Slides
Condensed Matter Physics
Revision Lecture 2

From 2011 Exam

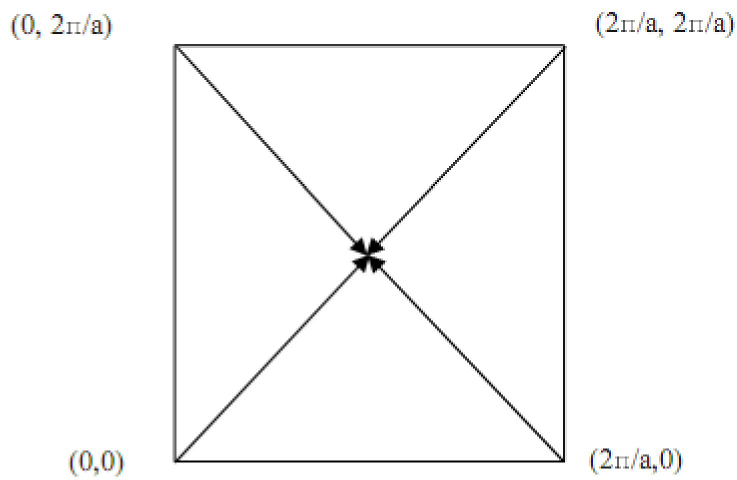
7. In the nearly free electron model electrons experience a weak periodic potential $V(\mathbf{r}) = V(\mathbf{r}+\mathbf{R})$, where $\mathbf{R}$ is any lattice vector. Show that $V_{\mathbf{k}-\mathbf{k}'} = \langle \mathbf{k}|V(r)|\mathbf{k}'\rangle$, where $|\mathbf{k}\rangle$ is the plane wave state, is non-zero only when $(\mathbf{k}-\mathbf{k}')$ is a reciprocal lattice vector $\mathbf{G}$. Using perturbation theory show that the secular equation is

$$(\varepsilon_0(\mathbf{k}) - E)\,(\varepsilon_0(\mathbf{k}+\mathbf{G}) - E) - |V_{\mathbf{G}}|^2 = 0\,,$$ [9]

where ε_0 is the energy of $|\mathbf{k}\rangle$. Consider a divalent two-dimensional metal with a square lattice, lattice parameter $a = 0.3\,\mathrm{nm}$, and one atom per primitive unit cell. The periodic potential has two Fourier components V_{10}, V_{11}, corresponding to $\mathbf{G} = (1,0)$ and $(1,1)$ respectively; both are negative, and $|V_{10}| > |V_{11}|$.

(i) Write down the secular equation and obtain an expression for the electron energies at $\mathbf{k} = (\pi/a, 0)$. [4]

(ii) As illustrated in the diagram, the state at $(\pi/a, \pi/a)$ is four-fold degenerate in the free electron approximation, with energy ε_0.



In the presence of the periodic potential the secular equation is

$$\begin{vmatrix} \varepsilon_0 - E & V_{10} & V_{11} & V_{10} \\ V_{10} & \varepsilon_0 - E & V_{10} & V_{11} \\ V_{11} & V_{10} & \varepsilon_0 - E & V_{10} \\ V_{10} & V_{11} & V_{10} & \varepsilon_0 - E \end{vmatrix} = 0 \, ,$$

which simplifies to

$$(\varepsilon_0 - E - V_{11})^2 \left[(\varepsilon_0 - E + V_{11})^2 - 4V_{10}^2 \right] = 0 \, .$$

Sketch the energy levels at $(\pi/a, \pi/a)$ and $(\pi/a, 0)$ in the same diagram. [4]

By considering overlapping energy bands, find the value of V_{10} at which the system becomes semiconducting given that $V_{11} = -0.2\,\text{eV}$. [8]