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**SECOND PUBLIC EXAMINATION**

**Honour School of Physics Part B: 3 and 4 Year Courses**

**Honour School of Physics and Philosophy Part B**

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# B3: VI. CONDENSED-MATTER PHYSICS

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**TRINITY TERM 2013**

**Monday, 10 June, 2.30 pm – 4.30 pm**

*Answer* **two** *questions.*

*Start the answer to each question in a* **fresh book.**

*A list of physical constants and conversion factors accompanies this paper.*

*The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.*

**Do NOT turn over until told that you may do so.**

**1.** A powder diffraction pattern from a silicon sample is obtained using a 10 keV collimated x-ray beam. The crystal structure of silicon is a cubic cell and it contains 8 atoms located at $[0,0,0]$, $\left[\frac{1}{2},\frac{1}{2},0\right]$, $\left[\frac{1}{2},0,\frac{1}{2}\right]$, $\left[0,\frac{1}{2},\frac{1}{2}\right]$, $\left[\frac{1}{4},\frac{1}{4},\frac{1}{4}\right]$, $\left[\frac{3}{4},\frac{3}{4},\frac{1}{4}\right]$, $\left[\frac{3}{4},\frac{1}{4},\frac{3}{4}\right]$, and $\left[\frac{1}{4},\frac{3}{4},\frac{3}{4}\right]$. Find the lattice type and determine the Miller indices of the two lines with the lowest scattering angle in the powder diffraction pattern. [7]

If the first two lines occur at an angle of 22.8° and 37.6°, determine the cubic lattice constant for silicon, and approximate the intensity ratio between these lines. [8]

The coefficient of linear thermal expansion for silicon is $3 \times 10^{-6}$ K$^{-1}$. Find the change in scattering angle for the first two lowest angle diffraction peaks due to a change in temperature from 300 K to 900 K. [7]

Discuss the experimental requirements to use these shifts for accurate temperature measurements. [3]

**2.** Consider a classical one-dimensional crystal made of a chain of alternating ions with charge $\pm q$ ($q = Ze$). Let $R$ be the nearest-neighbour distance (i.e. the distance between adjacent positive and negative ions) and $N$ the number of ion pairs (with $N$ a large number). In addition to the long-range electrostatic interaction between ions, a repulsive (short-range) interaction of the form $A/R^n$ (where $A$ is a real number and $n$ is an integer) also exists between nearest-neighbours only. Derive an expression for the total energy per ion pair. [Hint: $\sum_{\ell\neq 0}(-1)^{\ell-1}/|\ell| = 2\ln 2 \equiv \alpha$ .] [8]

Determine the equilibrium nearest-neighbour distance $R_0$ and find the mean energy per ion pair. [4]

Consider now the case of a one-dimensional metal, consisting of a chain of $N$ positive charges $+q$ separated by a distance $2R$ and immersed in a neutralizing background of electrons with density per unit length $n_e$. The electrostatic energy due to the interaction of the electrons with the ion cores and among themselves is

$$\mathcal{E}_{el} = -\frac{\alpha q^2}{4\pi\epsilon_0}\frac{N}{R} .$$

Assuming that the electrons form a non-interacting Fermi gas, calculate the Fermi energy. Write down an expression for the total kinetic energy of the electrons. [8]

Determine the electron density that minimizes the total energy (electrostatic plus kinetic) per ion. [5]

**3.** Consider a one-dimensional crystal of atoms of mass $m$. The nearest-neighbour interaction is modelled by springs with force constant $K$, while the next-nearest-neighbour interactions are modelled by springs with force constant $G$. Write down the equation of motion of the atoms. [3]

Derive the phonon dispersion relation and find the sound speed. Discuss the effect of $G > 0$ on the sound speed compared with $G = 0$. [12]

Assume there is also a force $-\Gamma \dot{u}_j$ acting on the $j$-th atom, with $u_j$ its displacement from the equilibrium position. Find an expression for the dispersion relation in the presence of this additional force. [6]

Comment on the time dependence of the phonon amplitude. [4]

**4.** Describe what are meant by *diamagnetism* and *paramagnetism*. [5]

Consider a paramagnetic solid formed by ions that have magnetic moment $\vec{\mu} = g\mu_B\vec{J}$, where $g$ is the Landé factor of the atom, $\mu_B$ is the Bohr magneton, and $\vec{J}$ is the total angular momentum. Along the $z$ direction, a weak external magnetic field $\vec{B} = B\hat{e}_z$ is applied to the solid. Assuming that the interaction Hamiltonian for a single ion is described as

$$\mathcal{H} = -\vec{\mu} \cdot \vec{B} \ ,$$

calculate the energy eigenvalues of $\mathcal{H}$. [5]

Estimate the average magnetic moment in the $z$ direction and show that, for large temperature $T$, it follows a Curie law relation $\langle \mu_z \rangle \sim C/T$. Determine the expression for the constant $C$.
[Hint: you may use either the approximation $\coth x \simeq \frac{1}{x} + \frac{x}{3}$ or the relation $\sum_{m=-J}^{J} m^2 = J(1+J)(1+2J)/3$ .] [10]

Give an estimate of the magnetic field above which the Curie law does not hold at room temperature. [2]

Iron does not behave as a paramagnet at room temperature. Discuss how the Curie law has to be modified in order to describe this regime. [3]