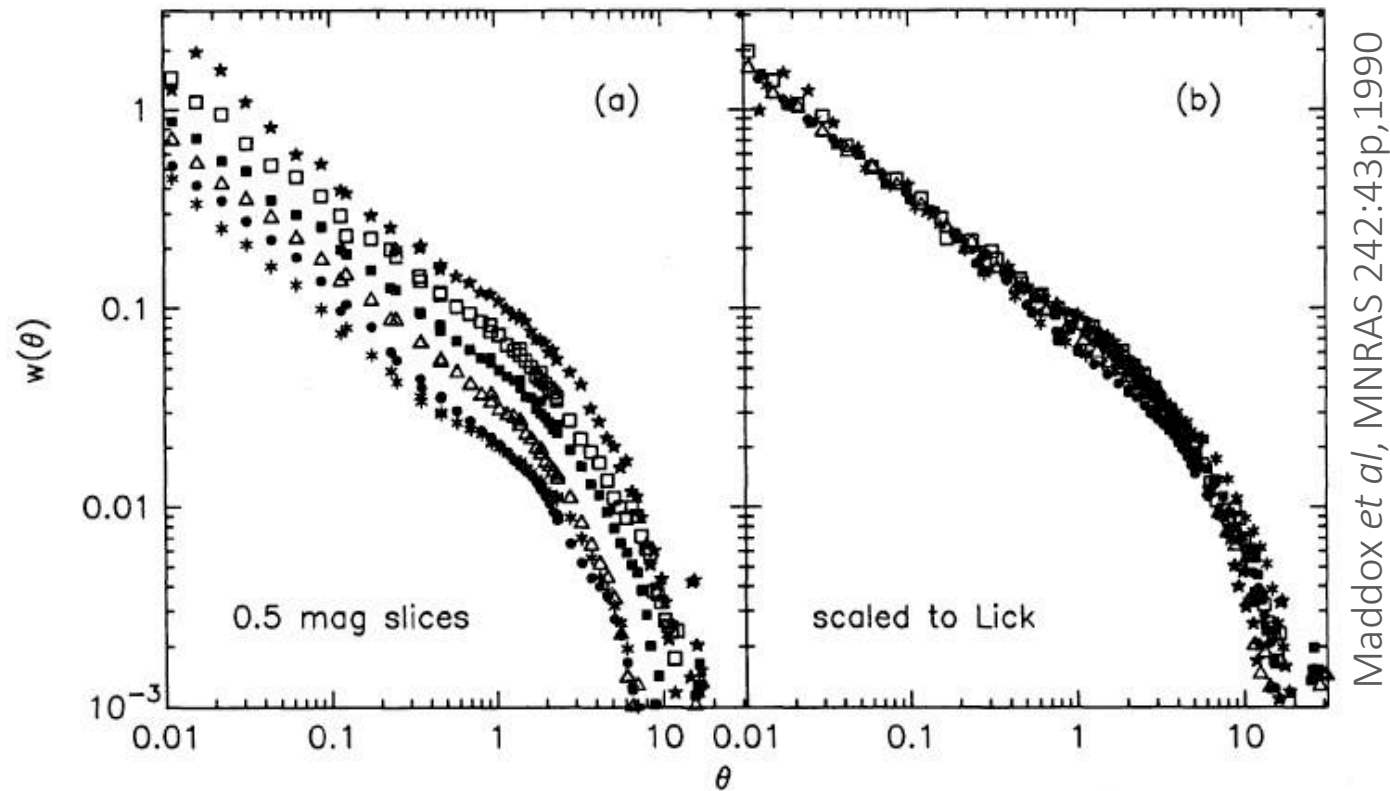


Figure 2. (a) Shows angular correlation functions for six 0.5 mag slices in the range  $17.5 \leq b_j \leq 20.5$ . (b) Shows the results from (a) scaled to the depth of the Lick survey as described in the text.

The **angular correlation function**  $w(\theta)$  - defined as the *excess* probability over average of finding two galaxies within an angle  $\theta$  of each other - does scale with the depth of the survey  $D_*$  as:  $w(\theta) = (r_0/D_*) W(\theta D_*/r_0)$  ... as is expected for a homogeneous distribution (with clustering scale  $r_0$ )  
 For a fractal distribution (with *no* intrinsic scale),  $w(\theta)$  would *not* change with  $D_*$

Equivalently the probability of finding 2 galaxies at a distance  $r$  from each other is:

$$dP_{1,2} = n^2[1 + \xi(r)]dV_1dV_2$$

The two-point correlation function (2PCF) typically falls as a power-law:

$$\xi(r) = \left(\frac{r_0}{r}\right)^\gamma$$

... so becomes *harder* to measure as the distribution tends towards homogeneity!

Easier to measure the conditional density:

$$\Gamma(r) = \frac{\langle n(\mathbf{r})n(\mathbf{r} + \mathbf{x}) \rangle}{\langle n \rangle}$$

related to the correlation function as:

$$\xi(r) = \frac{\Gamma(r)}{\langle n \rangle} - 1$$

But this assumes that  $\langle n \rangle$  is well-defined globally ... *not* true for fractal distribution

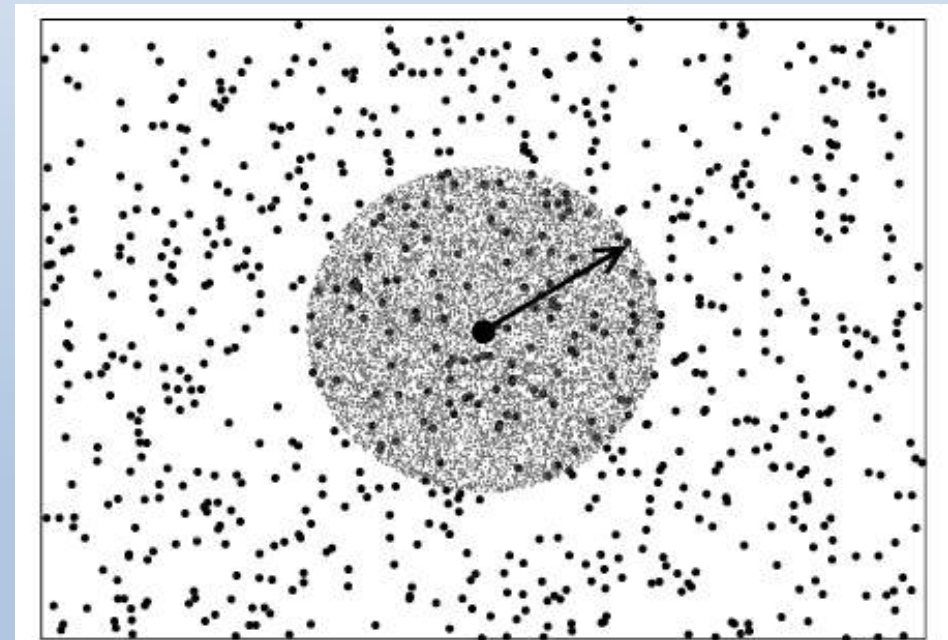


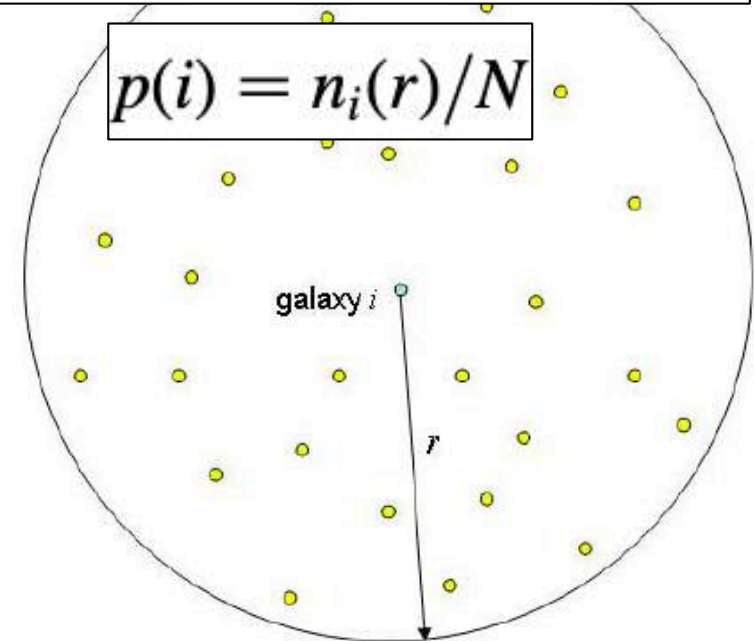
Fig. F.1. Estimation of the integrated conditional density: given a point of the distribution one counts the number of points contained in a ball of radius  $r$  and divides it by the volume  $V(r)$  of the ball. Repeating the procedure for all points in the sample for which the ball of radius  $r$  is inside the sample volume (full-shell estimator), and making the average of all these determinations, one obtains the estimation of  $\Gamma^*(r)$

HOWEVER USE OF THE 2-POINT CORRELATION FUNCTION IMPLICITLY ASSUMES HOMOGENEITY ON LARGE SCALES - IN ORDER TO DEFINE 'AVERAGE DENSITY'  
 But we ought to analyse the data without making any *prior* assumptions about the nature of the galaxy distribution (Sylos Labini, CQG 28:164003,2011)

Count number of galaxies in a spheres of different radius, centred on each galaxy in survey.

$$Z(q, r) = \frac{1}{N} \sum_{i=1}^N p_i(r)^{q-1} \propto r^{\tau(q)},$$

$$p(i) = n_i(r)/N$$



$$D_q = \frac{\tau(q)}{q-1}$$

Scaling exponent

Set of generalised dimensions

2PCF

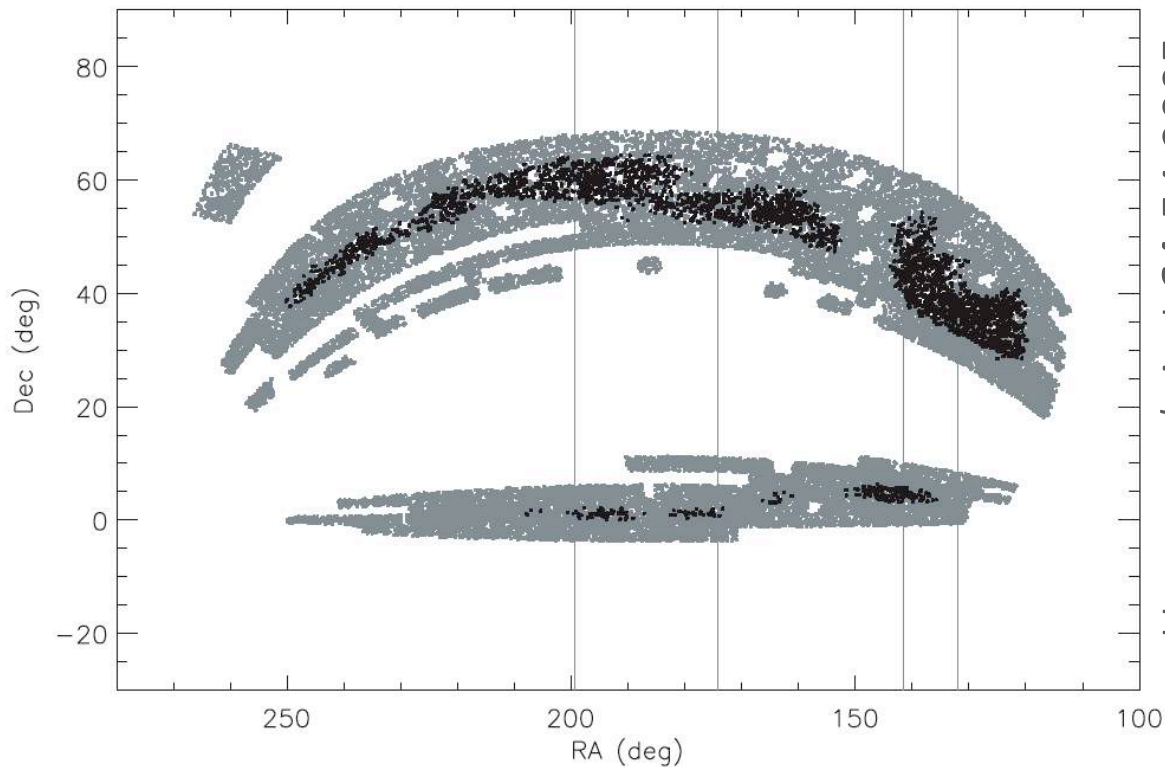
$$D_2 = 3 - \frac{d[\log \xi(r)]}{d[\log(r)]}$$

Correlation dimension

(Pan & Coles, MNRAS 318:L51,2000)

So  $\langle n \rangle \sim r^\alpha \Rightarrow \alpha = D_2$  ... and a homogeneous distribution has  $D_2 = 3$

THIS TEST WAS FIRST PERFORMED ON A SAMPLE OF 3658 LUMINOUS RED GALAXIES WITH  $0.2 < z < 0.4$  (OCCUPYING  $\sim 2 \text{ GPC}^3$ ) IN THE SLOAN DIGITAL SKY SURVEY



Hogg et al, ApJ 64:54, 2005

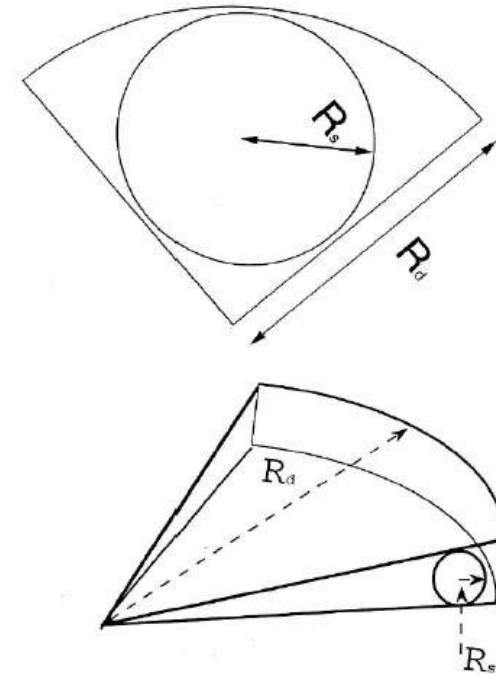
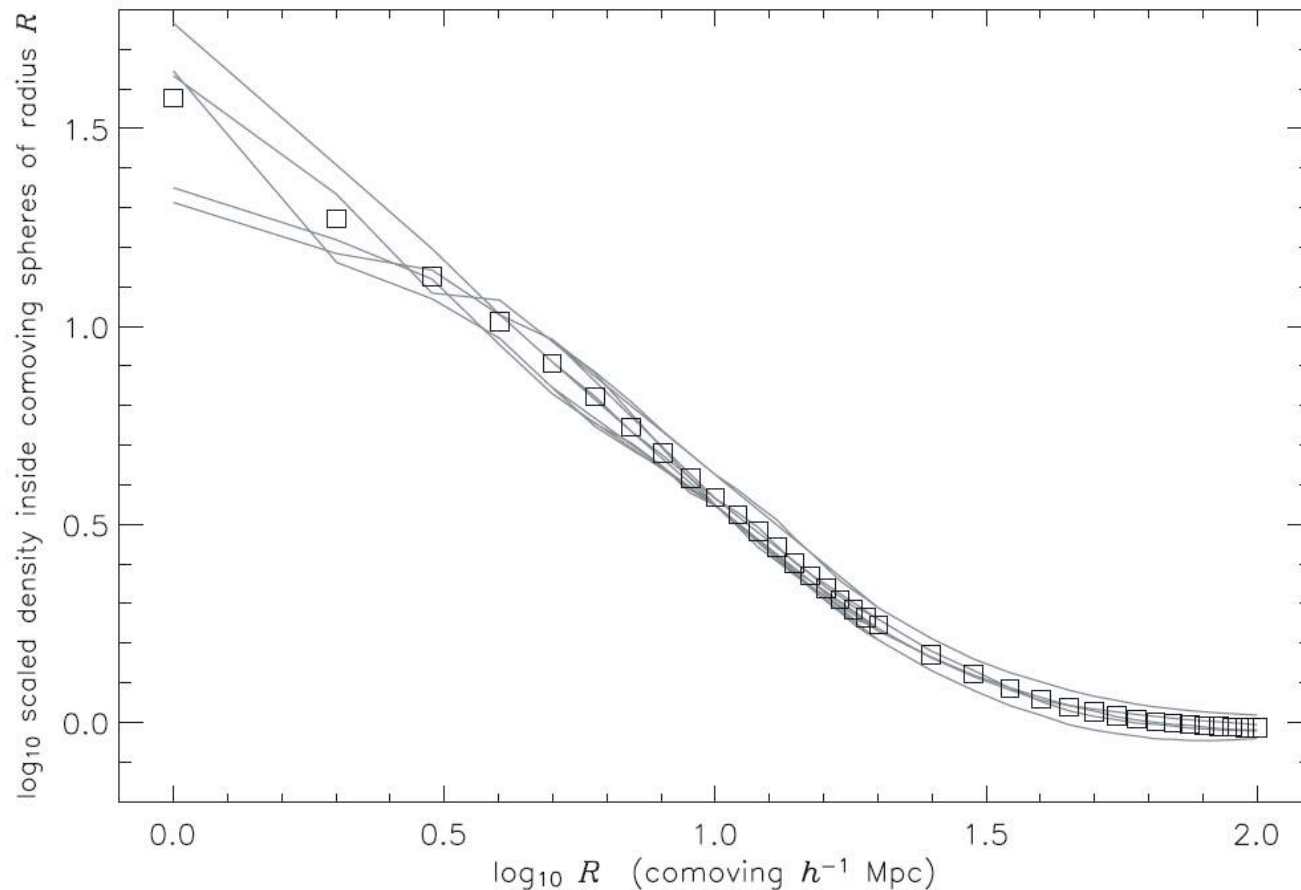


Fig. 9.6. *Upper part:* A typical cone diagram for a wide angle galaxy catalog (e.g. CfA, SSRS, Perseus-Pisces). The depth of the VL sample considered is  $R_{VL} = R_d$ . The *effective depth* is  $R_s$  and it corresponds to the radius of the maximum sphere fully contained in the sample volume ( $R_s \lesssim R_d$ ). *Bottom part:* A typical cone diagram for a narrow angle galaxy catalog (e.g. LCRS, ESP). In this case  $R_s \ll R_d$

In practice typical galaxy redshift surveys are *not* well-suited for this test to be carried out ... one must take care that the test sphere is contained *within* the (usually non-optimally shaped) survey volume, and also that luminosity selection does not introduce bias

ACTUAL COUNTS IN THE **SDSS** GROW AS  $\sim r^2$  ON SMALL SCALES,  
BUT TEND TO HOMOGENEITY BEYOND  $\sim 100$  Mpc ...



Hogg *et al*, ApJ 64:54, 2005

FIG. 2.—Average comoving number density (i.e., number counted divided by expected number from a homogeneous random catalog) of LRGs inside comoving spheres centered on the 3658 LRGs shown in Fig. 1, as a function of comoving sphere radius  $R$ . The average over all 3658 spheres is shown with squares, and the averages of each of the five R.A. quantiles are shown as separate lines. At small scales, the number density drops with radius, because the LRGs are clustered; at large scales, the number density approaches a constant, because the sample is homogeneous. (for a critique see Sylos-Labini *et al*, Europhys. Lett. 86:49001, 2009)



## IN THE WIGGLEZ SURVEY, THE HOMOGENEITY SCALE IS CLAIMED TO BE $\sim 80$ MPC

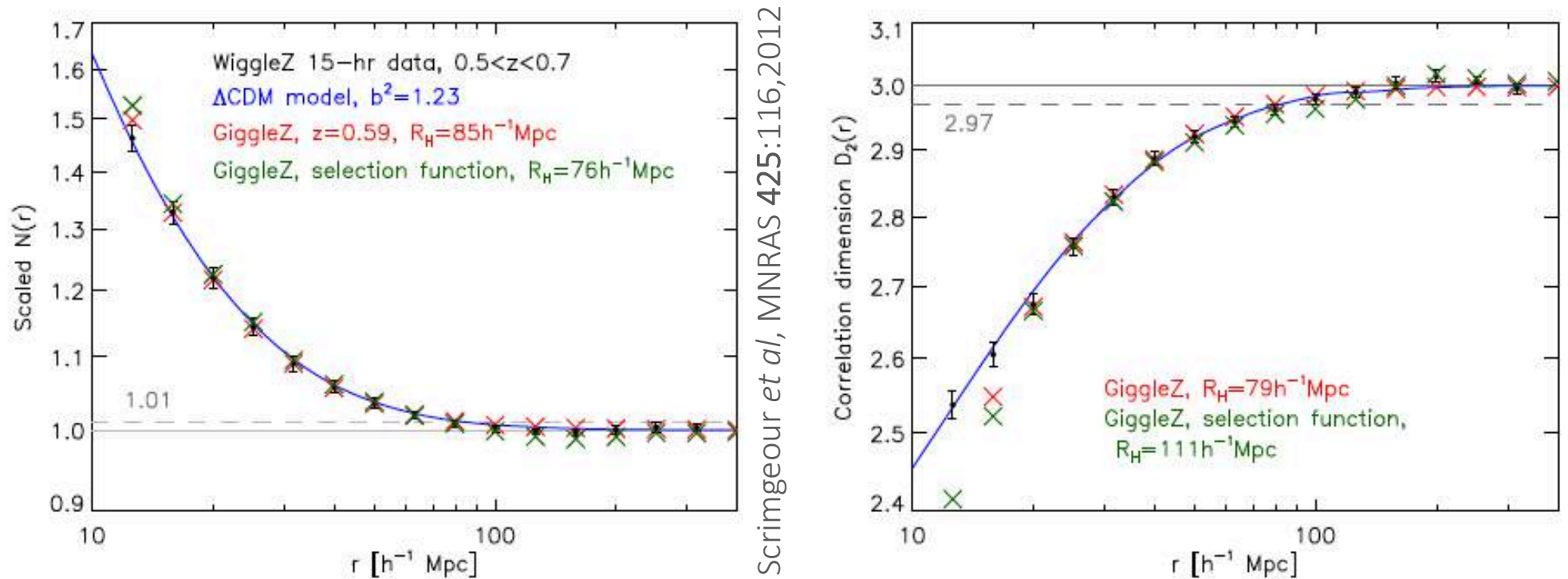


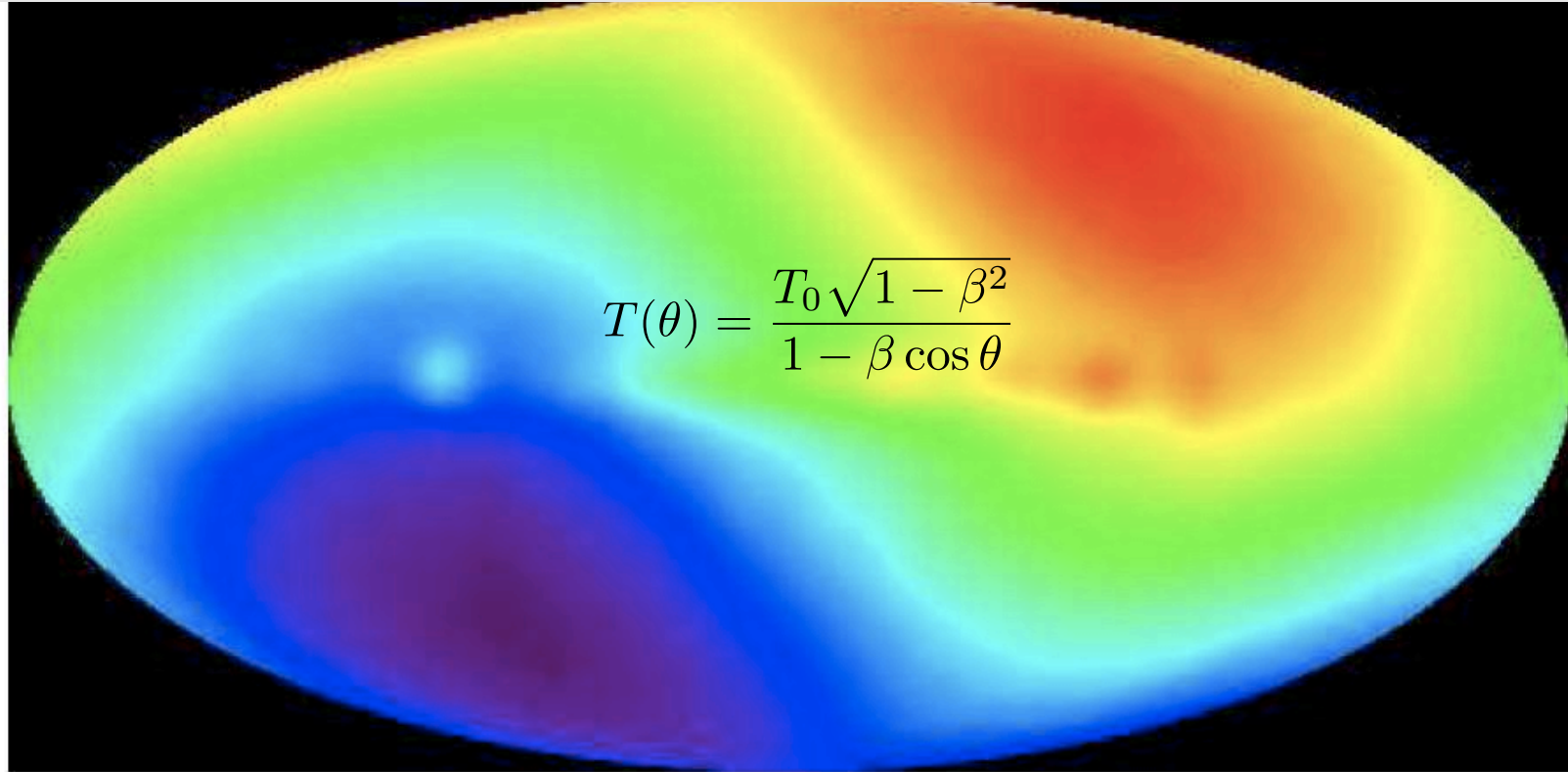
Figure 13. Comparison of the GigggleZ  $N$ -body simulation with WiggleZ, for the 15-hr region  $0.5 < z < 0.7$  redshift slice. The  $N(< r)$  results are shown on the left, and  $D_2(r)$  on the right. The WiggleZ data is shown as black data points, and a  $\Lambda$ CDM model is shown in blue. The results for the full GigggleZ box are shown as the red crosses. The green crosses show the results for the GigggleZ simulation sampled with the WiggleZ 15-hr  $0.5 < z < 0.7$  selection function. The measured homogeneity scale  $R_H$  is indicated for each.

However the survey volume of WiggleZ is rather awkward ... the biggest spheres are *not* fully contained and were (effectively) filled with galaxies drawn from a random distribution - so essentially *assuming* large-scale homogeneity!

If this is indeed true, there should be *no* coherent structures or flows on scales much larger than the homogeneity scale of  $\sim 100$  Mpc

IN FACT THE SKY IS *NOT* ISOTROPIC ... THE COSMIC MICROWAVE BACKGROUND EXHIBITS A CHARACTERISTIC DIPOLE ANISOTROPY

This is believed to be due to our 'peculiar' (non-Hubble) motion



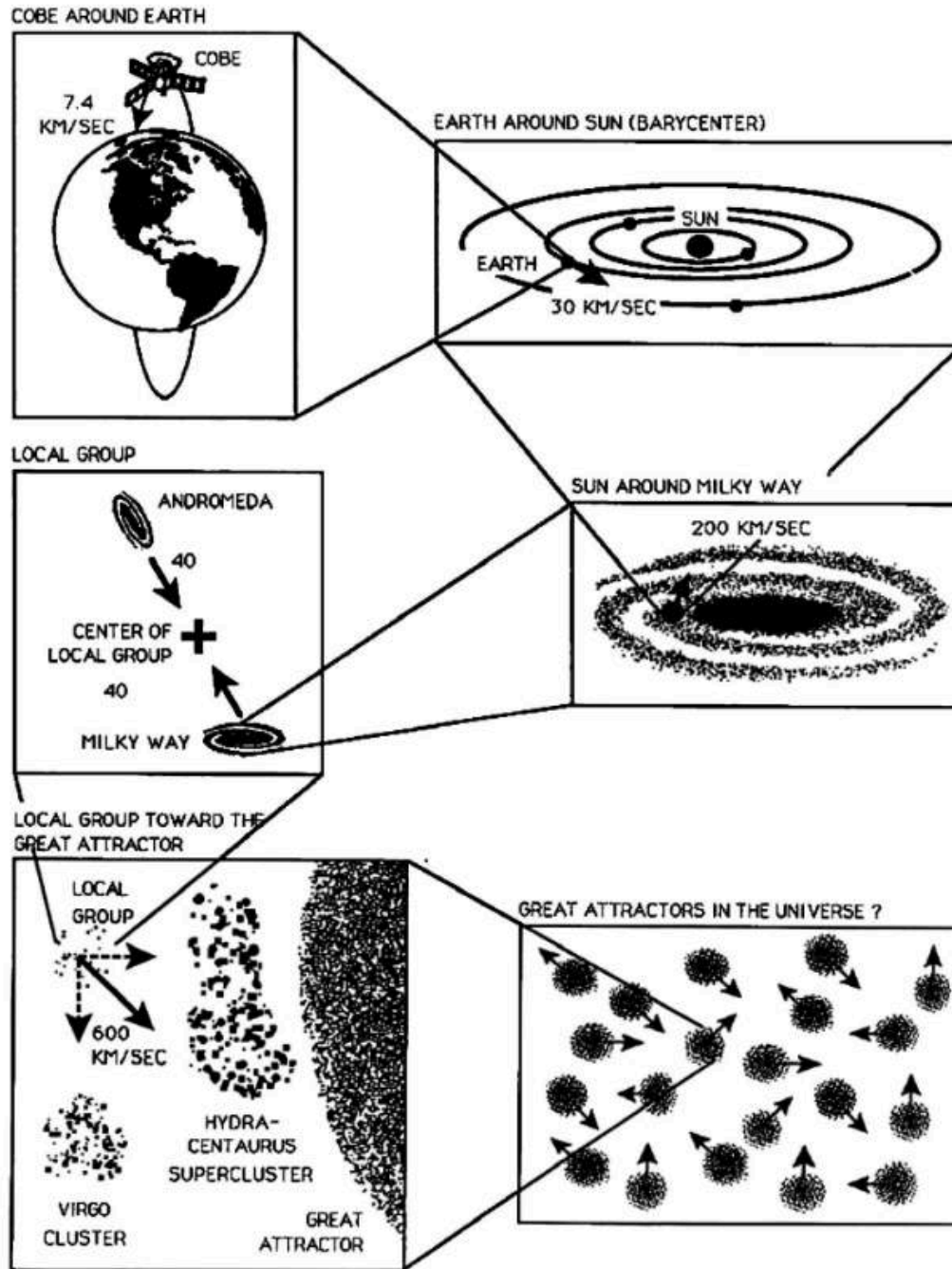
Bracewell & Conklin, Nature 219:1343,1968

Kogut et al, ApJ 419:1,1993

FIG. 18: The dipole in CMB as measured by the COBE satellite. The temperature range is  $T=2.721\text{K}$  (violet) to  $2.729\text{K}$  (red). The inferred dipole velocity of the Solar System is  $v = 368 \pm 2 \text{ km/s}$  and of the Local Group,  $v_{\text{LG}} = 627 \pm 22 \text{ km/s}$ . The CMB would be isotropic after we do a SR boost to this frame.

So the universe is *not* homogeneous locally ... but only on scales larger than the one where we converge to the CMB frame – how big is that scale?

## VELOCITY COMPONENTS OF THE OBSERVED CMB DIPOLE



## Peculiar Velocity of the Sun and its Relation to the Cosmic Microwave Background

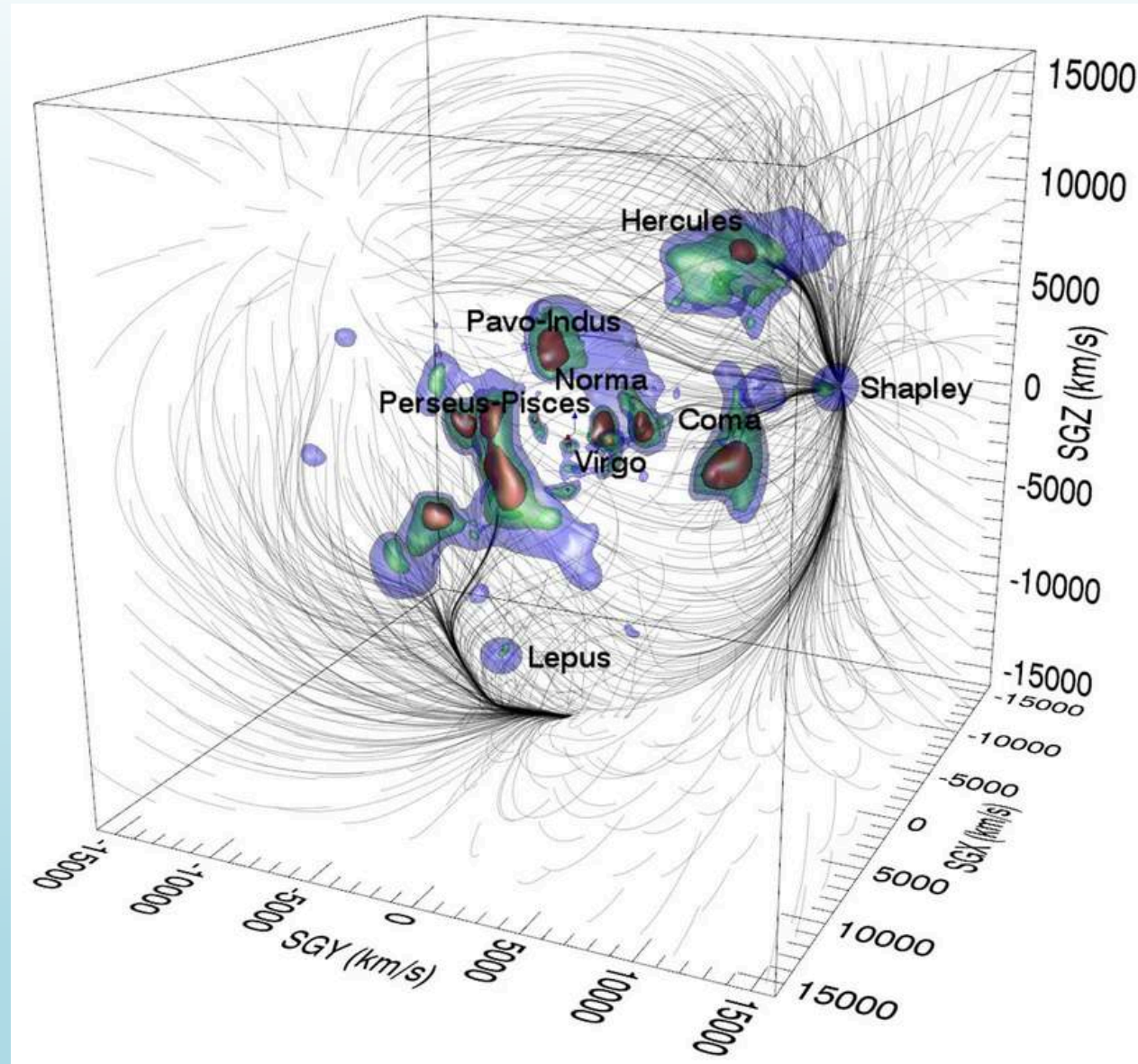
J. M. Stewart & D. W. Sciama

If the microwave blackbody radiation is both cosmological and isotropic, it will only be isotropic to an observer who is at rest in the rest frame of distant matter which last scattered the radiation. In this article an estimate is made of the velocity of the Sun relative to distant matter, from which a prediction can be made of the anisotropy to be expected in the microwave radiation. It will soon be possible to compare this prediction with experimental results.

NATURE 216, 748 (1967)

The predicted CMB dipole was found soon afterwards ... in broad *agreement* with expectations

STRUCTURE WITHIN A CUBE EXTENDING  $\sim 200$  MPC FROM OUR POSITION (SUPERGAL. COORD.)

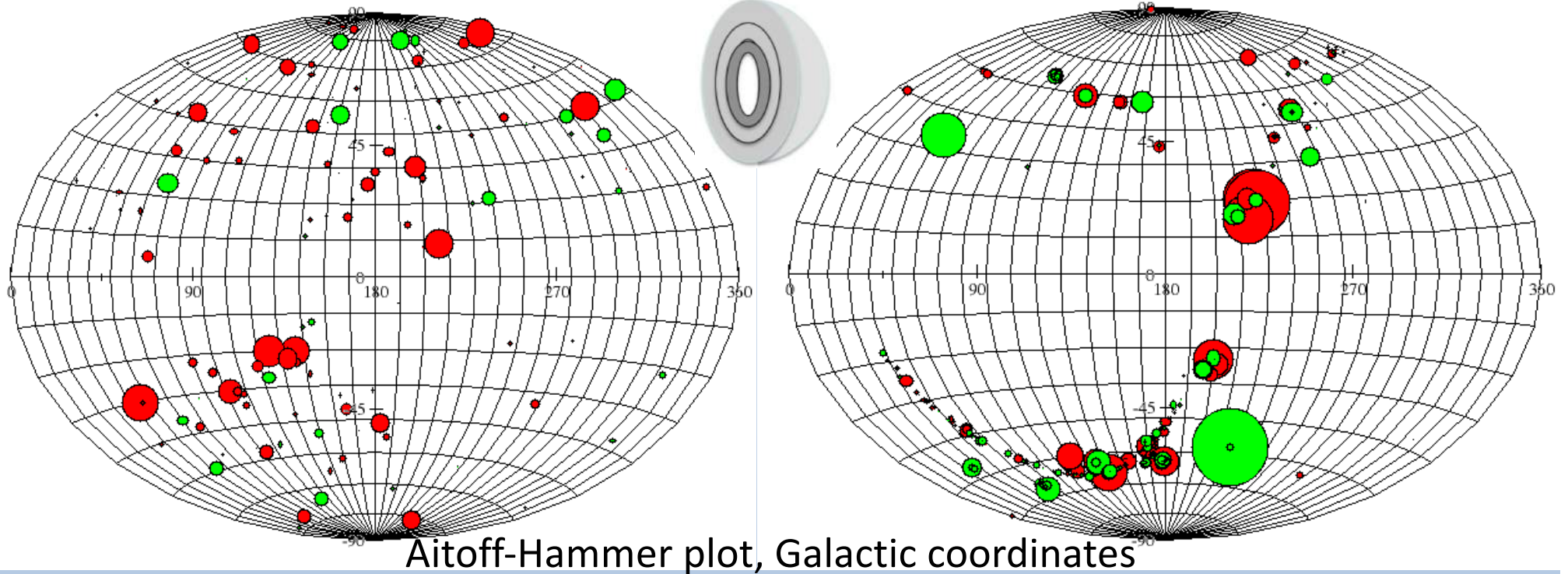


Tully, Courtois, Hoffman, Pomaredé, Nature 513:71, 2014

We appear to be moving towards the Shapley supercluster due to a ‘Great Attractor’ ... if so, our local ‘peculiar velocity’ should fall off as  $\sim 1/r$  as we “converge to the CMB frame” - in which the universe supposedly looks [Friedmann-Lemaître-Robertson-Walker](#)

WE CAN PERFORM *TOMOGRAPHY* OF THE HUBBLE FLOW BY TESTING IF THE HOST GALAXIES OF SUPERNOVAE ARE AT THE EXPECTED HUBBLE DISTANCES  
**RESIDUALS  $\Rightarrow$  'PECULIAR VELOCITY' FLOW IN LOCAL UNIVERSE**

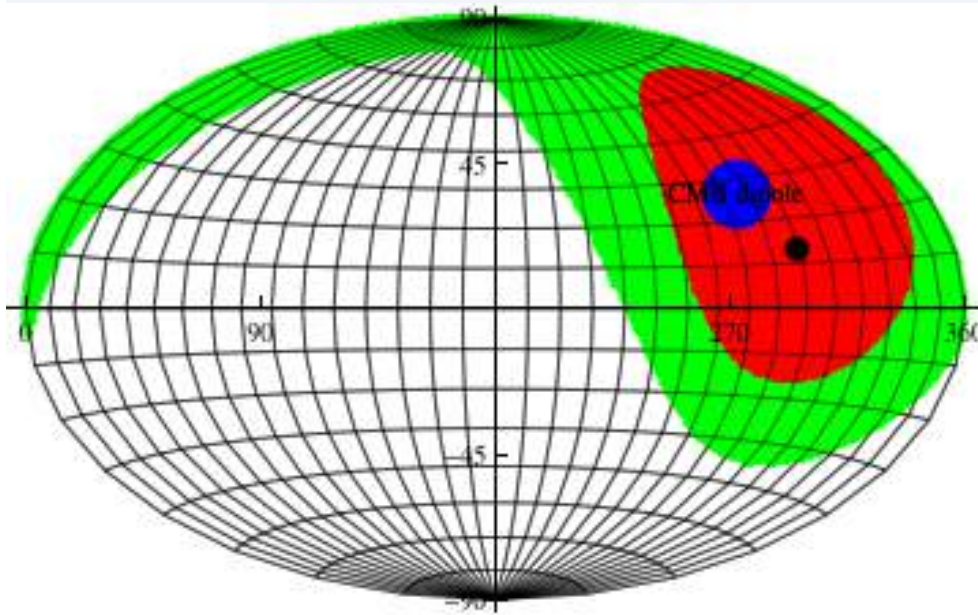
## UNION 2 COMPILATION OF 557 SNE IA



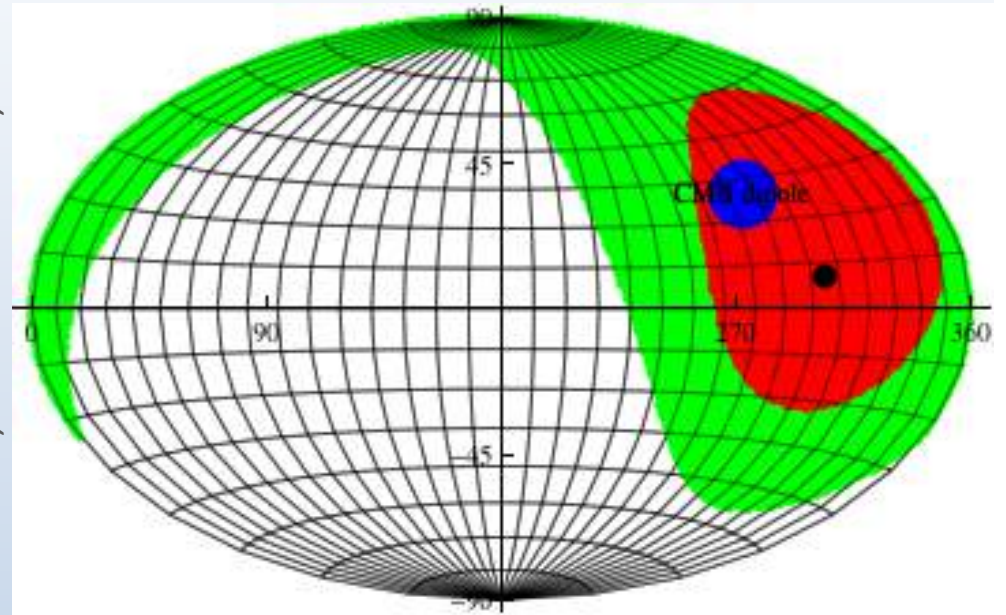
**Left panel:** The red spots represent the data points for  $z < 0.06$  with distance moduli  $\mu_{\text{data}}$  bigger than the values  $\mu_{\text{CDM}}$  predicted by  $\Lambda\text{CDM}$ , and the green spots are those with  $\mu_{\text{data}}$  less than  $\mu_{\text{CDM}}$ ; the spot size is a relative measure of the discrepancy. A dipole anisotropy is visible around the direction  $b = -30^\circ$ ,  $l = 96^\circ$  (red points) and its opposite direction  $b = 30^\circ$ ,  $l = 276^\circ$  (small green points), which is the direction of the CMB dipole.

**Right panel:** Same plot for  $z > 0.06$

# THE UNION 2 SN IA CATALOGUE EXHIBITS A DIPOLE ANISOTROPY IN THE SAME DIRECTION

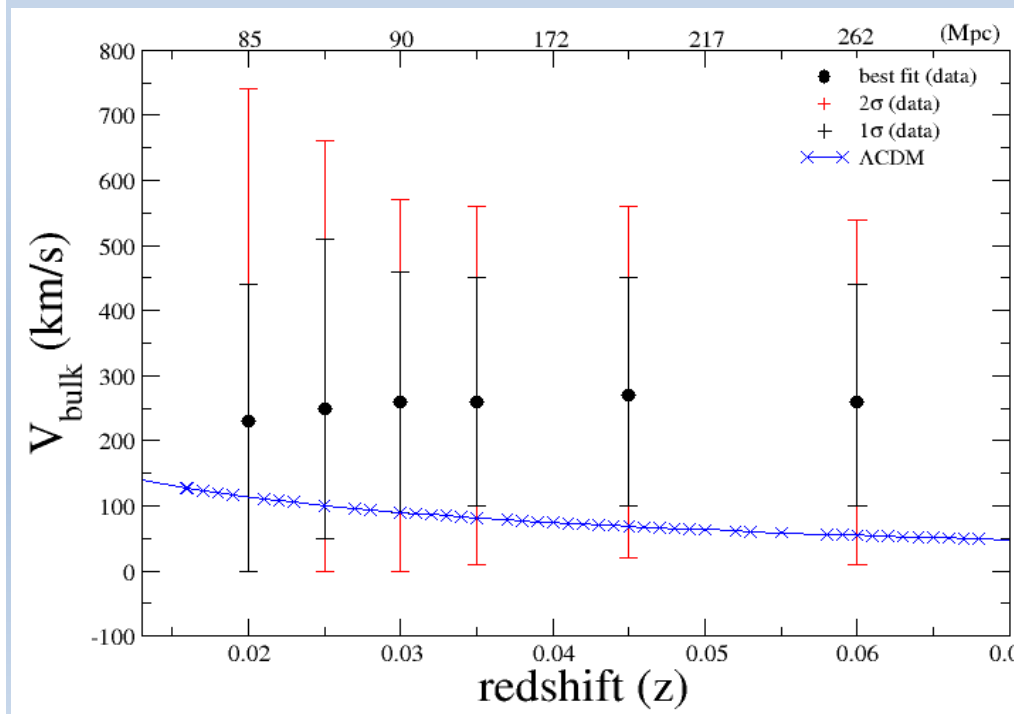


Colin et al, MNRAS 414:264,2011



$0.015 < z < 0.045$ ,  $v = 270$  km/s,  $l = 291$ ,  $b = 15$

$0.015 < z < 0.06$ ,  $v = 260$  km/s,  $l = 298$ ,  $b = 8$



Watkins *et al* MNRAS 392:743,2009 found an even higher bulk flow of  $v = 416 \pm 78$  km/s towards  $b = 60 \pm 6$ ,  $l = 282 \pm 11$  at a scale of  $\sim 100 h^{-1}$  Mpc ...

This is much higher than is expected for a gaussian random field ( $\Lambda$ CDM)

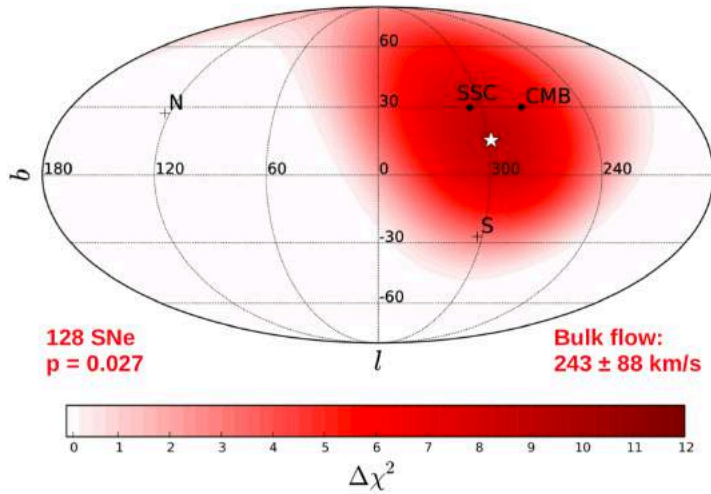
Moreover convergence to the CMB frame has *not* occurred even as far out as the Shapley supercluster ( $180 h^{-1}$  Mpc)

# Bulk Flow Analysis

## CONFIRMATION FROM

Dipole fit:  $0.015 < z < 0.035$

Full dataset: 279 SNe ( $z < 0.1$ ) from SNfactory & Union2 compilation



Bulk flow modeled as velocity dipole:

$$\vec{d}_L(z) = d_L(z) + \frac{(1+z)^2}{H(z)} \vec{n} \cdot \vec{v}_d$$

Best fit direction consistent with direction to Shapley

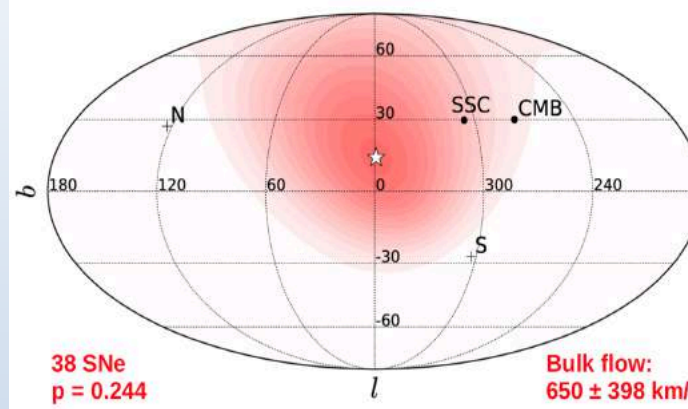
→ Amplitude matches previous studies

128 SNe  
 $p = 0.027$

Bulk flow:  
 $243 \pm 88$  km/s

# THE 'NEARBY SUPERNOVA FACTORY' SURVEY

Dipole fit:  $0.045 < z < 0.06$



No backside infall behind Shapley

- Contradicts Shapley as the main source of the bulk flow
- Results in this shell are driven by SNfactory data

38 SNe  
 $p = 0.244$

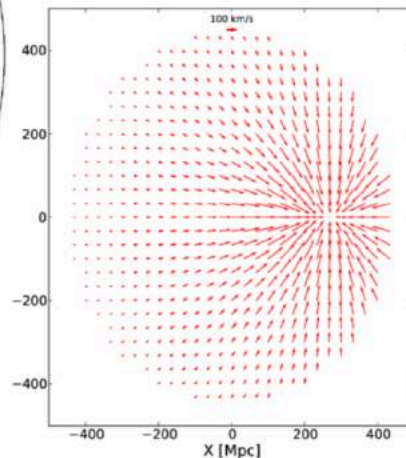
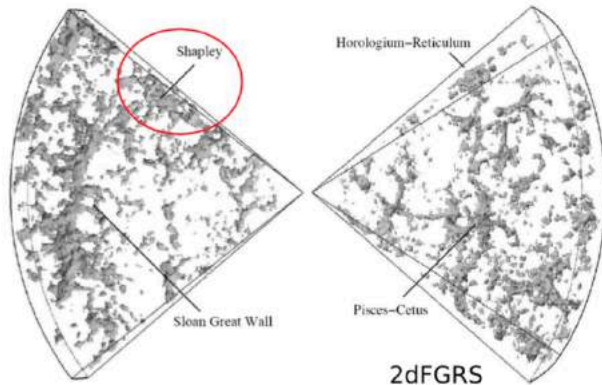
Bulk flow:  
 $650 \pm 398$  km/s

Need attractor mass of  $> 10^{17} M_{\text{Sun}}$  at  $\sim 300$  Mpc to account for the flow

Feindt *et al*, A&A 560:A90,2013

# Finding the Attractors

## Modeling the velocity field



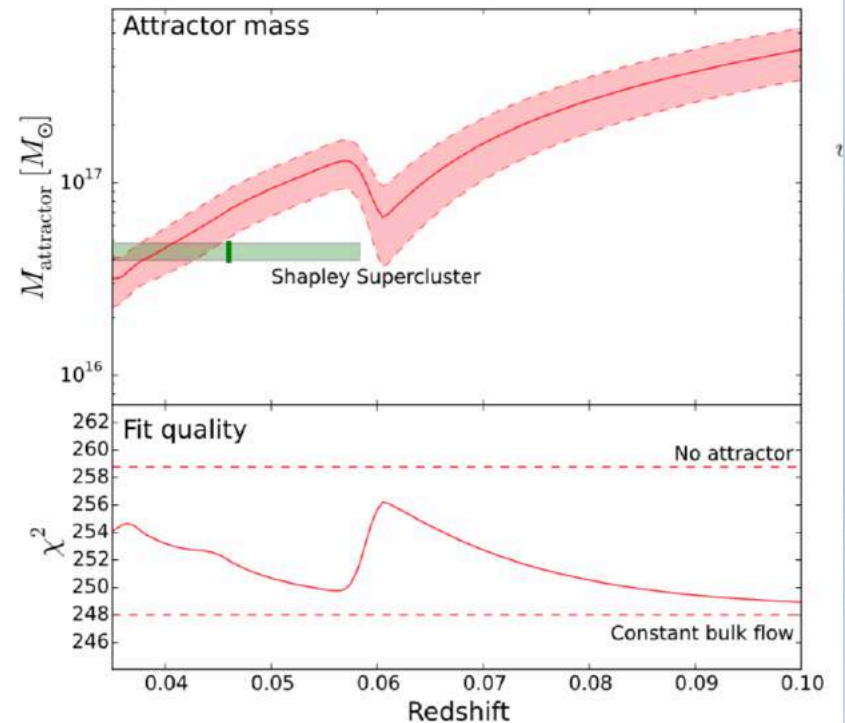
**Simplest model:**

Infall into spherical mass concentration

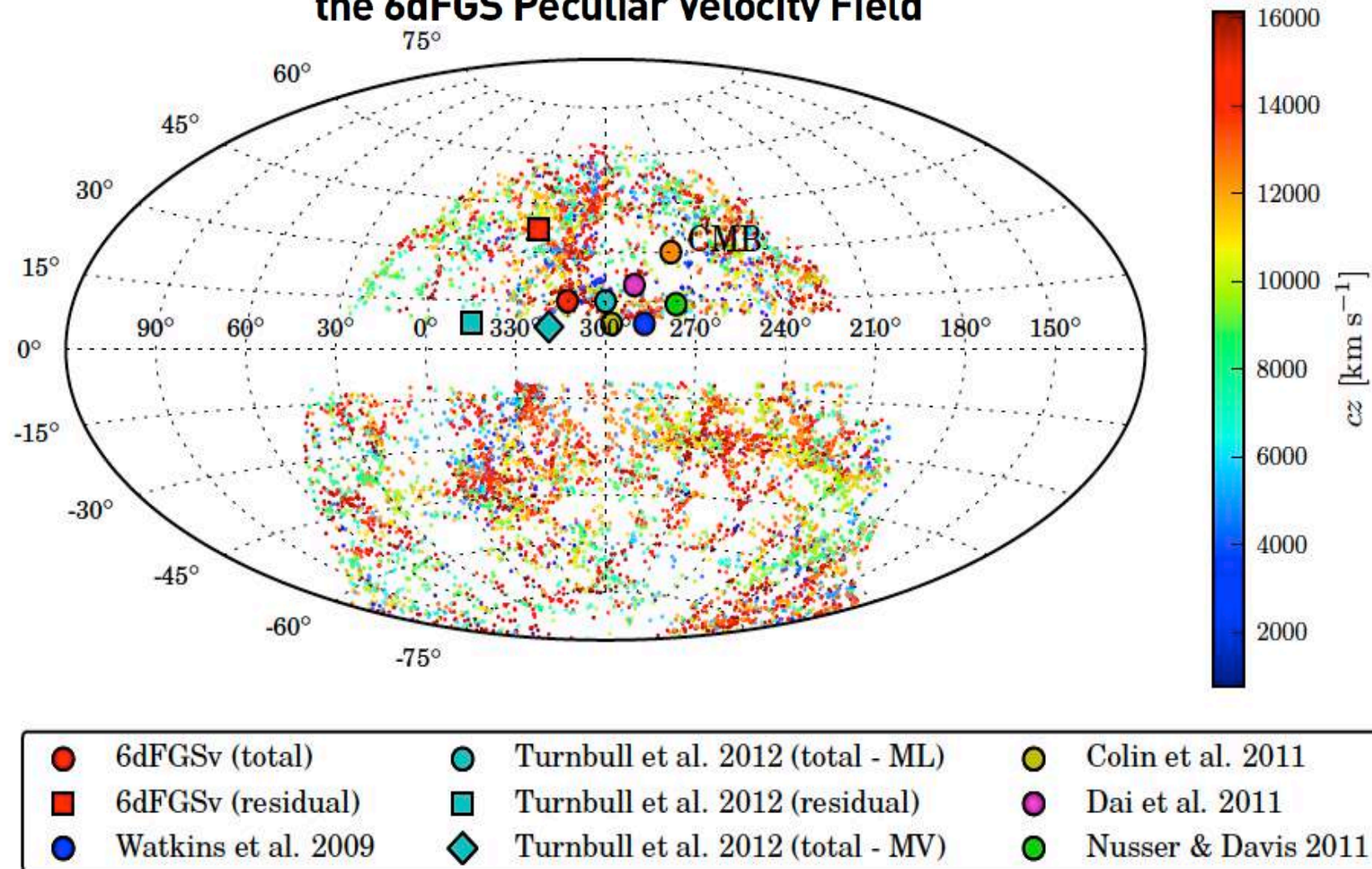
$$M_{\text{tot}} = \frac{4\pi}{3} R^3 \Omega_M \rho_{\text{crit}} (1 + \delta)$$

$$v_p(\vec{y}) = \frac{a\Omega_M^{0.55} H}{4\pi} \int \frac{\vec{y} - \vec{x}}{|\vec{y} - \vec{x}|^3} \delta(\vec{y}) d^3y$$

Courtesy: Ulrich Feindt



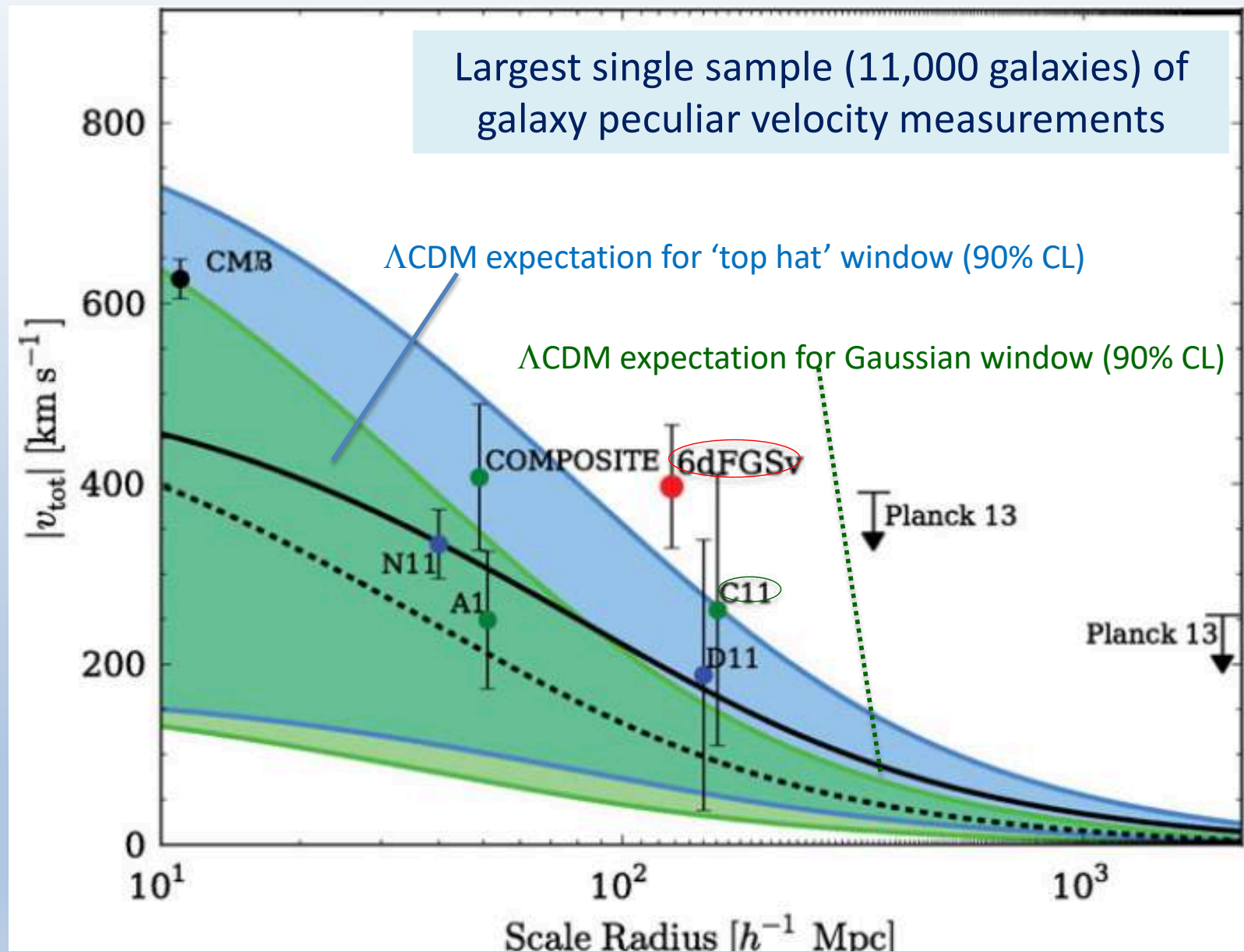
## First Cosmological Constraints from the 6dFGS Peculiar Velocity Field



**Figure 2:** Distribution of 6dFGSv galaxies in Galactic latitude ( $b$ ) and longitude ( $l$ ), shown in an equal-area Aitoff projection. Individual galaxies are colour-coded by their CMB frame redshift (in  $\text{km s}^{-1}$ ). The 6dFGSv bulk flow measurements are indicated in red for the total bulk flow of  $337 \pm 66 \text{ km s}^{-1}$  (circle) towards  $(313^\circ \pm 9^\circ, 15^\circ \pm 10^\circ)$  and the residual bulk flow of  $272 \pm 45 \text{ km s}^{-1}$  (square) in the direction of  $(326^\circ \pm 13^\circ, 37^\circ \pm 14^\circ)$ . The bulk flow measurements from various recent studies are also shown coloured according to the legend [12,13,14,15,16]. For reference we also show the direction of the Local Group motion with respect to the CMB in orange [17].



The 6dFGSv *confirms* the lack of convergence to the CMB frame  
... well beyond the 'scale of homogeneity'



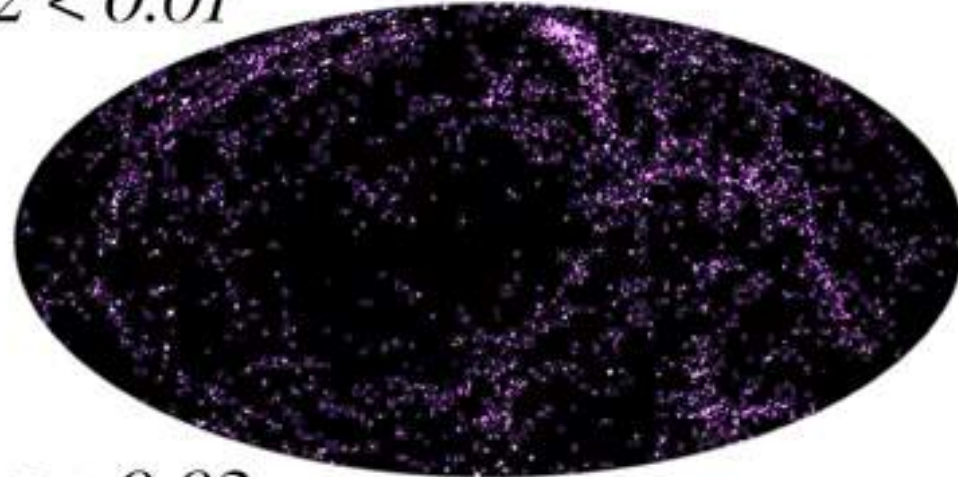
Magoulas, Springbob, Colless, Mould, et al (2016)

According to the 'Dark Sky'  $\Lambda$ CDM Hubble Volume simulations, <1% of Milky Way-like observers experience a bulk flow as large as is observed, extending out as far as is seen

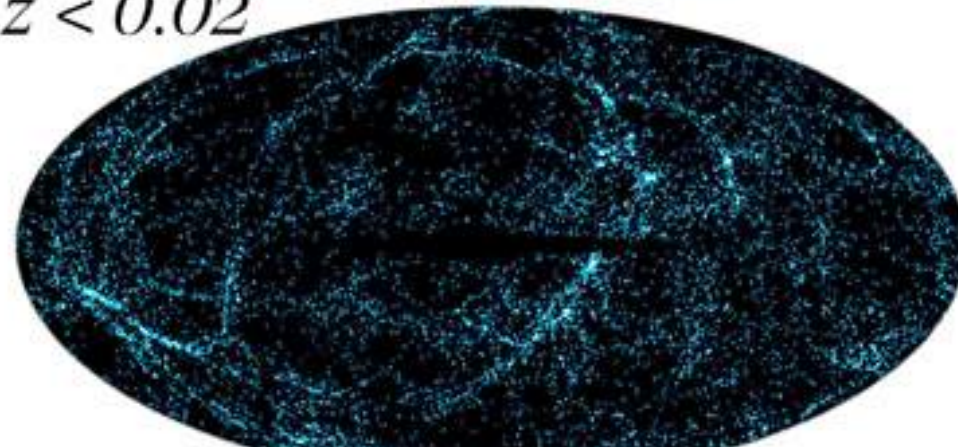
*Milky Way*



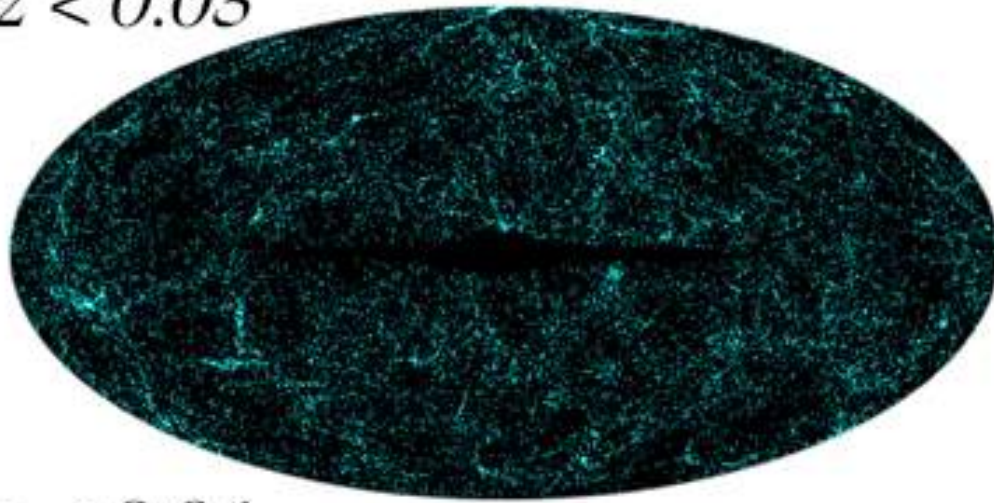
$z < 0.01$



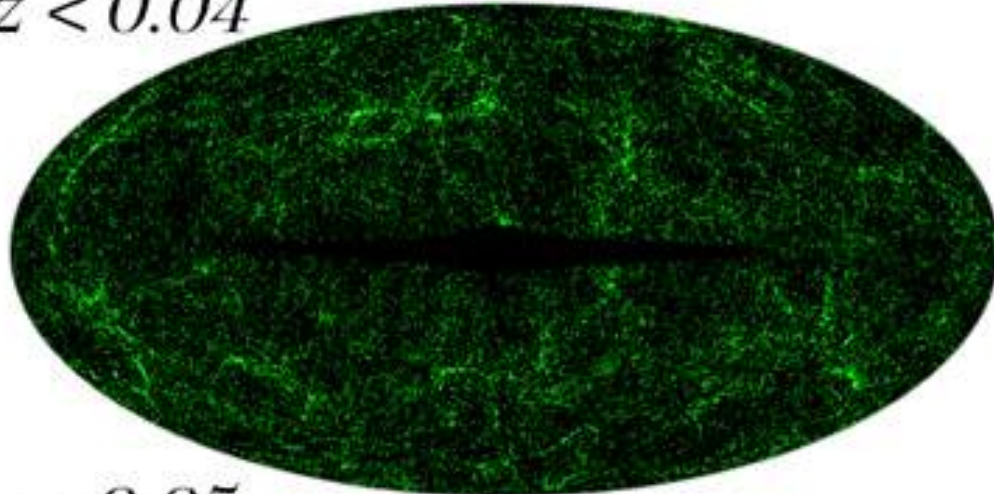
$0.01 < z < 0.02$



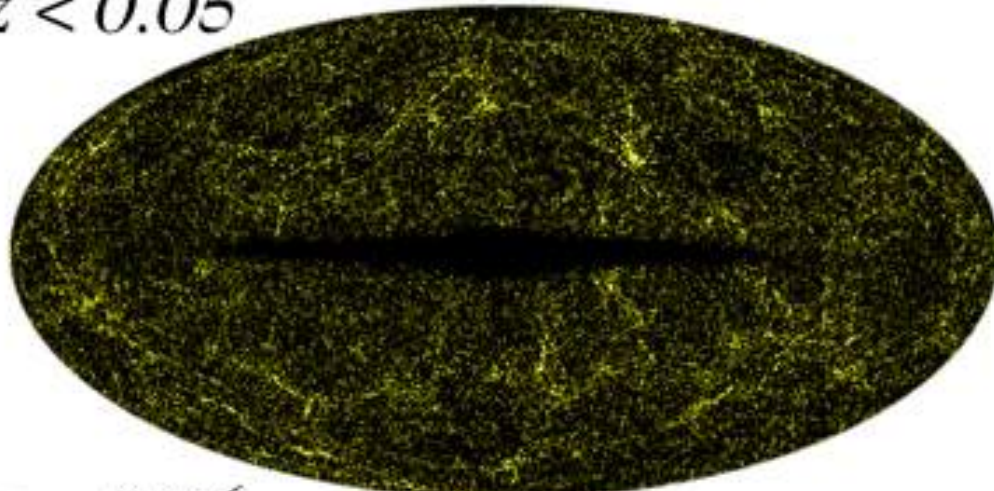
$0.02 < z < 0.03$



$0.03 < z < 0.04$



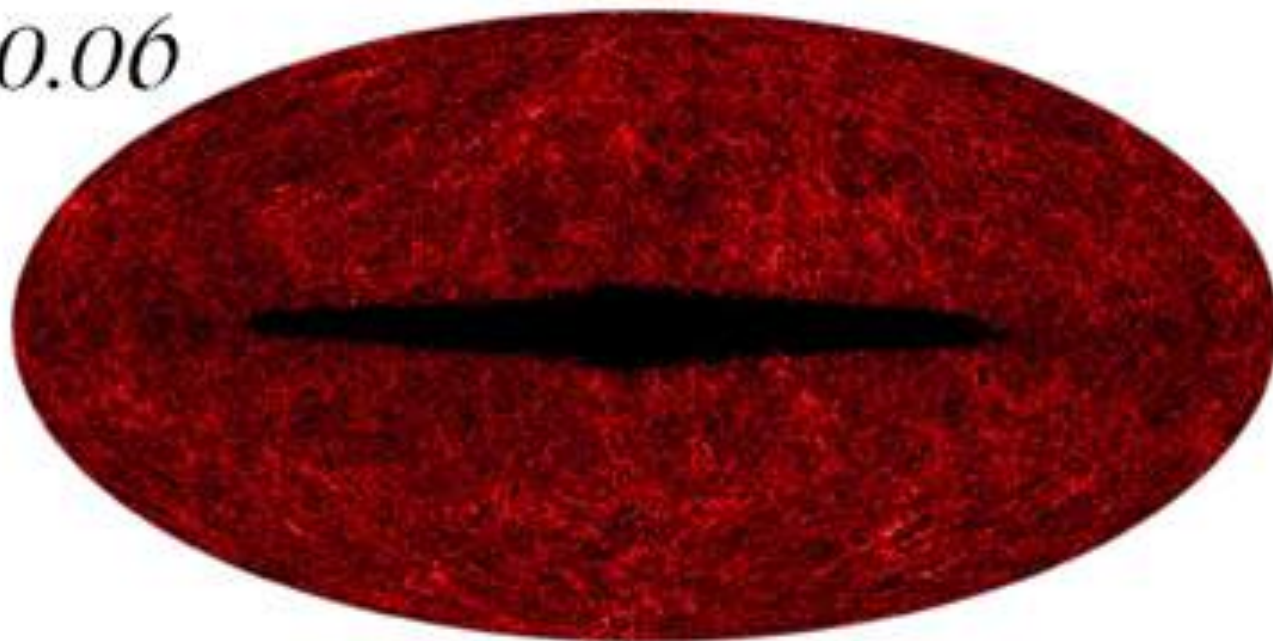
$0.04 < z < 0.05$



$0.05 < z < 0.06$



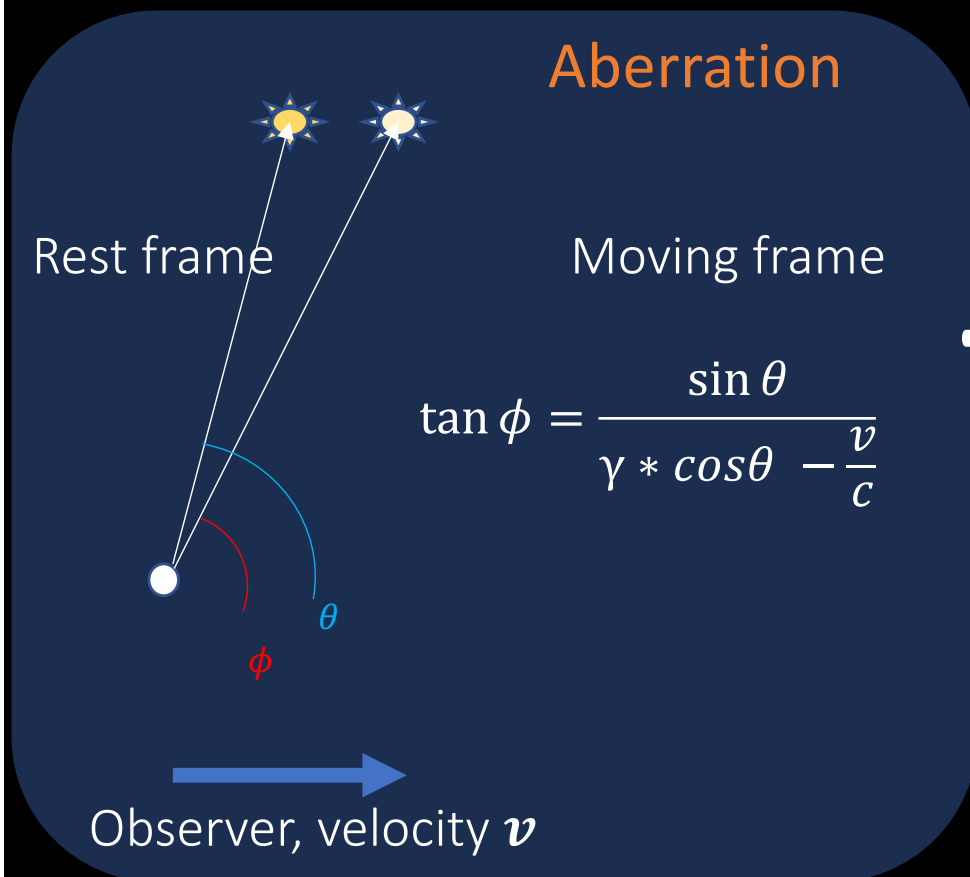
$z > 0.06$



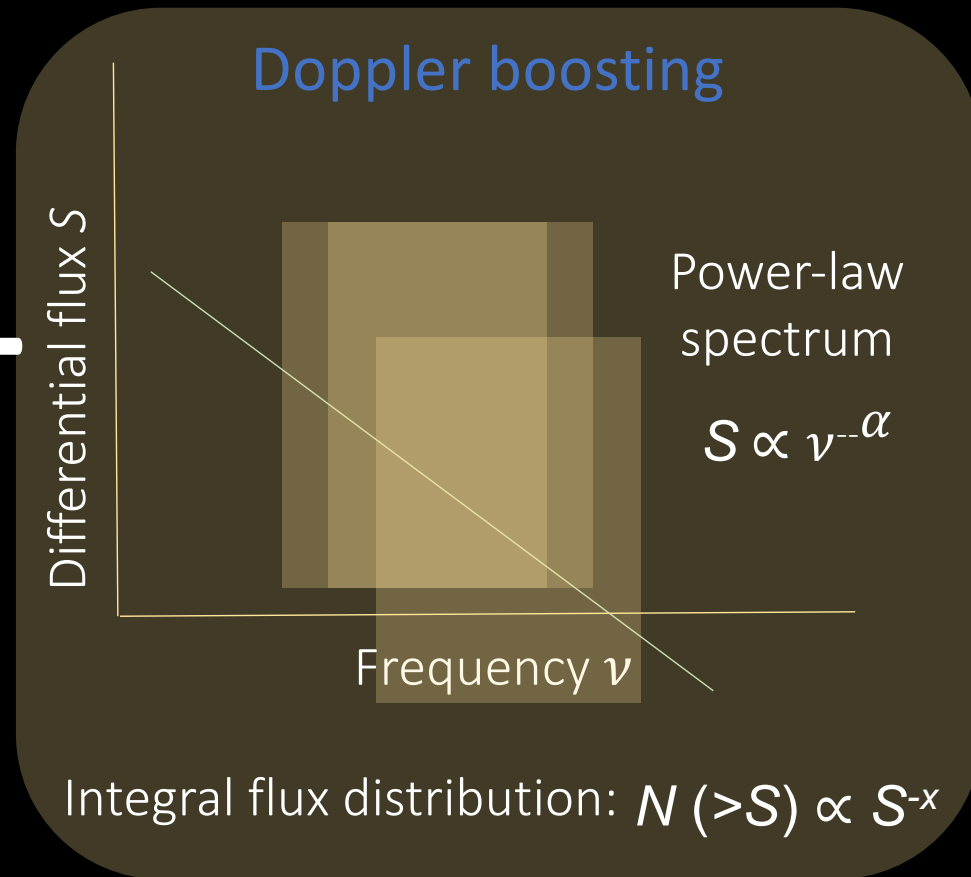
This is well beyond the 'scale of homogeneity' ... but convergence to the CMB frame has not yet occurred!

IS THE CMB DIPOLE REALLY DUE TO OUR MOTION WRT THE 'CMB FRAME'?  
 THEN WE SHOULD SEE *SIMILAR* DIPOLE IN THE DISTRIBUTION OF DISTANT SOURCES

$$\sigma(\theta)_{obs} = \sigma_{rest} \left[ 1 + \left[ 2 + x(1 + \alpha) \right] \frac{v}{c} \cos(\theta) \right]$$



+



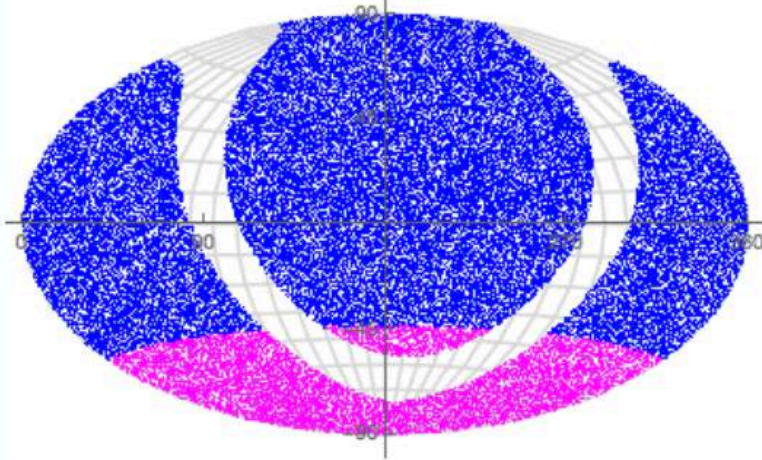
Flux-limited catalog  $\rightarrow$  *more* sources in direction of motion

(Ellis & Baldwin  
 MNRAS 206:377,1984)

**BUT THIS IS *NOT* WHAT IS ACTUALLY FOUND!**

DATA: NVSS+SUMSS

576461 Radio galaxies in 10 mJy <Flux< 1000 mJy



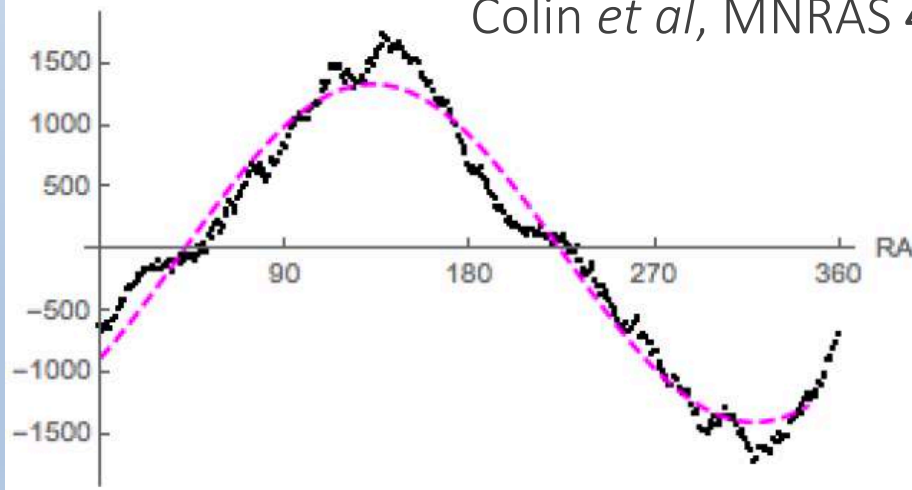
$$\vec{D}_C = \frac{1}{N} \sum_{i=1}^N \hat{r}_i$$

Statistical error  
 $1/\sqrt{N}$

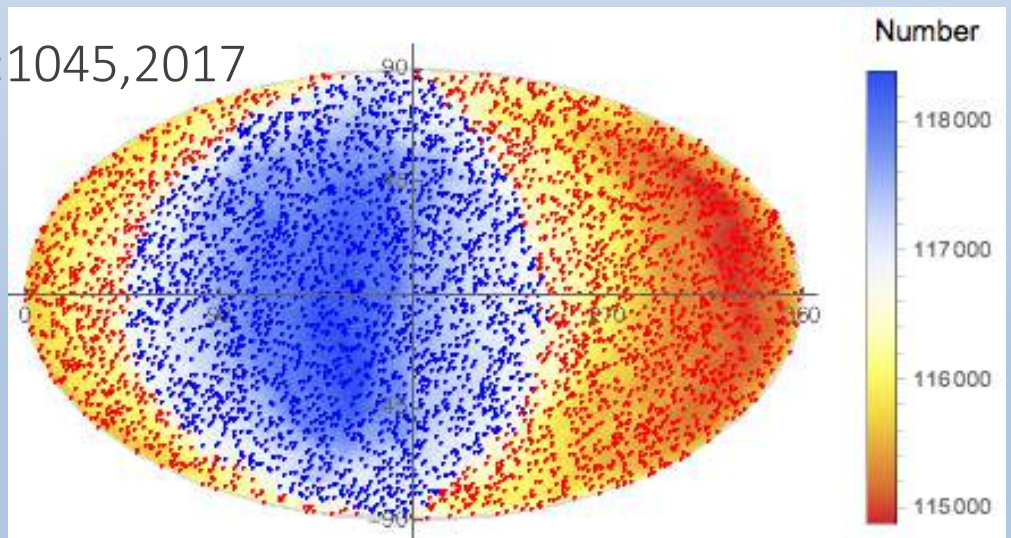
Add up unit vectors corresponding to directions in the sky for every source

$$\vec{D}_C = \frac{\hat{z}}{N} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sigma(\theta) \cos\theta \sin\theta d\theta d\phi$$

velocity (km/s)



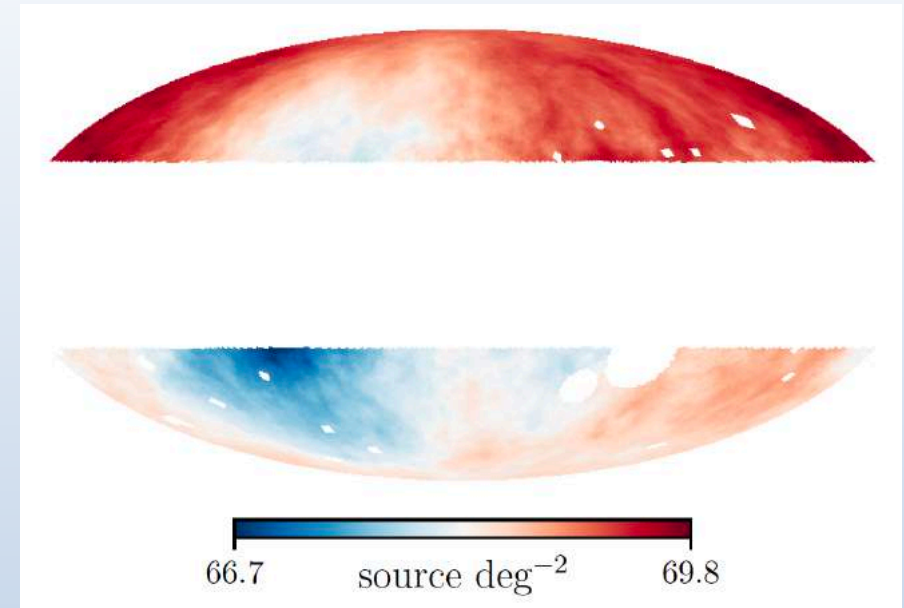
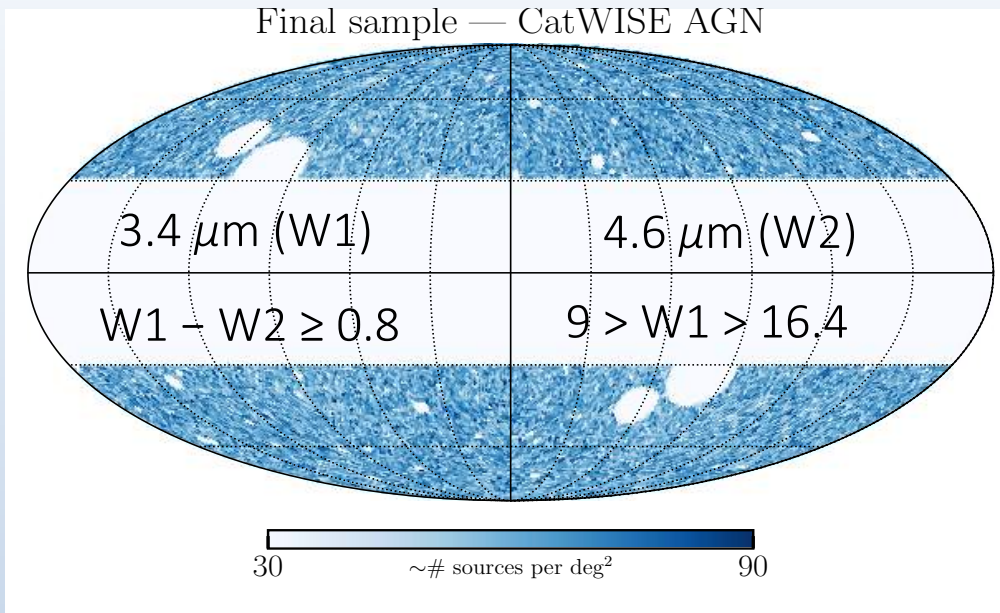
Colin *et al*, MNRAS 471:1045,2017



Velocity =  $1355 \pm 174$  km/s.... within  $10^\circ$  of CMB dipole (but **4x faster**)!

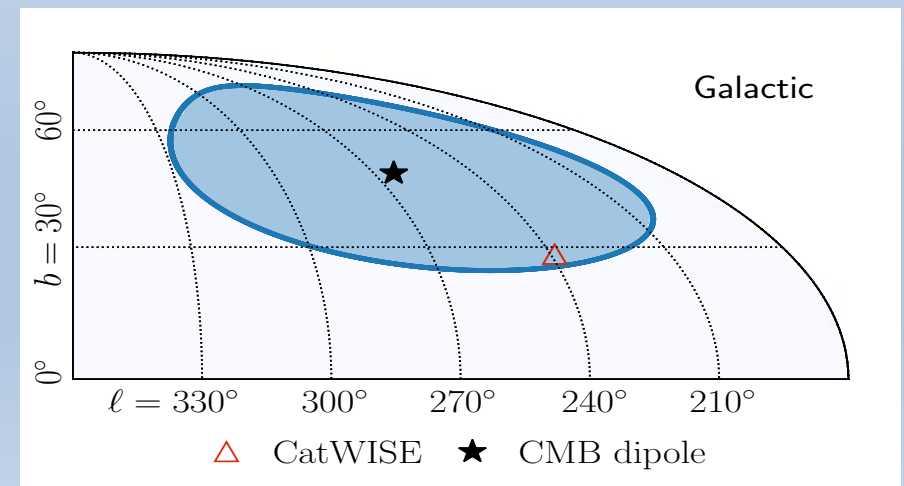
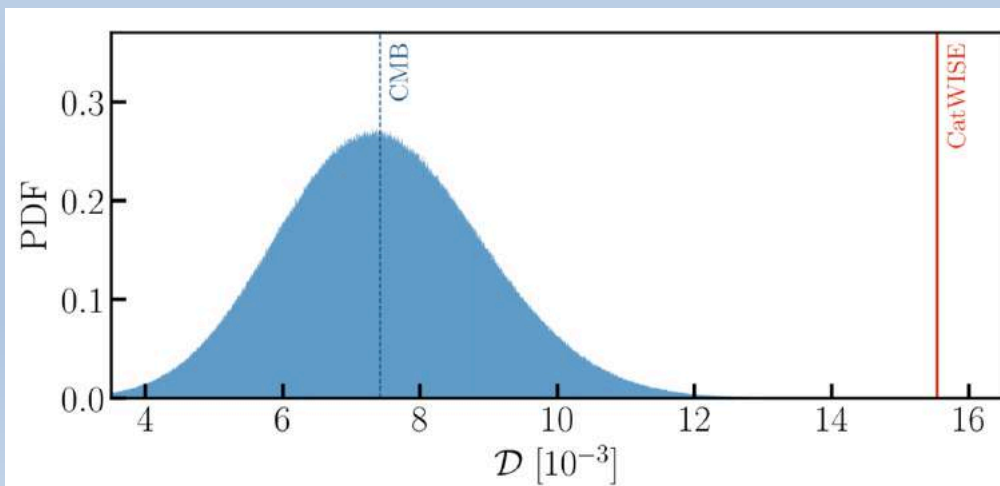
This calls into question the usual kinematic interpretation of the CMB dipole and raises the possibility of a ‘tilted universe’ (King & Ellis CMP 31:209,1973, Turner PRD 44:3737,1991)

# A VERY INTRIGUING RESULT: QUASAR REST FRAME $\neq$ CMB REST FRAME



We have now constructed a catalogue of 1.4 million quasars, with 99% at redshift  $> 0.1$

Colin *et al*, ApJL in press [arXiv:2009.14826]



The kinematic interpretation of the CMB dipole is *rejected* with  $p = 5 \times 10^{-7} \Rightarrow 4.9\sigma$

## On the expected anisotropy of radio source counts

G. F. R. Ellis<sup>\*</sup> and J. E. Baldwin<sup>†</sup> *Orthodox Academy of Crete, Kolymbari, Crete*

Received 1983 May 31; in original form 1983 March 31

**Summary.** If the standard interpretation of the dipole anisotropy in the microwave background radiation as being due to our peculiar velocity in a homogeneous isotropic universe is correct, then radio-source number counts must show a similar anisotropy. Conversely, determination of a dipole anisotropy in those counts determines our velocity relative to their rest frame; this velocity must agree with that determined from the microwave background radiation anisotropy. Present limits show reasonable agreement between these velocities.

### 4 Conclusion

Anisotropies in radio-source number counts can be used to determine a cosmological standard of rest. Current observations determine it to about  $\pm 500 \text{ km s}^{-1}$ , but accurate counts of fainter sources will reduce the error to a level comparable to that set by observations of the microwave background radiation. If the standards of rest determined by the MBR and the number counts were to be in serious disagreement, one would have to abandon either

- (a) the idea that the radio sources are at cosmological distances, or
- (b) the interpretation of the cosmic microwave radiation as relic radiation from the big bang, or
- (c) the standard FRW Universe models.

Thus comparison of these standards of rest provides a powerful consistency test of our understanding of the Universe.