Gauge Threshold Corrections for Local String Models

Joseph Conlon (Oxford University)

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Local vs Global

Model-building in string theory can either be local or global.

Global models:
- Canonical example is weakly coupled heterotic string.
- Model specification requires global consistency conditions.
- Relies on geometry of entire compact space
- Limit $V \rightarrow \infty$ also gives $\alpha_{SM} \rightarrow 0$: cannot separate string and Planck scales.
- Other examples: IIA/IIB intersecting brane worlds, M-theory on G2 manifolds
Local vs Global

Local models:

- Canonical example branes at singularity
- Model specification only requires knowledge of local geometry and local tadpole cancellation.
- Full consistency depends on existence of a compact embedding of the local geometry.
- Standard Model gauge and Yukawa couplings remain finite in the limit $\mathcal{V} \to \infty$.

It is possible to have $M_P \gg M_s$ by taking $\mathcal{V} \to \infty$.

- Examples: LARGE volume models, branes at singularities, IIB/F-theory GUTs.
Joseph Conlon (Oxford University)  

Gauge Threshold Corrections for Local String Models
Threshold corrections $\Delta_a(M, \bar{M})$ are difference between naive and actual gauge coupling running:

\[
\frac{1}{g_a^2(\mu)} = \frac{1}{g_a^2} \bigg|_0 + \beta_a \ln \left( \frac{M_s^2}{\mu^2} \right) + \Delta_a(M, \bar{M}).
\]

Arise from heavy KK/string/winding states.
Threshold Corrections in Supergravity

In supergravity, physical and holomorphic gauge couplings are related by Kaplunovsky-Louis formula:

\[
g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = \text{Re}(f_a(\Phi)) + \frac{b_a}{16\pi^2} \ln \left( \frac{M_P^2}{\mu^2} \right) + \frac{T(G)}{8\pi^2} \ln \left( g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) \right) + \left( \sum_r n_r T_a(r) - T(G) \right) \hat{K}(\Phi, \bar{\Phi}) - \sum_r \frac{T_a(r)}{8\pi^2} \ln \det Z^r(\Phi, \bar{\Phi}, \mu).
\]

(Holomorphic coupling)

(\(\beta\)-function running)

(NSVZ term)

(Kähler-Weyl anomaly)

(Konishi anomaly)

Relates *measurable* couplings and *holomorphic* couplings.
For local models in IIB

- Kähler potential $\hat{K}$ is given by

$$\hat{K} = -2 \ln \mathcal{V} + \ldots$$

- Matter kinetic terms $\hat{Z}$ are given by

$$\hat{Z} = \frac{f(\tau_{SM})}{\mathcal{V}^{2/3}}$$

Why? When we decouple gravity the physical couplings

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{\hat{Z}_\alpha \hat{Z}_\beta \hat{Z}_\gamma}}$$

should remain finite and be $\mathcal{V}$-independent.
\[ \hat{K} = -2 \ln \mathcal{V}, \quad \hat{Z} = \frac{f(\tau_{SM})}{\mathcal{V}^{2/3}} \]

- Local models require a LARGE bulk volume (\( \mathcal{V} \sim 10^4 \) for \( M_s \sim M_{GUT} \), \( \mathcal{V} \sim 10^{15} \) for \( M_s \sim 10^{11} \) GeV).
- Kähler and Konishi anomalies are formally one-loop-suppressed. However if volume is LARGE, both anomalies are enhanced by \( \ln \mathcal{V} \) factors.
- This implies the existence of large anomalous contributions to physical gauge couplings!
Plug in $\hat{K} = -2 \ln V$ and $\hat{Z} = \frac{1}{V^{2/3}}$ into Kaplunovsky-Louis formula.

We restrict to terms enhanced by $\ln V$ and obtain:

$$g_{\text{phys}}^{-2}(\Phi, \bar{\Phi}, \mu) = \text{Re}(f_a(\Phi)) + \frac{\left(\sum_r n_r T_a(r) - 3 T_a(G)\right)}{8\pi^2} \ln \left(\frac{M_P}{V^{1/3} \mu}\right)$$

$$g_{\text{phys}}^{-2}(\Phi, \bar{\Phi}, \mu) = \text{Re}(f_a(\Phi)) + \beta_a \ln \left(\frac{(RM_s)^2}{\mu^2}\right).$$

- Gauge couplings start running from an effective scale $RM_s$ rather than $M_s$.
- Universal $\text{Re}(f_a(\Phi))$ implies unification occurs at a super-stringy scale $RM_s$ rather than $M_s$.
- Want to check and understand this in string theory!
Threshold Corrections in String Theory

- Calculate using the **background field method**.
- Running gauge couplings are the 1-loop coefficient of

\[
\frac{1}{4g^2} \int d^4x \sqrt{g} F^a_{\mu \nu} F^{a,\mu \nu}
\]

- Turn on background magnetic field \( F_{23} = B \) and compute the quantised string spectrum.
- Use the string partition function to compute the 1-loop vacuum energy

\[
\Lambda = \Lambda_0 + \frac{1}{2} \left( \frac{B}{2\pi^2} \right)^2 \Lambda_2 + \frac{1}{4!} \left( \frac{B}{2\pi^2} \right)^4 \Lambda_4 + \ldots
\]

- From \( \Lambda_2 \) term we can extract beta function running and threshold corrections.
String theory 1-loop vacuum function given by partition function

$$\Lambda_{1-loop} = \frac{1}{2}(T + KB + A(B) + MS(B)).$$

- Require $O(B^2)$ term of this expansion.
- Background magnetic field only shifts moding of open string states.
- Torus and Klein Bottle amplitudes do not couple to open strings.
- Only annulus and Möbius strip amplitudes contribute at $O(B^2)$. 
We want examples of calculable local models with non-zero beta functions.

- The simplest such examples are (fractional) D3 branes at orbifold/orientifold singularities.
- String can be exactly quantised and all calculations can be performed explicitly.

We have studied

- D3 branes on $\mathbb{C}^3/\mathbb{Z}_4$, $\mathbb{C}^3/\mathbb{Z}_6$, $\mathbb{C}^3/\mathbb{Z}_6'$, $\mathbb{C}^3/\Delta_{27}$.
- D3/D7 systems on $\mathbb{C}^3/\mathbb{Z}_3$.
- D3/O3 on $\mathbb{C}^3/\{I\Omega, \mathbb{Z}_4\}$, $\mathbb{C}^3/\{I\Omega, \mathbb{Z}_6\}$, $\mathbb{C}^3/\{I\Omega, \mathbb{Z}_6'\}$. 

Joseph Conlon (Oxford University) 

Gauge Threshold Corrections for Local String Models
We separately evaluate each amplitude in the $\theta^N$ sector (e.g. $\mathbb{Z}_4$)

$$A(B) = \int_0^\infty \frac{dt}{2t} \text{STr} \left( \left( \frac{1 + \theta + \theta^2 + \theta^3}{4} \right) \left( 1 + \frac{(-1)^F}{2} \frac{1}{2} q^{(p^\mu p_\mu + m^2)/2} \right) \right)$$

$$M(B) = \int_0^\infty \frac{dt}{2t} \text{STr} \left( \left( \frac{1 + \theta + \theta^2 + \theta^3}{4} \right) \left( 1 + \frac{(-1)^F}{2} \frac{\Omega}{2} q^{(p^\mu p_\mu + m^2)/2} \right) \right)$$

- $\theta^0 = (1, 1, 1)$ is an ‘$\mathcal{N} = 4$’ sector.
- $\theta^1 = (1/4, 1/4, -1/2)$ and $\theta^3 = (-1/4, -1/4, 1/2)$ are ‘$\mathcal{N} = 1$’ sectors.
- $\theta^2 = (1/2, 1/2, 0)$ is an ‘$\mathcal{N} = 2$’ sector.

The amplitudes reduce to products of Jacobi $\vartheta$-functions with different prefactors.
$\mathcal{N} = 1$ amplitudes

\[ A_{\mathcal{N}=1}^{(k)} = -\int \frac{dt}{2t} \frac{1}{(2\pi^2 t)} \sum_{\alpha, \beta=0, 1/2} \frac{\eta_{\alpha\beta}}{2} \text{Tr} \left( \gamma_{\theta^k} \otimes \gamma_{\theta^k}^{-1} \frac{i(\beta_1 + \beta_2)}{2\pi^2} \frac{\vartheta \left[ \begin{array}{c} \alpha \\ \beta \end{array} \right] \left( \frac{i\epsilon t}{2} \right)}{\vartheta \left[ \begin{array}{c} 1/2 \\ 1/2 \end{array} \right] \left( \frac{i\epsilon t}{2} \right)} \right) \]

\[ \times \prod_{i=1}^{3} \left( -2 \sin \left( \pi \theta_i^k \right) \right) \frac{\vartheta \left[ \begin{array}{c} \alpha \\ \beta + \theta_i^k \end{array} \right]}{\vartheta \left[ \begin{array}{c} 1/2 \\ 1/2 + \theta_i^k \end{array} \right]} \]

\[ M_{\mathcal{N}=1}^{(k)} = 2 \int_0^\infty \frac{dt}{2t} \frac{1}{(2\pi^2 t)} \sum_{\alpha, \beta=0, 1/2} \frac{\eta_{\alpha\beta}}{2} \text{Tr} \left[ \frac{i}{2\pi^2} \beta \gamma_{\Omega_k^T} \gamma_{\Omega_k'}^{-T} \frac{\vartheta \left[ \begin{array}{c} \alpha \\ \beta \end{array} \right] \left( \frac{i\epsilon t}{2} \right)}{\vartheta \left[ \begin{array}{c} 1/2 \\ 1/2 \end{array} \right] \left( \frac{i\epsilon t}{2} \right)} \right] \]

\[ \times \prod_{i=1}^{3} \left( -2 \sin \left( \pi R_i^k \right) \right) \frac{\vartheta \left[ \begin{array}{c} \alpha \\ \beta + R_i^k \end{array} \right]}{\vartheta \left[ \begin{array}{c} 1/2 \\ 1/2 + R_i^k \end{array} \right]} \]
$N = 2$ amplitudes

$$A_{N=2}^{(k)} = - \int \frac{dt}{2t} \frac{1}{(2\pi^2 t)} \sum_{\alpha,\beta=0,1/2} \frac{\eta_{\alpha\beta}}{2} (-1)^{2\alpha} \text{Tr} \left( \gamma_{\theta^k} \otimes \gamma_{\theta^k}^{-1} \frac{i(\beta_1 + \beta_2)}{2\pi^2} \frac{\vartheta \left[ \frac{\alpha}{\beta} \right] (\frac{i\epsilon t}{2})}{\vartheta \left[ \frac{1/2}{1/2} \right] (\frac{i\epsilon t}{2})} \right)$$

$$\times \frac{\vartheta \left[ \frac{\alpha}{\beta} \right]}{\eta^3} \prod_{i=1}^{2} \frac{(-2\sin \pi \theta_{i}^k)}{\vartheta \left[ \frac{1/2}{1/2 + \theta_{i}^k} \right]}.$$
$\mathcal{N} = 1$ sector

$$A_{\mathcal{N}=1} + M_{\mathcal{N}=1} \xrightarrow{t \to \infty} \int \frac{dt}{2t} \left( \frac{B}{2\pi^2} \right)^2 \times \beta_{\mathcal{N}=1}$$

$$A_{\mathcal{N}=1} + M_{\mathcal{N}=1} \xrightarrow{t \to 0} \int \frac{dt}{t^2} \left( \frac{B}{2\pi^2} \right)^2 \left( 0 + O(e^{-1/t}) \right).$$

- In IR $t \to \infty$ limit, obtain contribution of $\mathcal{N} = 1$ sectors to $\beta$-functions.
- In UV $t \to 0$ limit, amplitudes vanish as local tadpole cancellation is enforced.
- Field theory running cut off at $M_s$ as string states contribute to amplitude.
\( \mathcal{N} = 2 \) sector

\[
\mathcal{A}_{\mathcal{N}=2} + \mathcal{M}_{\mathcal{N}=2} \xrightarrow{t \to \infty} \int \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \times \beta_{\mathcal{N}=2}^a
\]

\[
\mathcal{A}_{\mathcal{N}=2} + \mathcal{M}_{\mathcal{N}=2} \xrightarrow{t \to 0} \int \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \times \beta_{\mathcal{N}=2}^a
\]

- In IR \( t \to \infty \) limit, obtain contribution of \( \mathcal{N} = 2 \) sectors to \( \beta \)-functions.
- In UV \( t \to 0 \) limit, amplitudes unaltered.
- Consequence of \( \mathcal{N} = 2 \) susy: all open string oscillators non-BPS and decouple.
- How should we interpret this?
$\mathcal{N} = 2$ sector

\[
\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_{N=2}^a
\]

- Divergence in $t \to \infty$ limit is physical: this is the IR limit and we recover ordinary $\beta$-function running.
- Divergence in $t \to 0$ limit is unphysical: open string UV limit and this amplitude must be finite in a consistent string theory.
- Physical understanding of the divergence is best understood from closed string channel.
$\mathcal{N} = 2$ sector

Annulus amplitude:

Annulus amplitude in $t \rightarrow 0$ limit:
$\mathcal{N} = 2$ sector

- $t \to 0$ divergence corresponds to a source for a partially twisted RR 2-form.
- In the local model this propagates into the bulk of the Calabi-Yau.
- In a compact model this becomes a physical divergence and tadpole must be cancelled by bulk sink branes/O-planes.
- Tadpole is sourced locally but cancelled globally
The purely local computation omits the following worldsheets:
\( \mathcal{N} = 2 \) sector

- The purely local string computation includes all open string states for \( t > 1/(RM_s)^2 \), i.e. \( M < RM_s \).

However for \( t < 1/(RM_s)^2 \) we must include new winding states in the partition function.

- These are essential for global consistency but are omitted by a purely local computation.

- Threshold corrections become finite

\[
\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a \rightarrow \int_{1/(RM_s)^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a
\]

- Effective UV cutoff is actually \( RM_s \) and not \( M_s \)!
Summary and Matching Field Theory

Running takes the form

$$\frac{1}{g^2(\mu)} = \left. \frac{1}{g^2} \right|_0 + \beta_a \ln \left( \frac{M_s^2}{\mu^2} \right) + \beta_a^{\mathcal{N}=2} \ln \left( \frac{(RM_s)^2}{M_s^2} \right).$$

- $\mathcal{N} = 1$ sectors run from $M_s$ into the IR.
- $\mathcal{N} = 2$ sectors run from $RM_s$ into the IR.

How to reconcile with Kaplunovsky-Louis?

$$g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = \text{Re}(f_a(\Phi)) + \beta_a \ln \left( \frac{(RM_s)^2}{\mu^2} \right).$$
Summary and Matching Field Theory

- At the singularity holomorphic gauge couplings are

\[ f_a = S + s_{ak} M_k \]

- \( M_k \) is the twisted blow-up field.

- At 1-loop,

\[ M_k \rightarrow M_k + \frac{\alpha}{16\pi^2} \ln \mathcal{V} \]

and \( \langle M_k \rangle \neq 0 \).

- Coefficients \( s_{ak} \) are \( \mathcal{N} = 1 \) contributions to running \( \beta_{ak}^{\mathcal{N}=1} \).
Summary and Matching Field Theory

Running takes the form

\[
\frac{1}{g^2}(\mu) = \left. \frac{1}{g^2} \right|_0 + (\beta_{a}^{N=1} + \beta_{a}^{N=2}) \ln \left( \frac{M_s^2}{\mu^2} \right) + \beta_{a}^{N=2} \ln \left( \frac{(RM_s)^2}{M_s^2} \right).
\]

Kaplunovsky-Louis:

\[
g_{\text{phys}}^{-2}(\Phi, \bar{\Phi}, \mu) = S + \beta_{a}^{N=1} T + (\beta_{a}^{N=1} + \beta_{a}^{N=2}) \ln \left( \frac{(RM_s)^2}{\mu^2} \right).
\]

\(T\) obtains a one-loop vev at the singularity:

\[
\langle T \rangle = - \ln \left( \frac{(RM_s)^2}{M_s^2} \right).
\]
Summary and Matching Field Theory

For D3 @ orbifold singularities, there are no $\mathcal{N}=1$ contributions to $\beta$ functions (vanishing of twisted tadpoles).

- Gauge coupling unification occurs at $RM_s$.

For D3/D7 @ orbifold singularities, 33 and 37 worldsheets combine to give $\mathcal{N}=1$ contribution.

- In general no gauge coupling unification, for $\mathbb{Z}_3$ singularity unification at $M_s$.

For D3 @ orientifold singularities, annulus and Mobius worldsheets give $\mathcal{N}=1$ contribution.

- Running starts at $RM_s$, with non-universal shift at $M_s$: no gauge coupling unification.
Local GUTs

- Proposal to realise $SU(5)$ GUTs through branes on del Pezzo with hypercharged flux.
- Flux is quantised on cycle that is non-trivial in $H^2(dP, \mathbb{Z})$ but trivial in $H^2(CY, \mathbb{Z})$.
- Holomorphic gauge couplings are

\[
\begin{align*}
  f_{SU(3)} &= T_{dP} + h_3(F)S + \epsilon_3(U), \\
  f_{SU(2)} &= T_{dP} + h_2(F)S + \epsilon_2(U), \\
  f_{U(1)_Y} &= T_{dP} + h_1(F)S + \epsilon_Y(U).
\end{align*}
\]

- For gauge unification assume $h_3 = h_2 = h_1$ and $\epsilon_i$ is negligible.
Local GUTs

\[ \text{CY}_3 \]

\[ dP_n \]

\[ F_Y \]

\[ R \propto I_s \]
Local GUTs: Field Theory

\[ f_{SU(3)} = f_{SU(2)} = f_Y = T + h(F)S. \]

\[ g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = \text{Re}(f_a(\Phi)) + \beta_a \ln \left( \frac{(RM_s)^2}{\mu^2} \right). \]

- Kaplunovsky-Louis implies gauge unification occurs at \( RM_s \) rather than \( M_s \).
- Field redefinitions are universal and cannot alter universality of gauge kinetic functions or affect result.
- Bulk does not decouple and enters the unification scale.
Can understand this from the string perspective:

- $T_{dP}$ is an $\mathcal{N} = 1$ sector: associated to $SU(5)$ universal physics.
- GUT breaking comes from hypercharge flux associated to an $\mathcal{N} = 2$ sector.
- Locally $\mathcal{F}_Y$ sources tadpole divergence via

$$\int_{dP} C_{2,Y} \wedge \mathcal{F}_Y$$

- Globally $C_{2,Y}$ is absent: divergence regulated at scale $RM_s$ and gauge couplings run to this scale.
Local GUTs: Warping and Exotica

Can apply ideas to warped throats:

- Suppose GUT realised on del Pezzo down a warped throat.
- Hypercharge cycle trivialises outside throat.
- Finiteness of threshold corrections requires knowledge of tadpole cancellation.
- In closed string channel string must leave throat, in open string channel need to include string states reaching out of throat and into bulk - with mass $M_P$.
- Running continues until Planck scale - possibility of TeV warped throat with gauge unification close to $M_P$?
Conclusions

- Lots of interesting physics associated to running gauge couplings for local models.
- For $\mathcal{N} = 1$ sectors, gauge couplings run from $M_s$.
- For $\mathcal{N} = 2$ sectors, gauge couplings run from $RM_s$.
- For local GUTs with hypercharge flux breaking, unification scale is $RM_s$ and not $M_s$.

Bulk cannot be consistently decoupled from the gauge theory.