

Continuous Global Symmetries in String Compactifications

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This talk is explicitly based on the paper

[0805.4037 \(hep-th\)](#) (C. P. Burgess, JC, L-H. Hung, C. Kom, A. Maharana, F. Quevedo)

and also uses previous work on the LARGE volume scenario.

Talk Structure

1. Symmetries
2. Review of Banks-Dixon
3. Continuous Approximate Global Symmetrie
4. Hyper-Weak Gauge Groups

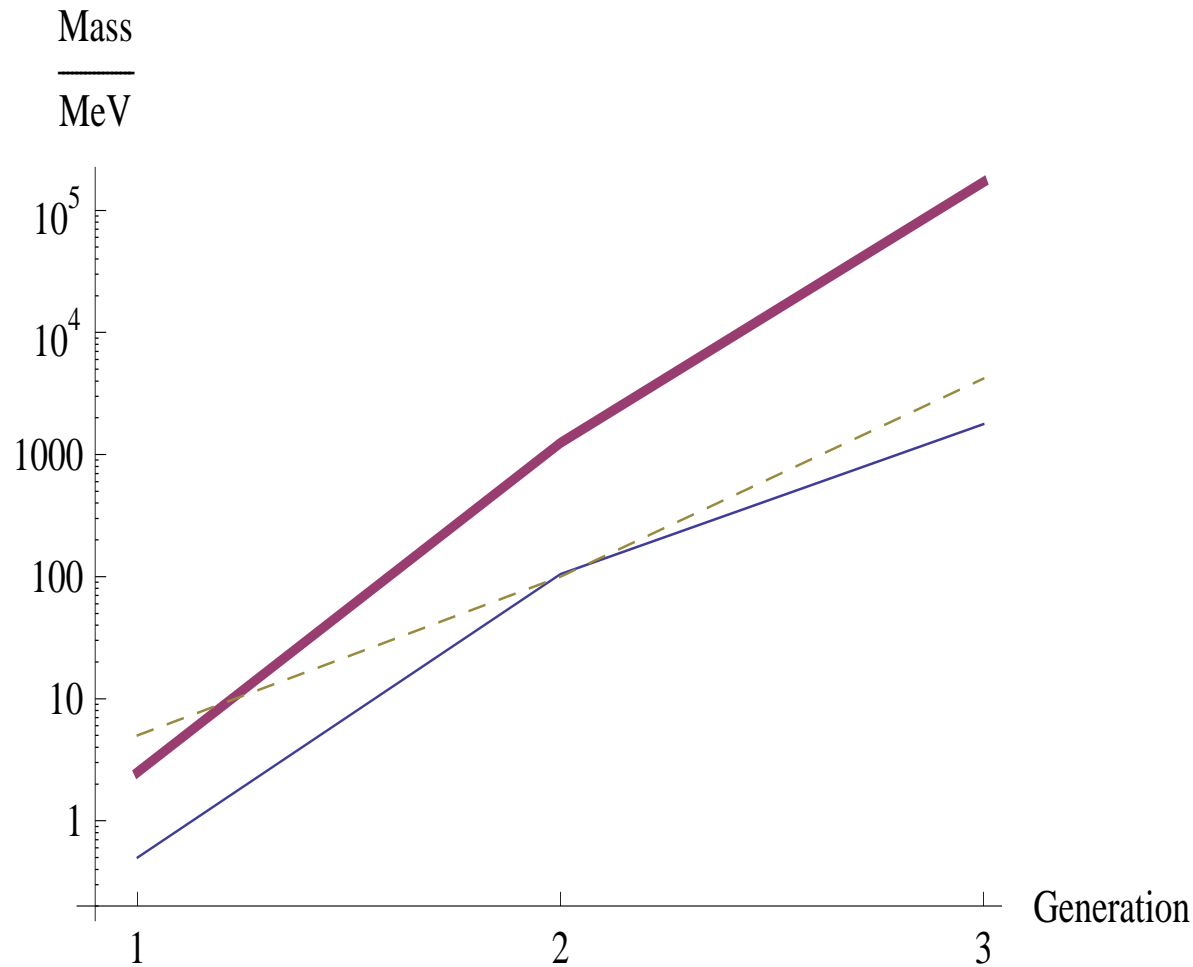
Symmetries

Symmetries are one of the deepest concepts in theoretical physics and play an essential role in the structure of the Standard Model.



Symmetries

Flavour symmetries are very attractive for explaining the fermion mass hierarchies.



Flavour Symmetries

In field theory symmetries are unconstrained.

In string theory there is a no-go theorem:

No continuous exact global symmetries arise in string compactifications. (Banks-Dixon 1988)

It is important to understand the reach of this statement as a constraint on effective field theories derived from string compactification.

Review of Banks-Dixon

In string theory spacetime symmetries come from worldsheet symmetries.

A worldsheet current generates a conserved charge

$$Q = \frac{1}{2\pi i} \oint [dz j(z) - d\bar{z} \bar{j}(\bar{z})]$$

The current j can be used to create vertex operators

$$V = \int d^2 z j \bar{\partial} X^\mu \exp(ik \cdot X), \quad \bar{V} = \int d^2 z \bar{j} \partial X^\mu \exp(ik \cdot X)$$

The operators V generate massless vectors that gauge the worldsheet symmetry.

Review of Banks-Dixon

Two exceptions:

1. Axionic symmetries: all world-sheet fields are uncharged and no world-sheet current exists.

Matter fields are also uncharged and transform trivially.

2. Lorentz symmetry: space-time is non-compact and the currents j and \bar{j} do not transform as conformal fields.

Lorentz symmetry is a global symmetry of non-compact extra dimensions.

What about open strings?

Review of Banks-Dixon

- For open strings a vertex operator is

$$V_{open} = \int_{boundary} dz V(z)$$

with $V(z)$ having conformal dimension $(1,0)$.

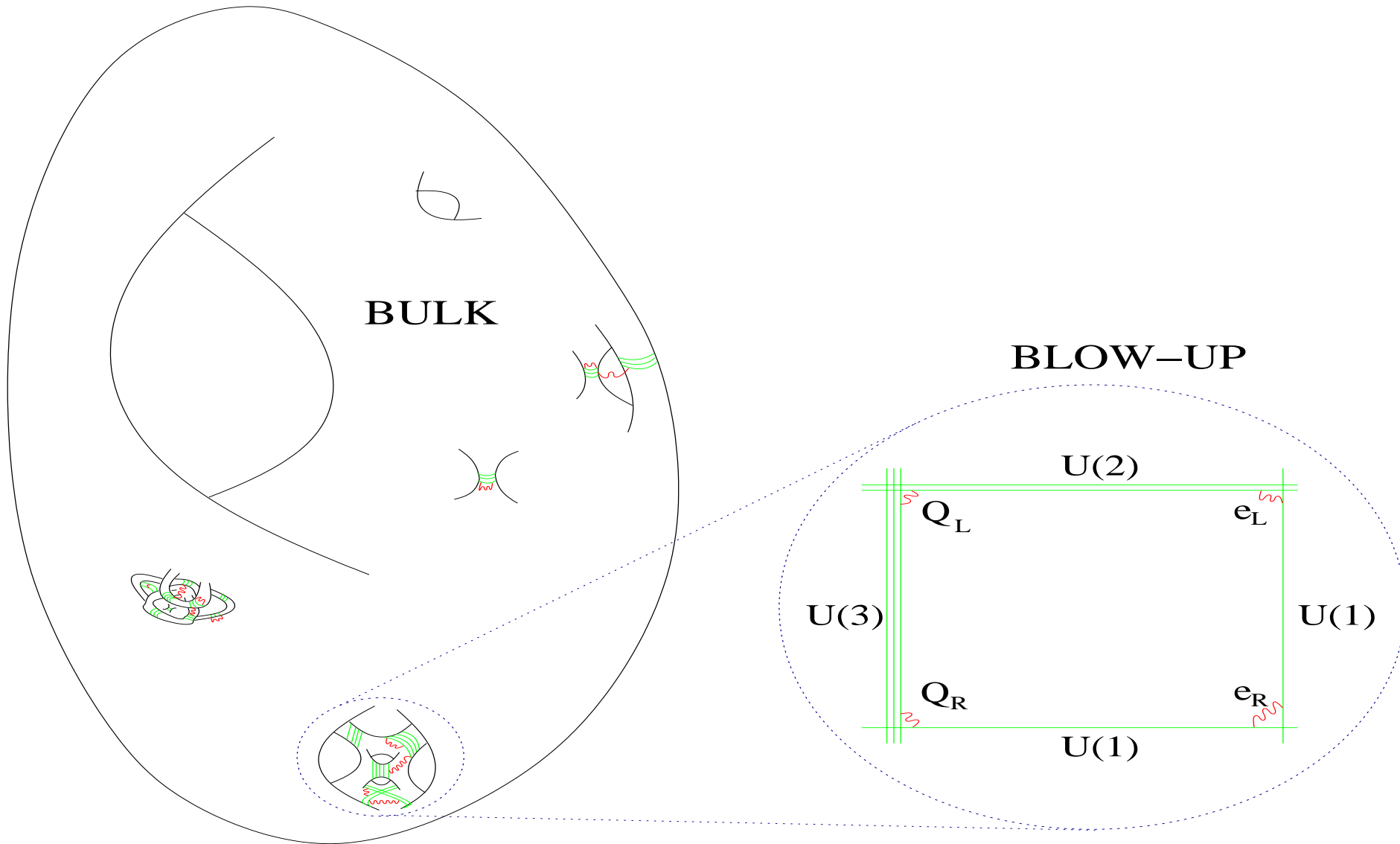
- The operator $A(z) = \partial X^\mu : \exp(ik.X) :$ already has conformal dimension $(1,0)$.
- There is no room to insert extra world-sheet currents in $A(z)$.

Symmetries of the worldsheet are not gauged by the open string sector.

Review of Banks-Dixon

- Closed string sector is needed to gauge open string symmetries.
- If closed string sector is approximately decoupled from open strings, open strings can feel approximate global symmetries.
- This is realised by local models of D-branes in approximately non-compact extra dimensions.
- **Approximate** Lorentz symmetry of non-compact space survives
as an **approximate** global symmetry
of branes in **approximately non-compact** spacetime.

LARGE Volume Models



LARGE Volume Models

- The stabilised volume is exponentially LARGE:

$$\mathcal{V} = W_0 e^{\frac{c}{g_s}}, \quad c \text{ an } \mathcal{O}(1) \text{ constant.}$$

- The Calabi-Yau has a ‘Swiss cheese’ structure.
- There is a large bulk cycle and a small blow-up cycle.
- The LARGE volume lowers the gravitino mass through

$$m_{3/2} = \frac{M_P W_0}{\mathcal{V}}.$$

- A volume of $\mathcal{V} \sim 10^{14} l_s^6$ generates TeV supersymmetry.

LARGE Volume Models

The Standard Model is assumed to be realised as a local D-brane construction on a blow-up cycle.

A volume $\mathcal{V} \sim 10^{14} l_s^6$ also generates

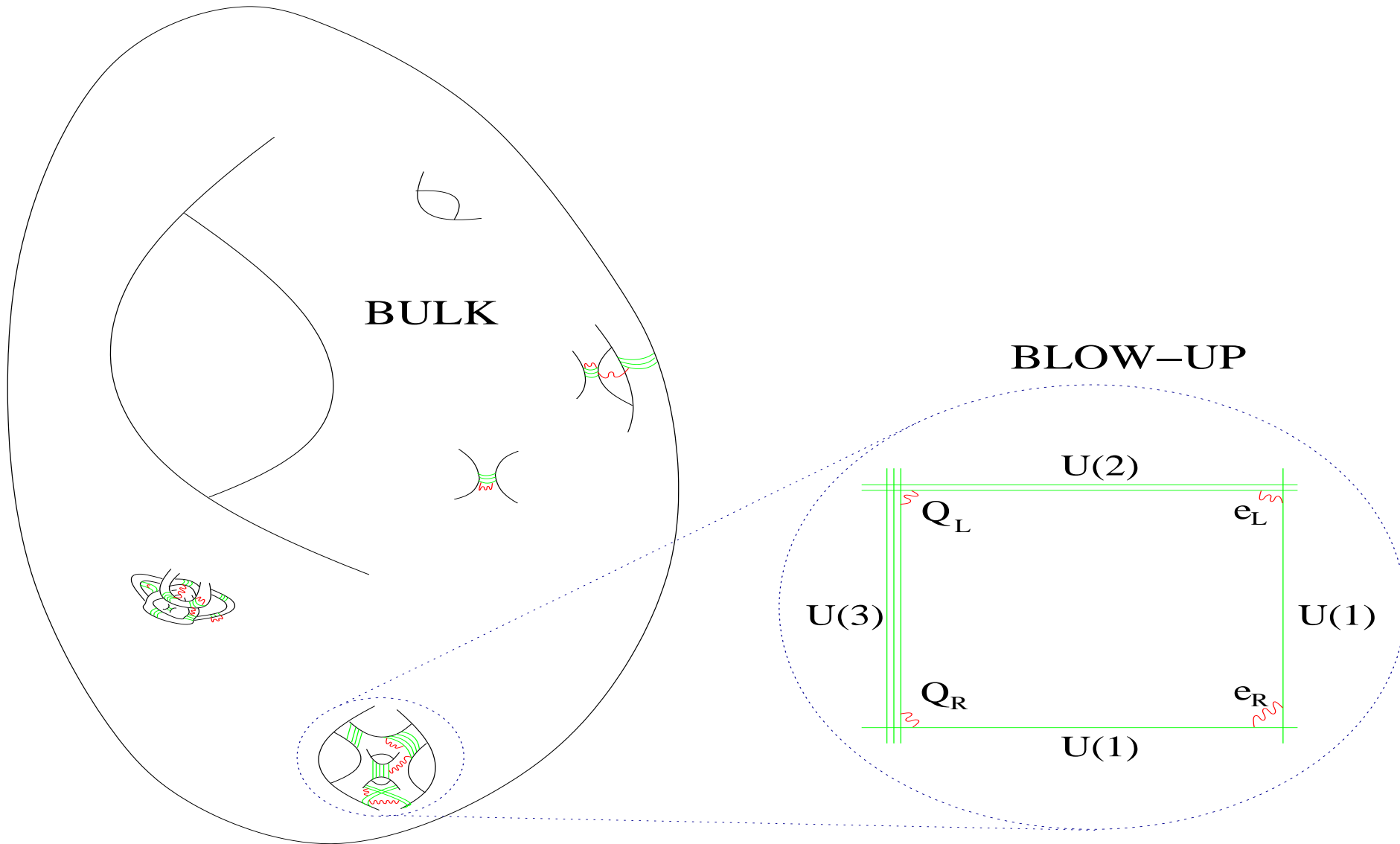
- The axion scale

$$f_a = M_{string} = \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11} \text{GeV}$$

- The neutrino suppression scale $\frac{1}{\Lambda} H_2 H_2 L L$

$$\Lambda \sim M_{string} \mathcal{V}^{1/6} \sim \frac{M_P}{\mathcal{V}^{1/3}} \sim 10^{14} \text{GeV}$$

LARGE Volume Models



Continuous Flavour Symmetries

- In LARGE volume models, the Standard Model is necessarily a local construction.
- The couplings of the Standard Model are determined by the local geometry and are insensitive to the bulk.
- In the limit $\mathcal{V} \rightarrow \infty$, the bulk decouples and all couplings and interactions of the Standard Model are set by the local geometry and metric.
- Global Calabi-Yau metrics are hard - local metrics are known!

Continuous Flavour Symmetries

It is a theorem that compact Calabi-Yaus have no continuous isometries.

Local Calabi-Yau metrics often have isometries. Examples:

1. Flat space \mathbb{C}^3 has an $SO(6)$ isometry.
2. The (resolved) orbifold singularity $\mathbb{C}^3/\mathbb{Z}_3 = \mathcal{O}_{\mathbb{P}^2}(-3)$ has an $SU(3)/\mathbb{Z}_3$ isometry.
3. The conifold geometry $\sum z_i^2 = 0$ has an $SU(2) \times SU(2) \times U(1)$ isometry.

Local metric isometries are (global) flavour symmetries of local brane constructions.

Caveat: no explicit brane construction realising SM

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Continuous Flavour Symmetries

- There are two scales in the geometry - the length scale of the local metric (set by R_s) and the size of the global metric (set by R_b).
- The rescaling $R_s \rightarrow \lambda R_s, R_b \rightarrow \lambda R_b$ is a pure rescaling of the global metric.
- The presence and goodness of the isometry is set by the ratio $\frac{R_s}{R_b}$.
- This determines the extent to which the local non-compact metric is a good approximation in the compact case.

Continuous Flavour Symmetries

- In the limit $\mathcal{V} \rightarrow \infty$ the flavour symmetry becomes exact and the space becomes non-compact.

New massless states exist as the bulk KK modes become massless.

- In the limit of $\mathcal{V} \gg 1$ but finite, the flavour symmetry is approximate, being softly broken.

$$g_{MN,\text{local}}(y) = g_{MN,\text{local},\mathcal{V} \rightarrow \infty}(y) + \delta g_{MN,\text{local},\mathcal{V} \text{ finite}}(y).$$

The full local metric is a perturbation on the non-compact metric.

Continuous Flavour Symmetries

- The breaking parameter is $\frac{R_s}{R_b}$.
- For LARGE volume models we need $\mathcal{V} \sim R_b^6 \sim 10^{14}$ to solve the hierarchy problem.

The breaking parameter is

$$\frac{R_s}{R_b} \sim \frac{1}{\mathcal{V}^{1/6}} \sim 0.01.$$

- This scale is not unattractive for the fermion mass spectrum.

Caveat: it is not clear which power of $\frac{R_s}{R_b}$ enters into the Yukawa couplings....

Continuous Flavour Symmetries

Phenomenological discussions of flavour symmetries start with a symmetry group

$$G_{SM} \times G_F = G_{SU(3) \times SU(2) \times U(1)} \times G_F.$$

Flavons Φ are charged under G_F and not under G_{SM} .

SM matter C_i is charged under both G_F and G_{SM} .

$$W = (\Phi_\alpha \Phi_\beta \Phi_\gamma \dots) C_i C_j C_k.$$

Flavon vevs break G_F and generate Yukawa textures.

The order parameter for G_F breaking is $\langle \Phi \rangle$.

What are the flavons in our case???

Continuous Flavour Symmetries

A puzzle:

- In 4d effective theory, flavour symmetry breaking is parametrised by the ratio $\frac{R_s}{R_b} = \frac{\tau_s^{1/4}}{\tau_b^{1/4}}$.

This sets the relative size of the bulk and local cycles.

- However $\frac{R_s}{R_b}$ is a real singlet while the flavour symmetry group is non-Abelian.
- $\frac{R_s}{R_b}$ can only be in a trivial representation of G_F .
- So there are no flavons in the 4d effective field theory!

Continuous Flavour Symmetries

- There are indeed no flavons in the 4d effective field theory!
- The approximate isometry comes from the full Calabi-Yau metric.

$$g_{MN,\text{local}}(y) = g_{MN,\text{local},\mathcal{V}\rightarrow\infty}(y) + \delta g_{MN,\text{local},\mathcal{V}\text{ finite}}(y).$$

- The flavon modes that are charged under G_F are the higher-dimensional (Kaluza-Klein) modes.

Yau's theorem implies that the vevs of KK modes are entirely set by the moduli vevs.

From a 4d perspective, it is the vevs of KK modes that break the flavour symmetry.

Continuous Flavour Symmetries

The bulk KK modes are massive but are hierarchically lighter than the local KK modes:

$$M_{KK,local} = \frac{M_S}{R_s}, \quad M_{KK,bulk} = \frac{M_S}{R_b}.$$

The local isometry is in no way a symmetry of the bulk.

- Bulk KK modes can be regarded as an infinite number of vector bosons ‘gauging’ the approximate local isometry.
- However bulk KK modes are never in the 4D EFT.
- The only locus in moduli space where bulk KK bosons are massless is the decompactification limit.

Continuous Flavour Symmetries

- (Approximate) flavour symmetries arise from (approximate) isometries of the non-compact Calabi-Yau.
- Symmetries occur for local brane models and do not hold for global models.
- Flavour symmetries may be Abelian or non-Abelian and are global symmetries within 4d EFT.
- The symmetries are exact at infinite volume and are broken for any finite value of the volume.
- The breaking parameter is (R_s/R_b) : LARGE volume implies a small breaking parameter for the flavour symmetry.

Warped Models and Speculation

Warped models:

- Highly warped space-times (RS/Klebanov-Strassler) also have isometries that are good local symmetries and are badly broken in the bulk.
- The quality of the symmetry is set by the strength of the warping.
- For brane constructions in warped throats, such isometries may serve as approximate global flavour symmetries.

Warped Models and Speculation

Speculation:

- Approximate global (flavour) symmetries can occur in both warped and LARGE volume geometries.
- Both geometries also generate hierarchies and have $M_{string} \ll M_{planck}$.
- The breaking of the symmetry is set by $\left(\frac{M_{string}}{M_{Planck}}\right)^k$.
- Is this a coincidence?
- Does the existence of geometric flavour symmetries in 4d effective field theory signal a cutoff $M_{string} \ll M_{planck}$?

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- Does the electron mass tell us the string scale??

Hyper-Weak Gauge Groups

- In LARGE volume models, the Standard Model is a local construction and branes only wrap small cycles.
- There are also bulk cycles associated to the overall volume. These have cycle size

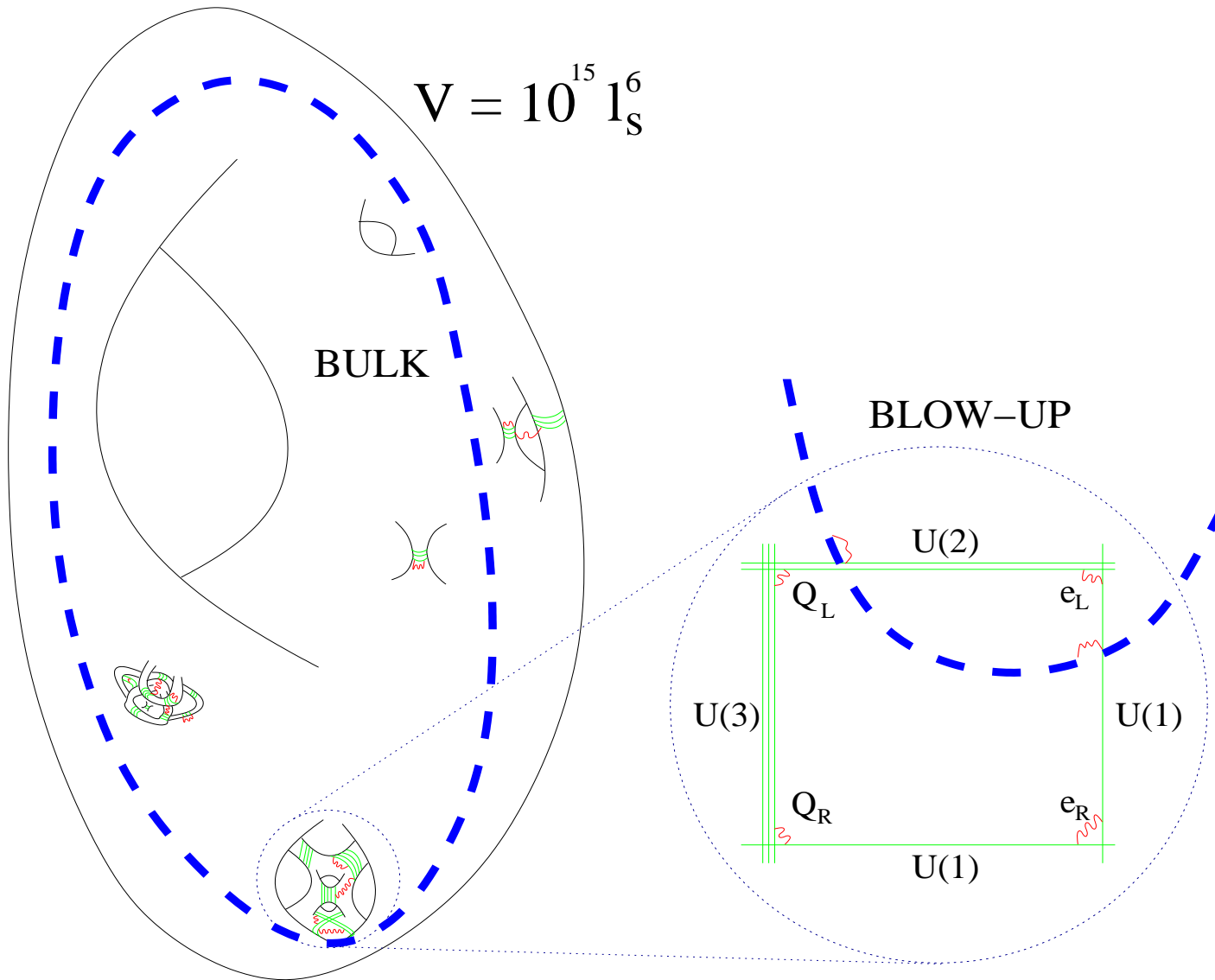
$$\tau_b \sim \mathcal{V}^{2/3} \sim 10^9.$$

- There is no reason not to have D7 branes wrapping these cycles!
- The gauge coupling for such branes is

$$\frac{4\pi}{g^2} = \tau_b$$

with $g \sim 10^{-4}$.

Hyper-Weak Gauge Groups



Hyper-Weak Gauge Groups

In LARGE volume models it is a generic expectation that there will exist additional gauge groups with very weak coupling

$$\alpha^{-1} \sim 10^9.$$

Two phenomenological questions to ask:

1. How heavy is the hyper-weak Z' gauge boson?
2. How does Standard Model matter couple to the hyper-weak force?

Hyper-Weak Gauge Groups

- Standard Model can couple directly to bulk D7.
- If weak-scale vevs of H_1, H_2 break the bulk D7 gauge group, then

$$M_{Z'} \sim gv \sim 10^{-4} \times 100\text{GeV} \sim 10\text{MeV}.$$

- Gauge group may couple directly to electrons, but with

$$g_e \sim 10^{-4}, \quad g_e^2 \sim 10^{-8}.$$

- In this case we have a new (relatively) light very weakly coupled gauge boson.

Conclusions

- Effective field theories of string compactifications can have continuous (approximate) global symmetries.
- Continuous global symmetries hold for local brane models.
- (Approximate) global symmetries come from an (approximately) non-compact space time.
- Required geometries are naturally realised in the LARGE volume models.
- The physics that generates the electroweak hierarchy may also generate the flavour hierarchy.