Hierarchy Problems in String Theory: The Power of Large Volume

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This talk represents edited highlights of

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Talk Structure

- Hierarchies in Nature
- String Phenomenology and Moduli Stabilisation
- Large Volume Models
- Susy Breaking and Soft Terms
- Axions
- Neutrino Masses
- Conclusions

Hierarchies in Nature

Nature likes hierarchies:

- The Planck scale, $M_P = 2.4 \times 10^{18}$GeV.
- The GUT/inflation scale, $M \sim 10^{16}$GeV.
- The axion scale, 10^9GeV $\lesssim f_a \lesssim 10^{12}$GeV
- The weak scale : $M_W \sim 100$GeV
- The QCD scale $\Lambda_{QCD} \sim 200$MeV - explained!
- The neutrino mass scale, 0.05eV $\lesssim m_\nu \lesssim 0.3$eV.
- The cosmological constant, $\Lambda \sim (10^{-3}\text{eV})^4$

These demand an explanation!

Hierarchies in Nature

This explanation should be

- natural - i.e. untuned
- explanatory - it should relate the different scales

In this talk I will enthusiastically advocate

- stabilised exponentially large extra dimensions ($\mathcal{V} \sim 10^{15} l_s^6$).
- an intermediate string scale $m_s \sim 10^{11}$GeV

as giving a natural, explanatory explanation for the axionic, weak and neutrino hierarchies.

The different hierarchies will come as different powers of the (large) volume.

String Phenomenology

In addition to the above hierarchies, also to be explained are

- the gauge groups
- the chiral spectrum
- the gauge couplings
- the number of space-time dimensions
- the flavour structure....

In fact, correctly explaining a limited subset of the above would be a huge success....

In this talk I focus on the hierarchies.

String Phenomenology

- String theory lives in ten dimensions.
- The world is four-dimensional, so ten-dimensional string theory needs to be compactified.
- To preserve $\mathcal{N} = 1$ supersymmetry, we compactify on a Calabi-Yau manifold.
- The gauge group and particle spectrum is determined by the Calabi-Yau geometry.
- Chirality can arise either from magnetic fluxes and magnetised branes (heterotic /type I / type IIB) or intersecting branes (type IIA).

String Phenomenology

- The spectrum of light particles is determined by higher-dimensional topology.
- (Technically, these are counted through sheaf cohomology.)
- As part of the spectrum, string compactifications generically produce many uncharged scalar particles.
- These moduli parametrise the size and shape of the extra dimensions.
- They also determine the four-dimensional gauge couplings.

Moduli Stabilisation

- Moduli are naively massless scalar fields which may take large classical vevs.
- They are uncharged and interact gravitationally.
- Such massless scalars generate unphysical fifth forces.
- The moduli need to be stabilised and given masses.
- Generating potentials for moduli is the popular field of moduli stabilisation.
- The large-volume models represent a particular (and appealing) moduli stabilisation scenario.

Moduli Stabilisation: Fluxes

- Flux compactifications involve non-vanishing flux fields on Calabi-Yau cycles.
- The fluxes carry a potential energy which depends on the geometry of the cycles.
- This energy generates a potential for the moduli associated with these cycles.
- In IIB compactifications, 3-form fluxes generate a superpotential

$$W = \int (F_3 + iSH_3) \wedge \Omega \equiv \int G_3 \wedge \Omega.$$

- This stabilises the dilaton and complex structure moduli.

Moduli Stabilisation: Fluxes

$$
\begin{aligned}
\hat{K} &= -2\ln\left(\mathcal{V}(T+\bar{T})\right) - \ln\left(i\int \Omega \wedge \bar{\Omega}(U)\right) - \ln\left(S+\bar{S}\right), \\
W &= \int G_3 \wedge \Omega\,(S,U). \\
V &= e^{\hat{K}}\left(\sum_{U,S} \hat{K}^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} + \sum_{T} \hat{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2\right) \\
&= e^{\hat{K}}\left(\sum_{U,S} \hat{K}^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W}\right)
\end{aligned}
$$

Stabilise S and U by solving $D_S W = D_U W = 0$.

Moduli Stabilisation: KKLT

$$\hat{K} = -2\ln(\mathcal{V}) - \ln\left(i\int \Omega \wedge \bar{\Omega}\right) - \ln\left(S + \bar{S}\right),$$
$$W = \int G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i}.$$

Non-perturbative effects (D3-instantons / gaugino condensation) allow the T-moduli to be stabilised by solving $D_T W = 0$.

For consistency, this requires

$$W_0 = \left\langle \int G_3 \wedge \Omega \right\rangle \ll 1.$$

Moduli Stabilisation: Large-Volume

$$\hat{K} = -2\ln\left(\mathcal{V} + \frac{\xi}{g_s^{3/2}}\right) - \ln\left(i\int \Omega \wedge \bar{\Omega}\right) - \ln\left(S + \bar{S}\right),$$

$$W = \int G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i}.$$

We include the leading α' corrections to the Kähler potential.

This leads to dramatic changes in the large-volume vacuum structure.

Moduli Stabilisation: Large-Volume

The simplest model $\mathbb{P}^4_{[1,1,1,6,9]}$ has two Kähler moduli.

$$\mathcal{V} = \left(\left(\frac{T_b + \bar{T}_b}{2} \right)^{3/2} - \left(\frac{T_s + \bar{T}_s}{2} \right)^{3/2} \right) \equiv \left(\tau_b^{3/2} - \tau_s^{3/2} \right).$$

If we compute the scalar potential, we get for $\mathcal{V} \gg 1$,

$$V = \frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s \tau_s}}{\mathcal{V}} - \frac{a_s |A_s W_0| \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{\xi |W_0|^2}{\mathcal{V}^3}.$$

The minimum of this potential can be found analytically.

Moduli Stabilisation: Large-Volume

The locus of the minimum satisfies

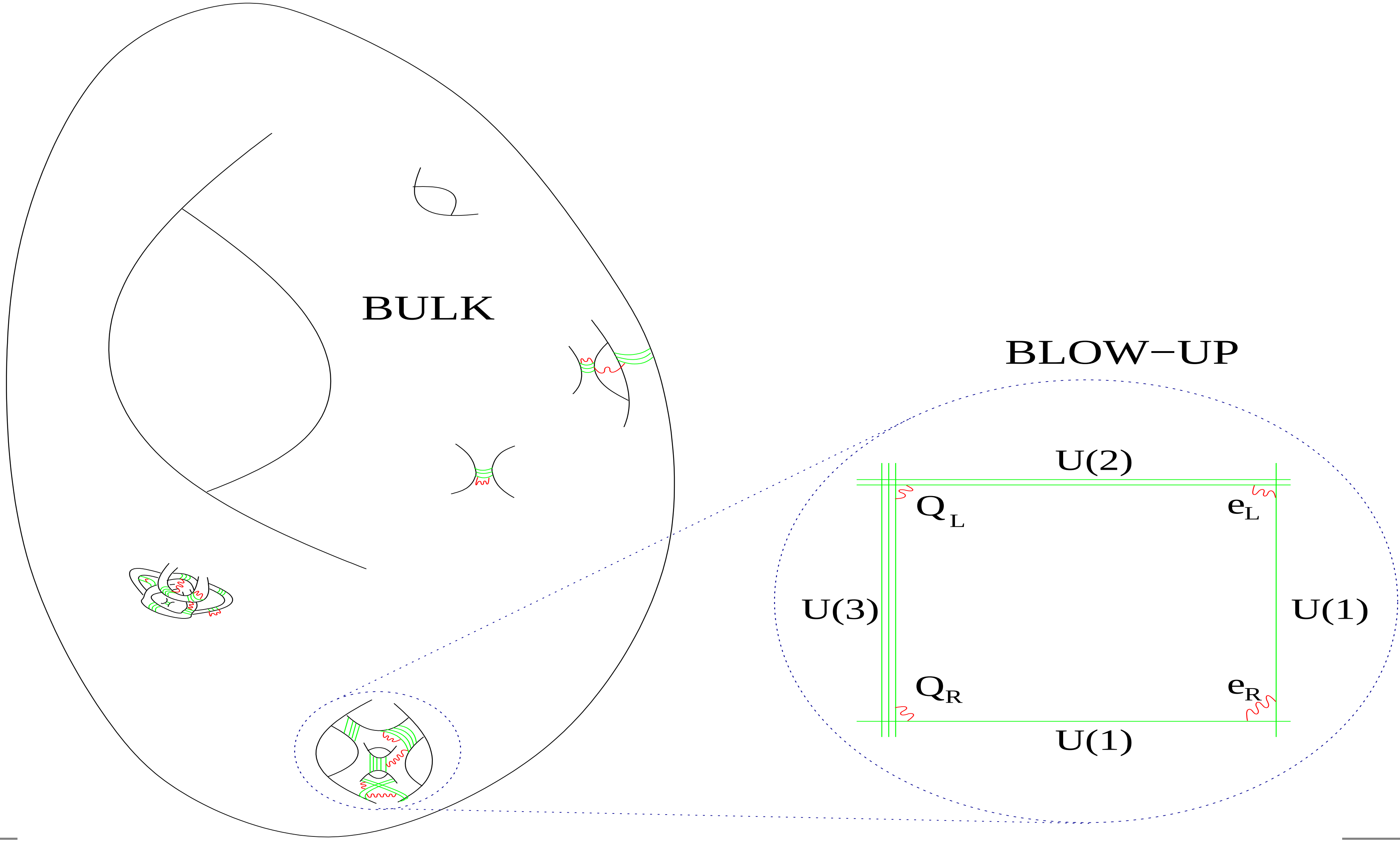
$$\mathcal{V} \sim |W_0| e^{c/g_s}, \qquad \tau_s \sim \ln \mathcal{V}.$$

The minimum is non-supersymmetric AdS and at exponentially large volume.

The large volume lowers the string scale and gravitino mass through

$$m_s = \frac{M_P}{\sqrt{\mathcal{V}}}, \qquad m_{3/2} = \frac{M_P}{\mathcal{V}}.$$

Moduli Stabilisation: Large-Volume



Summary of Large Volume Models

- The stabilised volume is naturally exponentially large.
- This lowers the gravitino mass through

$$m_{3/2} = \frac{M_P}{\mathcal{V}}.$$

- The minimum is non-supersymmetric and generates gravity-mediated soft terms.
- The weak hierarchy can be naturally generated through TeV-scale supersymmetry.

SUSY Breaking and Soft Terms

The MSSM is specified by its matter content (that of the supersymmetric Standard Model) plus soft breaking terms. These are

- Soft scalar masses, $m_i^2 \phi_i^2$
- Gaugino masses, $M_a \lambda^a \lambda^a$,
- Trilinear scalar A-terms, $A_{\alpha\beta\gamma}\phi^\alpha \phi^\beta \phi^\gamma$
- B-terms, BH_1H_2.

The soft terms break supersymmetry explicitly, but do not reintroduce quadratic divergences into the Lagrangian.

We want to compute these soft terms for the large volume models.

SUSY Breaking and Soft Terms

The soft terms are generated by the mechanism of supersymmetry breaking and how this is transmitted to the observable sector. Examples are

- Gravity mediation - hidden sector supersymmetry breaking
- Gauge mediation - visible sector supersymmetry breaking
- Anomaly mediaton - susy breaking through loop effects

These all have characteristic features and scales. Generally,

$$M_{soft} = \frac{M^2_{susy-breaking}}{M_{transmission}}.$$

SUSY Breaking and Soft Terms

Here, the moduli break susy and we have gravity-mediation.

In gravity-mediation, supersymmetry is

- broken in a hidden sector
- communicated to the observable sector through non-renormalisable supergravity contact interactions
- which are suppressed by M_P.

Naively,

$$m_{susy} \sim \frac{F^2}{M_P}$$

This requires $F \sim 10^{11}$GeV for TeV-scale soft terms.

Soft Terms

- We can compute the soft terms for the large-volume models...

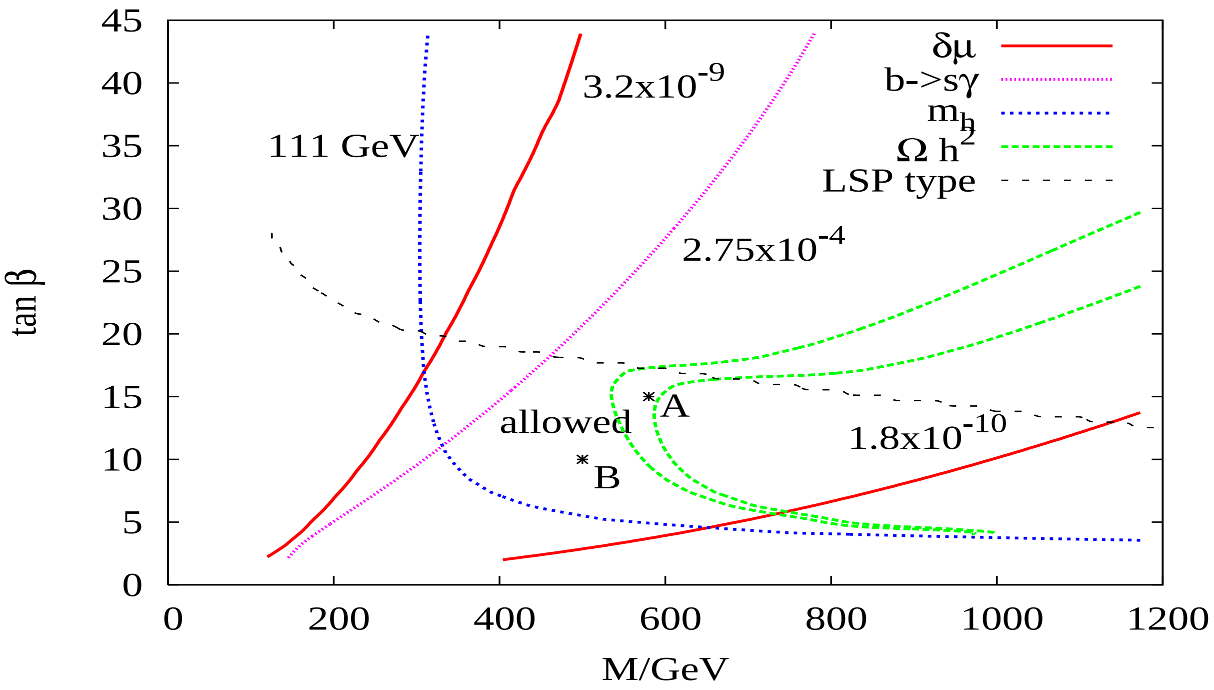
  (details skipped)

  ... we get

$$
\begin{aligned}
M_i &= \frac{F^s}{2\tau_s} \equiv M, \\
m_{\alpha\bar{\beta}} &= \frac{M}{\sqrt{3}}\tilde{K}_{\alpha\bar{\beta}}, \\
A_{\alpha\beta\gamma} &= -M\,\hat{Y}_{\alpha\beta\gamma}, \\
B &= -\frac{4M}{3}.
\end{aligned}
$$

Soft Terms: Phenomenology

- We run the soft terms to low energy using SoftSUSY.
- We scan over M and $\tan\beta$ and impose constraints from Ωh^2, $b \to s\gamma$, m_H, $g_\mu - 2$ and LSP type.



Axions

- Axions are a well-motivated solution to the strong CP problem.
- The QCD Lagrangian is

$$\mathcal{L}_{QCD} = \frac{1}{g^2} \int d^4x F^a_{\mu\nu} F^{a,\mu\nu} + \theta \int F^a \wedge F^a.$$

- The strong CP problem is that naively $\theta \in (-\pi, \pi)$ while experimentally, $|\theta| \lesssim 10^{-10}$.
- The axionic (Peccei-Quinn) solution is to promote θ to a dynamical field, $\theta(x)$.

Axions

- The canonical Lagrangian for θ is

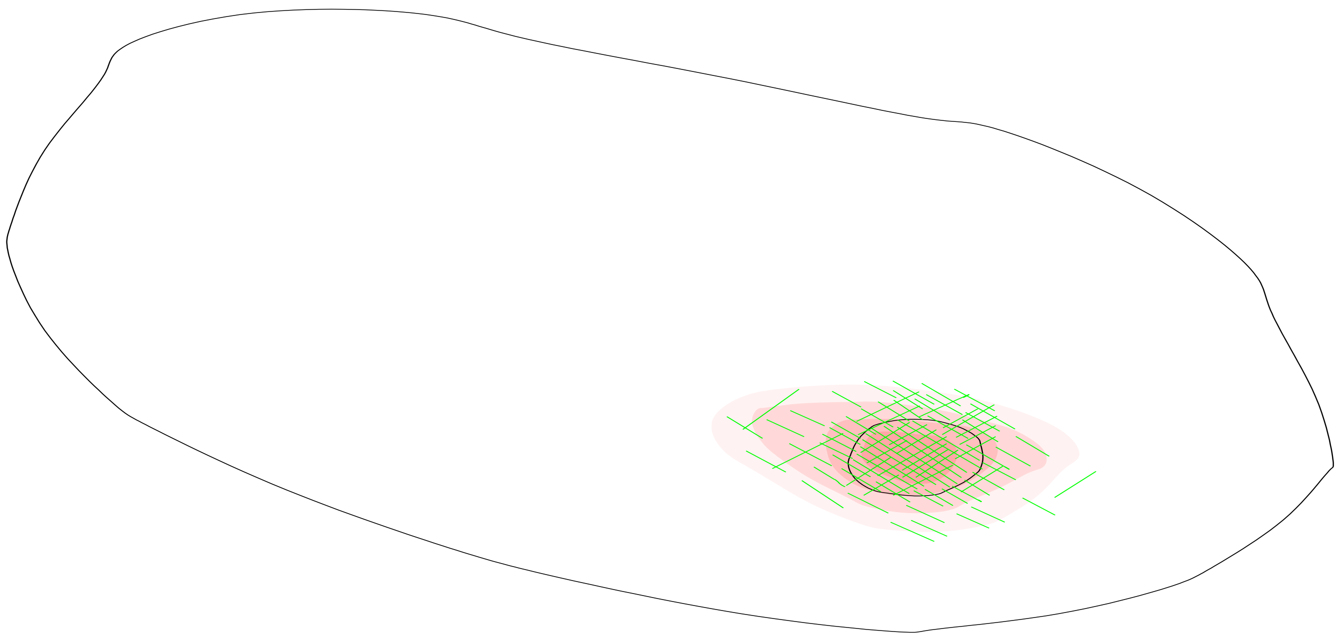
$$\mathcal{L} = \frac{1}{2}\partial_\mu \theta \partial^\mu \theta + \int \frac{\theta}{f_a} F^a \wedge F^a.$$

- f_a is called the axionic decay constant.
- Constraints on supernova cooling and direct searches imply $f_a \gtrsim 10^9$GeV.
- Avoiding the overproduction of axion dark matter prefers $f_a \lesssim 10^{12}$GeV.
- There exists an axion 'allowed window',

$$10^9\text{GeV} \lesssim f_a \lesssim 10^{12}\text{GeV}.$$

Axions

- The axionic coupling comes from the RR form in the brane Chern-Simons action.
- The axion decay constant f_a measures the coupling of the axion to matter.



Axions

- The coupling of the axion to matter is a local coupling and does not see the overall volume.
- This coupling can only see the string scale, and so

$$f_a \sim m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}.$$

(This is confirmed by a full analysis)

- This generates the axion scale,

$$f_a \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11}\text{GeV}.$$

Neutrino Masses

- The theoretical origin of neutrino masses is a mystery. Experimentally

$$0.05\text{eV} \lesssim m_\nu^H \lesssim 0.3\text{eV}.$$

- This corresponds to a Majorana mass scale

$$M_{\nu_R} \sim 3 \times 10^{14}\text{GeV}.$$

- Equivalently, this is the suppression scale Λ of the dimension five Standard Model operator

$$\mathcal{O} = \frac{1}{\Lambda} HHLL.$$

Neutrino Masses

- In the supergravity MSSM, consider the superpotential operator

$$\frac{\lambda}{M_P} H_2 H_2 L L \in W,$$

where λ is dimensionless.

- This gives rise to the Lagrangian terms,

$$\tilde{K}_{H_2} \partial_\mu H_2 \partial^\mu H_2 + \tilde{K}_L \partial_\mu L \partial^\mu L + e^{\hat{K}/2} \frac{\lambda}{M_P} H_2 H_2 L L.$$

- This corresponds to the *physical* coupling

$$e^{\hat{K}/2} \frac{\lambda}{M_P} \frac{\langle H_2 H_2 \rangle L L}{(\tilde{K}_{H_2} \tilde{K}_{H_2} \tilde{K}_L \tilde{K}_L)^{\frac{1}{2}}}.$$

Neutrino Masses

- We have the coupling

$$e^{\hat{K}/2}\frac{\lambda}{M_P}\frac{\langle H_2 H_2\rangle LL}{(\tilde{K}_{H_2}\tilde{K}_{H_2}\tilde{K}_L\tilde{K}_L)^{\frac{1}{2}}}.$$

- Once the Higgs receives a vev, this generates neutrino masses.
- However, to compute the mass scale we need to know $\hat{K}_{H_2}$ and $\hat{K}_L$...

Computing the Kähler Metric

- I now describe new techniques for computing the matter metric $\tilde{K}_{\alpha\bar{\beta}}$ for chiral matter fields on a Calabi-Yau.
- These techniques will apply to chiral bifundamental D7/D7 matter in IIB compactifications.
- I will compute the modular weights of the matter metrics from the modular dependence of the (physical) Yukawa couplings.

I start by reviewing Yukawa couplings in supergravity.

Yukawa Couplings in Supergravity

- In supergravity, the Yukawa couplings arise from the Lagrangian terms (Wess & Bagger)

$$\begin{aligned}\mathcal{L}_{kin} + \mathcal{L}_{yukawa} &= \tilde{K}_\alpha \partial_\mu C^\alpha \partial^\mu \bar{C}^{\bar{\alpha}} + e^{\hat{K}/2}\, \partial_\beta \partial_\gamma W\, \psi^\beta \psi^\gamma \\ &= \tilde{K}_\alpha \partial_\mu C^\alpha \partial^\mu \bar{C}^{\bar{\alpha}} + e^{\hat{K}/2} Y_{\alpha\beta\gamma} C^\alpha \psi^\beta \psi^\gamma\end{aligned}$$

(This assumes diagonal matter metrics, but we can relax this)

- The matter fields are not canonically normalised.
- The *physical* Yukawa couplings are given by

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}}.$$

Computing $\tilde{K}_{\alpha\bar{\beta}}$ (I)

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}}.$$

- We know the modular dependence of $\hat{K}$:

$$\hat{K} = -2\ln(\mathcal{V}) - \ln\left(i \int \Omega \wedge \bar{\Omega}\right) - \ln(S + \bar{S})$$

- We compute the modular dependence of $\tilde{K}_\alpha$ from the modular dependence of $\hat{Y}_{\alpha\beta\gamma}$. We work in a power series expansion and determine the leading power λ,

$$\tilde{K}_\alpha \sim (T + \bar{T})^\lambda k_\alpha(\phi) + (T + \bar{T})^{\lambda - 1} k'_\alpha(\phi) + \ldots$$

λ is the modular weight of the field T.

Computing $\tilde{K}_{\alpha\bar{\beta}}$ (II)

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}}.$$

- We are unable to compute $Y_{\alpha\beta\gamma}$. If $Y_{\alpha\beta\gamma}$ depends on a modulus T, knowledge of $\hat{Y}_{\alpha\beta\gamma}$ gives no information about the dependence $\tilde{K}_{\alpha\bar{\beta}}(T)$.
- Our results will be restricted to those moduli that do not appear in the superpotential.
- This holds for the T-moduli (Kähler moduli) in IIB compactifications. In perturbation theory,

$$\partial_{T_i} Y_{\alpha\beta\gamma} \equiv 0.$$

Computing $\tilde{K}_{\alpha\bar{\beta}}$ (III)

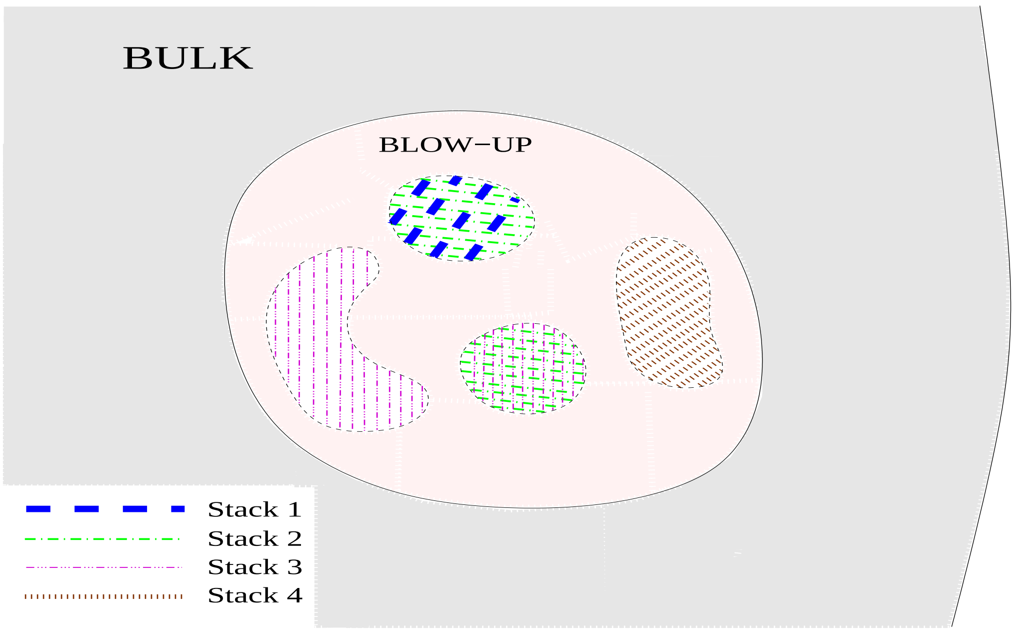
$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}}.$$

- We know $\hat{K}(T)$.
- If we can compute $\hat{Y}_{\alpha\beta\gamma}(T)$, we can then deduce $\tilde{K}_\alpha(T)$.
- We can compute $\hat{Y}_{\alpha\beta\gamma}(T)$ for IIB compactifications through wavefunction overlap.

We now describe the computation of $\hat{Y}_{\alpha\beta\gamma}$ for bifundamental matter on a stack of magnetised D7-branes.

The brane geometry

- We consider a stack of (magnetised) branes all wrapping an identical cycle.
- Chiral fermions stretch between differently magnetised branes.



Computing the physical Yukawas $\hat{Y}_{\alpha\beta\gamma}$

- We use a simple computational technique:

- Physical Yukawa couplings are determined by the triple overlap of normalised wavefunctions.

- These wavefunctions can be computed (in principle) by dimensional reduction of the brane action. At low energies, this DBI action reduces to Super Yang-Mills.

Computing the physical Yukawas $\hat{Y}_{\alpha\beta\gamma}$

Consider the higher-dimensional term

$$\mathcal{L} \sim \int_{\mathbb{M}_4 \times \Sigma} d^n x \sqrt{g}\, \bar{\lambda}\Gamma^i (A_i + \partial_i)\lambda$$

Dimensional reduction of this gives the kinetic terms

$$\bar{\psi}\partial\psi$$

and the Yukawa couplings

$$\bar{\psi}\phi\psi$$

- The physical Yukawa couplings are set by the combination of the above!

Kinetic Terms

$$\mathcal{L} \sim \int_{\mathbb{M}_4 \times \Sigma} d^n x \sqrt{g}\, \bar{\lambda} \Gamma^i (A_i + \partial_i) \lambda$$

- Dimensional reduction gives

$$\lambda = \psi_4 \otimes \psi_6, \qquad A_i = \phi_4 \otimes \phi_6.$$

and the reduced kinetic terms are

$$\left(\int_\Sigma \psi_6^\dagger \psi_6 \right) \int_{\mathbb{M}_4} \bar{\psi}_4 \Gamma^\mu (A_\mu + \partial_\mu) \psi_4$$

- Canonically normalised kinetic terms require

$$\int_\Sigma \psi_6^\dagger \psi_6 = 1.$$

Yukawa Couplings

$$\mathcal{L} \sim \int_{\mathbb{M}_4 \times \Sigma} d^n x \, \bar{\lambda} \Gamma^i (A_i + \partial_i) \lambda$$

- Dimensional reduction gives the four-dimensional interaction

$$\left(\int_\Sigma \bar{\psi}_6 \Gamma^I \phi_{I,6} \psi_6 \right) \int_{\mathbb{M}_4} d^4 x \, \phi_4 \bar{\psi}_4 \psi_4 .$$

- The physical Yukawa couplings are determined by the (normalised) overlap integral

$$\hat{Y}_{\alpha\beta\gamma} = \left(\int_\Sigma \bar{\psi}_6 \Gamma^I \phi_{I,6} \psi_6 \right) .$$

The Result

$$\int_\Sigma \psi_6^\dagger \psi_6 = 1, \qquad \hat{Y}_{\alpha\beta\gamma} = \left(\int_\Sigma \bar{\psi}_6 \Gamma^I \phi_{I,6} \psi_6 \right).$$

- For canonical kinetic terms,

$$\psi_6(y) \sim \frac{1}{\sqrt{\text{Vol}(\Sigma)}}, \qquad \Gamma^I \phi_{I,6} \sim \frac{1}{\sqrt{\text{Vol}(\Sigma)}},$$

and so

$$\hat{Y}_{\alpha\beta\gamma} \sim \text{Vol}(\Sigma) \cdot \frac{1}{\sqrt{\text{Vol}(\Sigma)}} \cdot \frac{1}{\sqrt{\text{Vol}(\Sigma)}} \cdot \frac{1}{\sqrt{\text{Vol}(\Sigma)}} \sim \frac{1}{\sqrt{\text{Vol}(\Sigma)}}$$

- This gives the scaling of $\hat{Y}_{\alpha\beta\gamma}(T)$.

Application: Large-Volume Models

- The physical Yukawa coupling

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}}$$

  is local and thus independent of $\mathcal{V}$.
- As $\hat{K} = -2 \ln \mathcal{V}$, we can deduce *simply from locality* that

$$\tilde{K}_\alpha \sim \frac{1}{\mathcal{V}^{2/3}}.$$

- As this is for a Calabi-Yau background, this is already non-trivial!

Application: Large-Volume Models

- We can go further. With all branes wrapping the same 4-cycle,

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}} \sim \frac{1}{\sqrt{\tau_s}}.$$

- We can then deduce that

$$\tilde{K}_{\alpha\bar{\beta}} \sim \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}} k_{\alpha\bar{\beta}}(U, \bar{U}).$$

- We also have the dependence on τ_s!

Neutrino Masses

- We have the coupling

$$e^{\hat{K}/2}\frac{\lambda}{M_P}\frac{\langle H_2 H_2\rangle LL}{(\tilde{K}_{H_2}\tilde{K}_{H_2}\tilde{K}_L\tilde{K}_L)^{\frac{1}{2}}}.$$

- Once the Higgs receives a vev, this generates neutrino masses.
- However, to compute the mass scale we need to know $\tilde{K}_{H_2}$ and $\tilde{K}_L$ ...done!
-

$$\tilde{K}_{\alpha\bar{\beta}} \sim \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}} k_{\alpha\bar{\beta}}(\phi).$$

Neutrino Masses

- Using the large-volume result $\tilde{K}_\alpha \sim \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}}$, this becomes

$$\frac{\lambda \mathcal{V}^{1/3}}{\tau_s^{2/3} M_P} \langle H_2 H_2 \rangle LL.$$

- Use $\mathcal{V} \sim 10^{15}$ (to get $m_{3/2} \sim 1$TeV) and $\tau_s \sim 10$:

$$\frac{\lambda}{10^{14}\text{GeV}} \langle H_2 H_2 \rangle LL$$

- With $\langle H_2 \rangle = \frac{v}{\sqrt{2}} = 174$GeV, this gives

$$m_\nu = \lambda(0.3\,\text{eV}).$$

Large Volumes are Power-ful

In large-volume models, an exponentially large volume appears naturally ($\mathcal{V} \sim e^{\frac{c}{g_s}}$). The scales that naturally appear are

- Susy-breaking: $m_{soft} \sim \frac{M_P}{\mathcal{V}} \sim 10^3$GeV
- Axions: $f_a \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11}$GeV
- Neutrinos/dim-5 operators: $\Lambda \sim \frac{M_P}{\mathcal{V}^{1/3}} \sim 10^{14}$GeV
- All three scales (plus flavour universality) are yoked in an attractive fashion.
- The origin of all three hierarchies is the exponentially large volume.

Summary

- I have advocated large volumes and an intermediate string scale as an approach to the hierarchies present in nature.
- For these models, I described how to compute modular weights for chiral matter metrics.
- The soft terms are computable and are flavour-universal at leading order.
- The weak, axionic and neutrino scales are yoked in an attractive fashion.
- The different hierarchies come as different powers of the volume.


"That's all Folks!"