# Gauge Threshold Corrections for Local String Models

Joseph Conlon (Oxford University)

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# Local vs Global

Model-building in string theory can either be local or global.

Global models:

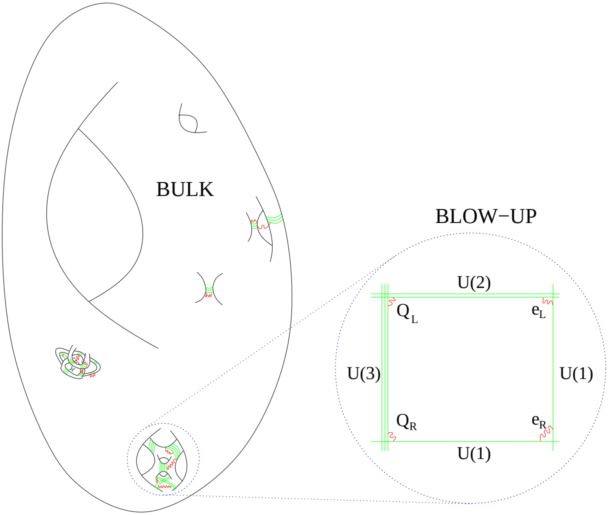
- Canonical example is weakly coupled heterotic string.
- Model specification requires global consistency conditions.
- Relies on geometry of entire compact space
- Limit $\mathcal{V} \to \infty$ also gives $\alpha_{SM} \to 0$: cannot separate string and Planck scales.
- Other examples: IIA/IIB intersecting brane worlds, M-theory on G2 manifolds

# Local vs Global

Local models:

- Canonical example branes at singularity
- Model specification only requires knowledge of local geometry and local tadpole cancellation.
- Full consistency depends on existence of a compact embedding of the local geometry.
- Standard Model gauge and Yukawa couplings remain finite in the limit $\mathcal{V} \to \infty$.

  It is possible to have $M_P \gg M_s$ by taking $\mathcal{V} \to \infty$.
- Examples: LARGE volume models, branes at singularities, IIB/F-theory GUTs.


BULK
BLOW−UP
U(2)
$Q_L$
$e_L$
U(3)
U(1)
$Q_R$
$e_R$
U(1)

# Threshold Corrections

Threshold corrections $\Delta_a(M, \bar{M})$ are difference between naive and actual gauge coupling running:

$$\frac{1}{g_a^2(\mu)} = \left.\frac{1}{g_a^2}\right|_0 + \beta_a \ln\left(\frac{M_s^2}{\mu^2}\right) + \Delta_a(M, \bar{M}).$$

Arise from heavy KK/string/winding states.

# Threshold Corrections in Supergravity

In supergravity, physical and holomorphic gauge couplings are related by Kaplunovsky-Louis formula:

$$
\begin{aligned}
g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = & \quad \mathrm{Re}(f_a(\Phi)) && \text{(Holomorphic coupling)} \\
& + \frac{b_a}{16\pi^2} \ln \left( \frac{M_P^2}{\mu^2} \right) && \text{($\beta$-function running)} \\
& + \frac{T(G)}{8\pi^2} \ln g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) && \text{(NSVZ term)} \\
& + \frac{(\sum_r n_r T_a(r) - T(G))}{16\pi^2} \hat{K}(\Phi, \bar{\Phi}) && \text{(Kähler-Weyl anomaly)} \\
& - \sum_r \frac{T_a(r)}{8\pi^2} \ln \det Z^r(\Phi, \bar{\Phi}, \mu). && \text{(Konishi anomaly)}
\end{aligned}
$$

Relates *measurable* couplings and *holomorphic* couplings.

For local models in IIB

- Kähler potential $\hat{K}$ is given by

$$\hat{K} = -2 \ln \mathcal{V} + \dots$$

- Matter kinetic terms $\hat{Z}$ are given by

$$\hat{Z} = \frac{f(\tau_{SM})}{\mathcal{V}^{2/3}}$$

Why? When we decouple gravity the physical couplings

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{\hat{Z}_\alpha \hat{Z}_\beta \hat{Z}_\gamma}}$$

should remain finite and be $\mathcal{V}$-independent.

$$\hat{K} = -2\ln\mathcal{V}, \qquad \hat{Z} = \frac{f(\tau_{SM})}{\mathcal{V}^{2/3}}$$

- Local models require a LARGE bulk volume ($\mathcal{V} \sim 10^4$ for $M_s \sim M_{GUT}$, $\mathcal{V} \sim 10^{15}$ for $M_s \sim 10^{11}\mathrm{GeV}$).
- Kähler and Konishi anomalies are formally one-loop suppressed.
  However if volume is LARGE, both anomalies are enhanced by $\ln\mathcal{V}$ factors.
- This implies the existence of large anomalous contributions to *physical* gauge couplings!

Plug in $\hat{K} = -2 \ln \mathcal{V}$ and $\hat{Z} = \frac{1}{\mathcal{V}^{2/3}}$ into Kaplunovsky-Louis formula.

We restrict to terms enhanced by $\ln \mathcal{V}$ and obtain:

$$g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = \mathrm{Re}(f_a(\Phi)) + \frac{\left(\sum_r n_r T_a(r) - 3 T_a(G)\right)}{8\pi^2} \ln\left(\frac{M_P}{\mathcal{V}^{1/3}\mu}\right)$$

$$g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = \mathrm{Re}(f_a(\Phi)) + \beta_a \ln\left(\frac{(RM_s)^2}{\mu^2}\right).$$

- Gauge couplings start running from an effective scale $RM_s$ rather than $M_s$.
- Universal $\mathrm{Re}(f_a(\Phi))$ implies unification occurs at a super-stringy scale $RM_s$ rather than $M_s$.
- Want to check and understand this in string theory!

# Threshold Corrections in String Theory

- Calculate using the background field method.
- Running gauge couplings are the 1-loop coefficient of

$$\frac{1}{4g^2}\int d^4x\sqrt{g}F^a_{\mu\nu}F^{a,\mu\nu}$$

- Turn on background magnetic field $F_{23} = B$ and compute the quantised string spectrum.
- Use the string partition function to compute the 1-loop vacuum energy

$$\Lambda = \Lambda_0 + \frac{1}{2}\left(\frac{B}{2\pi^2}\right)^2 \Lambda_2 + \frac{1}{4!}\left(\frac{B}{2\pi^2}\right)^4 \Lambda_4 + \dots$$

- From $\Lambda_2$ term we can extract beta function running and threshold corrections.

String theory 1-loop vacuum function given by partition function

$$\Lambda_{1-loop} = \frac{1}{2}(T + KB + A(B) + MS(B)).$$

- Require $\mathcal{O}(B^2)$ term of this expansion.
- Background magnetic field only shifts moding of open string states.
- Torus and Klein Bottle amplitudes do not couple to open strings.
- Only annulus and Möbius strip amplitudes contribute at $\mathcal{O}(B^2)$.

We want examples of calculable local models with non-zero beta functions.

- The simplest such examples are (fractional) D3 branes at orbifold/orientifold singularities.
- String can be exactly quantised and all calculations can be performed explicitly.

We have studied

- D3 branes on $\mathbb{C}^3/\mathbb{Z}_4$, $\mathbb{C}^3/\mathbb{Z}_6$, $\mathbb{C}^3/\mathbb{Z}_6'$, $\mathbb{C}^3/\Delta_{27}$.
- D3/D7 systems on $\mathbb{C}^3/\mathbb{Z}_3$.
- D3/O3 on $\mathbb{C}^3/\{I\Omega, \mathbb{Z}_4\}$, $\mathbb{C}^3/\{I\Omega, \mathbb{Z}_6\}$, $\mathbb{C}^3/\{I\Omega, \mathbb{Z}_6'\}$.

We separately evaluate each amplitude in the $\theta^N$ sector (e.g $\mathbb{Z}_4$)

$$\mathcal{A}(B) = \int_0^\infty \frac{dt}{2t} \,\mathrm{STr}\left( \frac{(1+\theta+\theta^2+\theta^3)}{4} \frac{1+(-1)^F}{2} \frac{1}{2}\, q^{(p^\mu p_\mu + m^2)/2} \right)$$

$$\mathcal{M}(B) = \int_0^\infty \frac{dt}{2t} \,\mathrm{STr}\left( \frac{(1+\theta+\theta^2+\theta^3)}{4} \frac{1+(-1)^F}{2} \frac{\Omega}{2}\, q^{(p^\mu p_\mu + m^2)/2} \right)$$

- $\theta^0 = (1,1,1)$ is an '$\mathcal{N}=4$' sector.
- $\theta^1 = (1/4, 1/4, -1/2)$ and $\theta^3 = (-1/4, -1/4, 1/2)$ are '$\mathcal{N}=1$' sectors.
- $\theta^2 = (1/2, 1/2, 0)$ is an '$\mathcal{N}=2$' sector.

The amplitudes reduce to products of Jacobi $\vartheta$-functions with different prefactors.

# $\mathcal{N} = 1$ amplitudes

$$\mathcal{A}^{(k)}_{\mathcal{N}=1} = -\int \frac{dt}{2t} \frac{1}{(2\pi^2 t)} \sum_{\alpha,\beta=0,1/2} \frac{\eta_{\alpha\beta}}{2} \mathrm{Tr} \left( \gamma_{\theta^k} \otimes \gamma^{-1}_{\theta^k} \frac{i(\beta_1 + \beta_2)}{2\pi^2} \frac{\vartheta\left[\begin{array}{c} \alpha \\ \beta \end{array}\right]\left(\frac{i\epsilon t}{2}\right)}{\vartheta\left[\begin{array}{c} 1/2 \\ 1/2 \end{array}\right]\left(\frac{i\epsilon t}{2}\right)} \right)$$

$$\times \prod_{i=1}^{3} \frac{\left(-2\sin\pi\theta^k_i\right) \vartheta\left[\begin{array}{c} \alpha \\ \beta + \theta^k_i \end{array}\right]}{\vartheta\left[\begin{array}{c} 1/2 \\ 1/2 + \theta^k_i \end{array}\right]}.$$

$$\mathcal{M}^{(k)}_{\mathcal{N}=1} = 2\int_0^\infty \frac{dt}{2t} \frac{1}{(2\pi^2 t)} \sum_{\alpha,\beta=0,1/2} \frac{\eta_{\alpha\beta}}{2} \mathrm{Tr} \left[ \frac{i}{2\pi^2} \beta \gamma_{\Omega'_k} \gamma^{-T}_{\Omega'_k} \frac{\vartheta\left[\begin{array}{c} \alpha \\ \beta \end{array}\right]\left(\frac{i\epsilon t}{2}\right)}{\vartheta\left[\begin{array}{c} 1/2 \\ 1/2 \end{array}\right]\left(\frac{i\epsilon t}{2}\right)} \right]$$

$$\times \prod_{i=1}^{3} \left(-2\sin\left(\pi R^k_i\right)\right) \frac{\vartheta\left[\begin{array}{c} \alpha \\ \beta + R^k_i \end{array}\right]}{\vartheta\left[\begin{array}{c} 1/2 \\ 1/2 + R^k_i \end{array}\right]}.$$

# $\mathcal{N} = 2$ amplitudes

$$\mathcal{A}^{(k)}_{\mathcal{N}=2} = -\int \frac{dt}{2t}\frac{1}{(2\pi^2 t)} \sum_{\alpha,\beta=0,1/2} \frac{\eta_{\alpha\beta}}{2}(-1)^{2\alpha}\,\mathrm{Tr}\left(\gamma_{\theta^k}\otimes\gamma^{-1}_{\theta^k}\frac{i(\beta_1+\beta_2)}{2\pi^2}\frac{\vartheta\left[\begin{array}{c}\alpha\\ \beta\end{array}\right]\left(\frac{i\epsilon t}{2}\right)}{\vartheta\left[\begin{array}{c}1/2\\ 1/2\end{array}\right]\left(\frac{i\epsilon t}{2}\right)}\right)$$

$$\times\frac{\vartheta\left[\begin{array}{c}\alpha\\ \beta\end{array}\right]}{\eta^3}\prod_{i=1}^{2}\frac{\left(-2\sin\pi\theta_i^k\right)\vartheta\left[\begin{array}{c}\alpha\\ \beta+\theta_i^k\end{array}\right]}{\vartheta\left[\begin{array}{c}1/2\\ 1/2+\theta_i^k\end{array}\right]}.$$

$$\mathcal{M}^{(k)}_{\mathcal{N}=2} = 2\int \frac{dt}{2t}\frac{1}{(2\pi^2 t)} \sum_{\alpha,\beta=0,1/2} \frac{\eta_{\alpha\beta}}{2}(-1)^{2\alpha}\,\mathrm{Tr}\left(\frac{i}{2\pi^2}\beta\gamma_{\Omega'_k}\gamma^{-T}_{\Omega'_k}\frac{\vartheta\left[\begin{array}{c}\alpha\\ \beta\end{array}\right]\left(\frac{i\epsilon t}{2}\right)}{\vartheta\left[\begin{array}{c}1/2\\ 1/2\end{array}\right]\left(\frac{i\epsilon t}{2}\right)}\right)$$

$$\times\frac{\vartheta\left[\begin{array}{c}\alpha\\ \beta\end{array}\right]}{\eta^3}\prod_{i=1}^{2}\frac{\left(-2\sin\pi R_i^k\right)\vartheta\left[\begin{array}{c}\alpha\\ \beta+R_i^k\end{array}\right]}{\vartheta\left[\begin{array}{c}1/2\\ 1/2+R_i^k\end{array}\right]}.$$

# $\mathcal{N} = 1$ sector

$$\mathcal{A}_{\mathcal{N}=1} + \mathcal{M}_{\mathcal{N}=1} \xrightarrow[t \to \infty]{} \int \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \times \beta_a^{\mathcal{N}=1}$$

$$\mathcal{A}_{\mathcal{N}=1} + \mathcal{M}_{\mathcal{N}=1} \xrightarrow[t \to 0]{} \int \frac{dt}{t^2} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \left( 0 + \mathcal{O}(e^{-1/t}) \right).$$

- In IR $t \to \infty$ limit, obtain contribution of $\mathcal{N} = 1$ sectors to $\beta$-functions.
- In UV $t \to 0$ limit, amplitudes vanish as local tadpole cancellation is enforced.
- Field theory running cut off at $M_s$ as string states contribute to amplitude.

# $\mathcal{N} = 2$ sector

$$\mathcal{A}_{\mathcal{N}=2} + \mathcal{M}_{\mathcal{N}=2} \xrightarrow[t\to\infty]{} \int \frac{dt}{2t}\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2 \times \beta_a^{\mathcal{N}=2}$$

$$\mathcal{A}_{\mathcal{N}=2} + \mathcal{M}_{\mathcal{N}=2} \xrightarrow[t\to 0]{} \int \frac{dt}{2t}\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2 \times \beta_a^{\mathcal{N}=2}$$

- In IR $t \to \infty$ limit, obtain contribution of $\mathcal{N} = 2$ sectors to $\beta$-functions.
- In UV $t \to 0$ limit, amplitudes unaltered.
- Consequence of $\mathcal{N} = 2$ susy: all open string oscillators non-BPS and decouple.
- How should we interpret this?

# $\mathcal{N} = 2$ sector

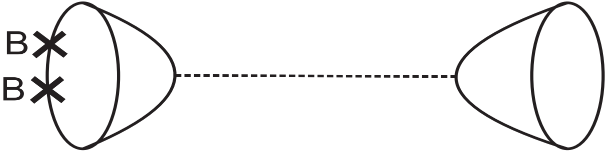
$$\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a^{\mathcal{N}=2}$$

- Divergence in $t \to \infty$ limit is physical: this is the IR limit and we recover ordinary $\beta$-function running.
- Divergence in $t \to 0$ limit is unphysical: open string UV limit and this amplitude must be finite in a consistent string theory.
- Physical understanding of the divergence is best understood from closed string channel.

# $\mathcal{N} = 2$ sector

Annulus amplitude:


Annulus amplitude in $t \to 0$ limit:

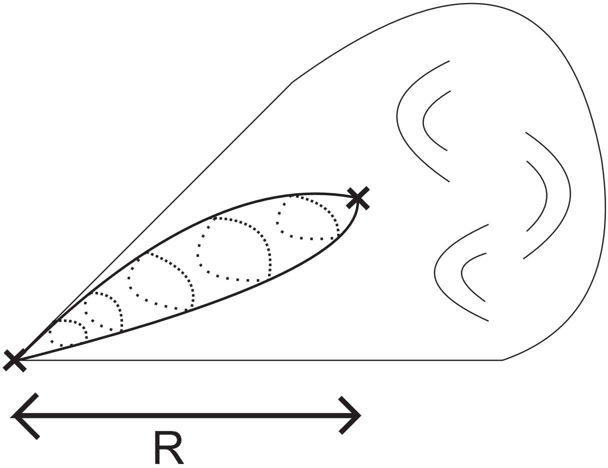


# $\mathcal{N} = 2$ sector

- $t \to 0$ divergence corresponds to a source for a partially twisted RR 2-form.
- In the local model this propagates into the bulk of the Calabi-Yau.
- In a compact model this becomes a physical divergence and tadpole must be cancelled by bulk sink branes/O-planes.
- Tadpole is sourced locally but cancelled globally

# $\mathcal{N} = 2$ sector

The purely local computation omits the following worldsheets:



## $\mathcal{N} = 2$ sector

- The purely local string computation includes all open string states for $t > 1/(RM_s)^2$, i.e. $M < RM_s$.

  However for $t < 1/(RM_s)^2$ we must include new winding states in the partition function.
- These are essential for global consistency but are omitted by a purely local computation.
- Threshold corrections become finite

$$\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a \to \int_{1/(RM_s)^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a$$

- Effective UV cutoff is actually $RM_s$ and *not* $M_s$.!

# Summary and Matching Field Theory

Running takes the form

$$\frac{1}{g^2}(\mu) = \frac{1}{g^2}\Big|_0 + \beta_a \ln\left(\frac{M_s^2}{\mu^2}\right) + \beta_a^{\mathcal{N}=2} \ln\left(\frac{(RM_s)^2}{M_s^2}\right).$$

- $\mathcal{N} = 1$ sectors run from $M_s$ into the IR.
- $\mathcal{N} = 2$ sectors run from $RM_s$ into the IR.

How to reconcile with Kaplunovsky-Louis?

$$g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = \text{Re}(f_a(\Phi)) + \beta_a \ln\left(\frac{(RM_s)^2}{\mu^2}\right).$$

# Summary and Matching Field Theory

- At the singularity holomorphic gauge couplings are

$$f_a = S + s_{ak} M_k$$

- $M_k$ is the twisted blow-up field.
- At 1-loop,

$$M_k \rightarrow M_k + \frac{\alpha}{16\pi^2} \ln \mathcal{V}$$

  and $\langle M_k \rangle \neq 0$.
- Coefficients $s_{ak}$ are $\mathcal{N} = 1$ contributions to running $\beta_{ak}^{\mathcal{N}=1}$.

# Summary and Matching Field Theory

Running takes the form

$$\frac{1}{g^2}(\mu) = \frac{1}{g^2}\Big|_0 + (\beta_a^{\mathcal{N}=1} + \beta_a^{\mathcal{N}=2}) \ln\left(\frac{M_s^2}{\mu^2}\right) + \beta_a^{\mathcal{N}=2} \ln\left(\frac{(RM_s)^2}{M_s^2}\right).$$

Kaplunovsky-Louis:

$$g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = S + \beta_a^{\mathcal{N}=1} T + (\beta_a^{\mathcal{N}=1} + \beta_a^{\mathcal{N}=2}) \ln\left(\frac{(RM_s)^2}{\mu^2}\right).$$

$T$ obtains a one-loop vev at the singularity:

$$\langle T \rangle = -\ln\left(\frac{(RM_s)^2}{M_s^2}\right).$$

# Summary and Matching Field Theory

For D3 @ orbifold singularities, there are no $\mathcal{N} = 1$ contributions to $\beta$ functions (vanishing of twisted tadpoles).

- Gauge coupling unification occurs at $RM_s$.

For D3/D7 @ orbifold singularities, 33 and 37 worldsheets combine to give $\mathcal{N} = 1$ contribution.

- In general no gauge coupling unification, for $\mathbb{Z}_3$ singularity unification at $M_s$.

For D3 @ orientifold singularities, annulus and Mobius worldsheets give $\mathcal{N} = 1$ contribution.

- Running starts at $RM_s$, with non-universal shift at $M_s$: no gauge coupling unification.

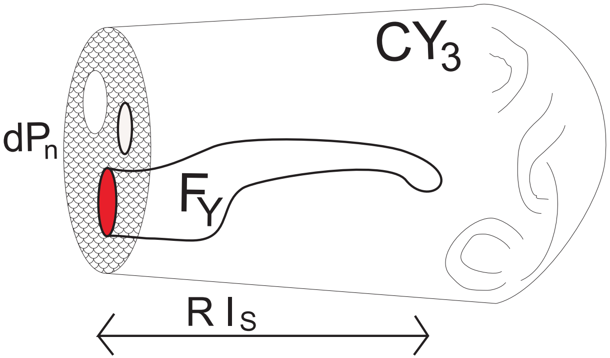
# Local GUTs

- Proposal to realise $SU(5)$ GUTs through branes on del Pezzo with hypercharged flux.
- Flux is quantised on cycle that is non-trivial in $H^2(dP, \mathbb{Z})$ but trivial in $H^2(CY, \mathbb{Z})$.
- Holomorphic gauge couplings are

$$
\begin{aligned}
f_{SU(3)} &= T_{dP} + h_3(F)S + \epsilon_3(U), \\
f_{SU(2)} &= T_{dP} + h_2(F)S + \epsilon_2(U), \\
f_{U(1)_Y} &= T_{dP} + h_1(F)S + \epsilon_Y(U).
\end{aligned}
$$

- For gauge unification assume $h_3 = h_2 = h_1$ and $\epsilon_i$ is negligible.

# Local GUTs



# Local GUTs: Field Theory

$$f_{SU(3)} = f_{SU(2)} = f_Y = T + h(F)S.$$

$$g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = \mathrm{Re}(f_a(\Phi)) + \beta_a \ln\left(\frac{(RM_s)^2}{\mu^2}\right).$$

- Kaplunovsky-Louis implies gauge unification occurs at $RM_s$ rather than $M_s$.
- Field redefinitions are universal and cannot alter universality of gauge kinetic functions or affect result.
- Bulk does not decouple and enters the unification scale.

# Local GUTs: String Theory

Can understand this from the string perspective:

- $T_{dP}$ is an '$\mathcal{N} = 1$' sector: associated to $SU(5)$ universal physics.
- GUT breaking comes from hypercharge flux associated to an '$\mathcal{N} = 2$' sector.
- Locally $\mathcal{F}_Y$ sources tadpole divergence via

$$\int_{dP} \mathcal{C}_{2,Y} \wedge \mathcal{F}_Y$$

- Globally $C_{2,Y}$ is absent: divergence regulated at scale $RM_s$ and gauge couplings run to this scale.

# Local GUTs: Warping and Exotica

Can apply ideas to warped throats:

- Suppose GUT realised on del Pezzo down a warped throat.
- Hypercharge cycle trivialises outside throat.
- Finiteness of threshold corrections requires knowledge of tadpole cancellation.
- In closed string channel string must leave throat, in open string channel need to include string states reaching out of throat and into bulk - with mass $M_P$.
- Running continues until Planck scale - possibility of TeV warped throat with gauge unification close to $M_P$?

# Conclusions

- Lots of interesting physics associated to running gauge couplings for local models.
- For $\mathcal{N} = 1$ sectors, gauge couplings run from $M_s$.
- For $\mathcal{N} = 2$ sectors, gauge couplings run from $RM_s$.
- For local GUTs with hypercharge flux breaking, unification scale is $RM_s$ and not $M_s$.

  Bulk cannot be consistently decoupled from the gauge theory.