Anomaly Mediation in String Theory

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Work in progress with Mark Goodsell and Eran Palti

Definition

This talk is about computing anomaly-mediated gaugino masses in string theory.

'Anomaly-mediated' will mean any mass term of generic 1-loop order

$$M_{\lambda,anomaly} \sim \frac{g^2 m_{3/2}}{16\pi^2}$$

Comparison:

$$M_{\lambda,tree} : \mathcal{O}(m_{3/2})$$

$$M_{\lambda,running} : \mathcal{O}(m_{3/2}) \frac{g^2}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\int d^4x d^2\theta\, f(\Phi) W_\alpha W^\alpha = \int d^4x\, \mathrm{Re}(f(\Phi)) F_{\mu\nu} F^{\mu\nu} + \int d^4x\, F_{f(\Phi)} \lambda\lambda + \ldots .$$

Gaugino masses and gauge couplings are closely related by supersymmetry.

Anomalous gaugino masses are closely related to gauge threshold corrections.

Gauge threshold corrections (anomalous gauge couplings) have been well studied in string theory.

Anomalous Gauge Couplings

In supergravity, physical and holomorphic gauge couplings are related by Kaplunovsky-Louis formula:

$$
\begin{aligned}
g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = \quad & \mathrm{Re}(f_a(\Phi)) && \text{(Holomorphic coupling)} \\
& + \frac{b_a}{16\pi^2} \ln\left(\frac{M_P^2}{\mu^2}\right) && \text{(β-function running)} \\
& + \frac{T(G)}{8\pi^2} \ln g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) && \text{(NSVZ term)} \\
& + \frac{(\sum_r n_r T_a(r) - T(G))}{16\pi^2} \hat{K}(\Phi, \bar{\Phi}) && \text{(Kähler-Weyl anomaly)} \\
& - \sum_r \frac{T_a(r)}{8\pi^2} \ln \det Z^r(\Phi, \bar{\Phi}, \mu). && \text{(Konishi anomaly)}
\end{aligned}
$$

Relates *measurable* couplings and *holomorphic* couplings.

This expression is well studied within string theory via gauge threshold corrections.

Anomalous Gaugino Masses

$$m_{1/2} = -\frac{g^2}{16\pi^2}\left[-\left(T_G - T_R\right) K_i F^i - \frac{2T_R}{d_R} F^i \partial_i \left(\ln \det Z\right) + 2T_G F^I \partial_I \ln\left(\frac{1}{g_0^2}\right)\right]$$

$(Kaplunovsky, Louis)(deAlwis)$

$$m_{1/2} = -\frac{g_a^2}{16\pi^2}\left[3T(G) - T(R)\right] m_{3/2}.$$

$(Giudice, Luty, Murayama, Rattazzi), (Randall, Sundrum)$

$$m_{1/2} = -\frac{g^2}{16\pi^2}\left[\left(3T_G - T_R\right) m_{3/2} - \left(T_G - T_R\right) K_i F^i - \frac{2T_R}{d_R} F^i \partial_i \left(\ln \det Z\right)\right]$$

$(Bagger, Moroi, Poppitz), (Gaillard, Nelson)$

$$m_{1/2} = -\frac{g^2}{16\pi^2}\left[\left(3T_G - T_R\right) m_{3/2} - \left(T_G - T_R\right) K_i F^i - \frac{2T_R}{d_R} F^i \partial_i \left(\ln \det Z\right) + 2T_G F^I \partial_I \ln\left(\frac{1}{g_0^2}\right)\right].$$

Also see (Dine, Seiberg).

Motivations

$$m_{1/2} = -\frac{g^2}{16\pi^2}\left[(3T_G - T_R)\, m_{3/2} - (T_G - T_R)\, K_i F^i - \frac{2T_R}{d_R} F^i \partial_i\, (\ln \det Z) + 2T_G F^I \partial_I \ln\left(\frac{1}{g_0^2}\right)\right].$$

Why study in string theory?

- Expression is ultraviolet sensitive (depends on K and Z)
- Expression is classically undetermined (not invariant under $K \rightarrow K + h + \bar{h}$, $W \rightarrow e^h W$).
- Not complete agreement about precise form of expression (in particular $m_{3/2}$ term)
- Previous stringy argument for no anomalous gaugino masses (Antoniadis, Taylor)

Motivations

Why study in string theory?

In no-scale susy breaking models (occurs in e.g. LARGE volume scenario)

$$\begin{aligned} K &= -3\ln(T+\bar{T}) + \frac{Q\bar{Q}}{T+\bar{T}}, \\ W &= W_0, \end{aligned}$$

there is a cancellation of anomaly-mediated terms

$$m_{1/2} = -\frac{g^2}{16\pi^2}\left[(3T_G - T_R)\,m_{3/2} - (T_G - T_R)\,K_i F^i - \frac{2T_R}{d_R} F^i \partial_i \left(\ln \det Z\right)\right] = 0.$$

The existence of this cancellation is phenomenologically important for the magnitude and structure of soft terms.

The Model

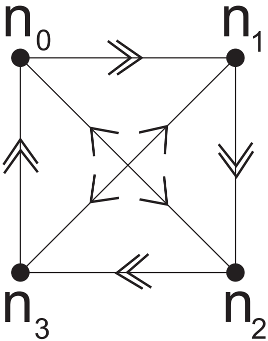
Where to compute?

We want examples of calculable models with non-zero beta functions.

- The simplest such examples are (fractional) D3 branes at orbifold singularities.
- String can be exactly quantised and all calculations can be performed explicitly.
- Orbifold singularities only involve annulus amplitudes further simplifying the computations.
- Will focus on D-branes at $\mathbb{C}^3/\mathbb{Z}_4$.
- Orbifold action is $(z_1, z_2, z_3) \rightarrow (\omega z_1, \omega z_2, \omega^2 z_3)$ with $\omega = e^{2\pi i/4}$.

The Model

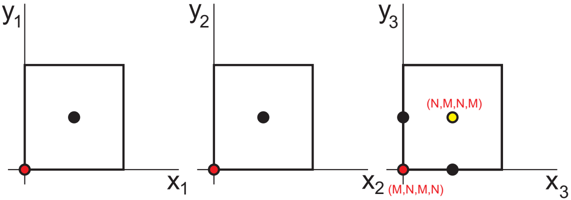
- The quiver for $\mathbb{C}^3/\mathbb{Z}_4$ is:



- Anomaly cancellation requires $n_0 = n_2$, $n_1 = n_3$.
- This sources a twisted tadpole in the $\mathcal{N} = 2$ sector set by $n_0 - n_1$.

The Model

Can embed the model in a compact space:



Kähler potential is given by

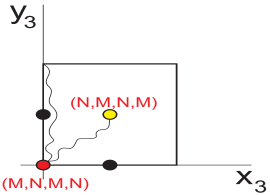
$$K = -\ln(S+\bar{S}) - \sum_{I=1}^{3} \ln\left((T_I+\bar{T}_I)(U_I+\bar{U}_I) - \frac{1}{6}(\Phi_I+\bar{\Phi}_I)^2\right) \; .$$

$$K_{\Phi_I \bar{\Phi}_{\bar{I}}} \equiv Z_I = \frac{1}{(T_I+\bar{T}_I)(U_I+\bar{U}_I)} \; .$$

Twisted tadpole cancellation requires extra brane stacks on other orbifold fixed points.

Caveat: Uncancelled D3 tadpole in N=4 sector, should not be relevant for questions involving β-functions.

The Model



An important quantity is the winding mode partition function:

$$Z_{winding}(t) = \mathrm{Tr}_{CP}(\Theta^2)\left(\sum_{n,m} e^{-(n^2R_1^2+m^2R_2^2)\alpha' t} - \sum_{n,m} e^{-((n+\frac{1}{2})^2R_1^2+(m+\frac{1}{2})^2R_2^2)\alpha' t}\right).$$

This satisfies

$$Z(t) \to 0 + \mathcal{O}(e^{-1/R^2 t}) \text{ as } t \to 0 \text{ (tadpole cancellation)}$$

$$Z(t) \to b_a \text{ as } t \to \infty \text{ (IR beta function)}$$

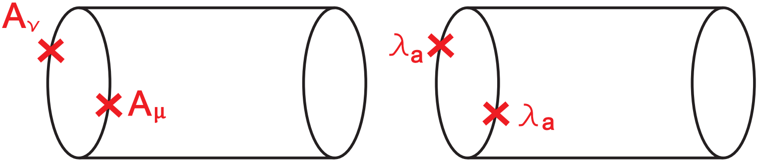
Computations

Have computed the following amplitudes:

- $\langle A^a_\mu A^a_\nu \rangle, \langle \lambda^a \lambda^a \rangle -$ gauge threshold corrections
- $\langle \psi_i \psi_j \phi_k \rangle -$ Yukawa threshold corrections
- $\langle \phi_i \phi_j \phi_k \lambda_a \lambda_a \rangle -$ Tree-level brane-to-brane susy breaking
- $\langle H_3 \lambda_a \lambda_a \rangle -$ Anomalous gaugino masses from NS-NS fluxes
- $\langle F_3 \lambda_a \lambda_a \rangle -$ Anomalous gaugino masses from RR fluxes

In general, need to compute with off-shell momenta and go on-shell only at the end of the computation (to account for finite $\frac{0}{0}$ terms such as $\frac{k_i \cdot k_j}{k_i \cdot k_j}$).

Gauge Thresholds

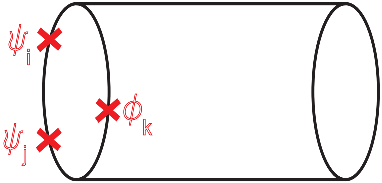


$$\mathcal{A} \sim \int \frac{dt}{t} \mathrm{Tr}(\Theta^2) Z_{winding}(t)$$

$$\frac{1}{g^2}(\mu) = \frac{1}{g_0^2} + \frac{b_a}{16\pi^2} \ln\left(\frac{(RM_s)^2}{\mu^2}\right)$$

Same as previous results for these models (JC, Palti)- couplings run from the winding scale $M_W = RM_s$.

Yukawa Thresholds



cf work by Abel, Schofield

Basic vertex operators

$$\begin{aligned} \mathcal{V}^a_{-\frac{1}{2}}(u_1, k_1, z_1) &= t^a e^{-\phi/2} S^{\pm}(z_1) e^{ik_1\cdot X(z_1)} e^{iq_1\cdot H(z_1)}, \\ \mathcal{V}^b_{-\frac{1}{2}}(u_2, k_2, z_2) &= t^b e^{-\phi/2} S^{\pm}(z_2) e^{ik_2\cdot X(z_2)} e^{iq_2\cdot H(z_2)}, \\ \mathcal{V}^c_{-1}(u_3, k_3, z_3) &= t^c e^{-\phi} e^{ik_3\cdot X(z_3)} e^{iq_3\cdot H(z_3)}. \end{aligned}$$

Amplitude is

$$\mathcal{A} = \int \frac{dt}{t} \int dz_1 dz_2 dz_3 \left\langle V^a_{-1/2}(u_1, k_1, z_1)\, \mathcal{V}^b_{\frac{1}{2}}(u_2, k_2, z_2)\, V^c_0(\varphi, k_3, z_3) \right\rangle .$$

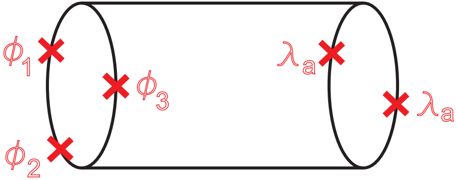
Yukawa Thresholds

Overall result is

$$\mathcal{A} \sim \int \frac{dt}{t} \mathrm{Tr}_L(t^a t^b t^c \theta^2) \mathrm{Tr}_R(\theta^2) \prod_i (-2 \sin \pi\theta_i) \, Z(t),$$

- Logarithmic running associated to wavefunction renormalisation.
- Result comes only from $\mathcal{N} = 2$ sector (θ^2 sector) - as for gauge thresholds.
- Running starts from super-stringy scale $M_W = RM_s$ - as for gauge thresholds.

Brane-to-Brane Susy Breaking

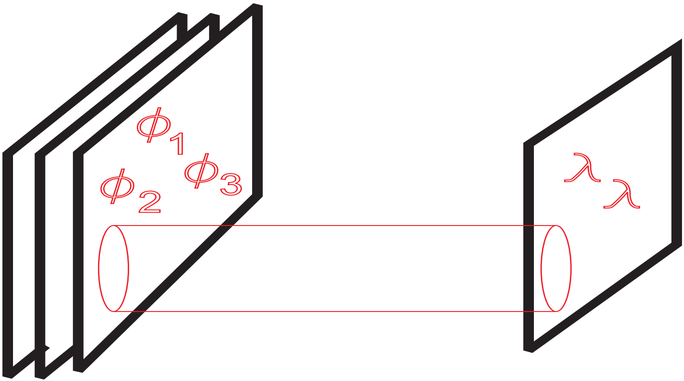


This is the correlator $\langle W\lambda\lambda\rangle$ where W is a field theory superpotential living on branes.

It gives gaugino masses induced by susy breaking vevs for ϕ_1, ϕ_2, ϕ_3.

Although this is an annulus diagram this in fact gives supergravity tree-level susy breaking (understand via closed string diagram).

Brane-to-Brane Susy Breaking



Brane-to-Brane Susy Breaking

$$\mathcal{A} \sim (k_2 \cdot k_3)(k_4 \cdot k_5) \int \frac{dt}{t^3} \int dz_1 \dots dz_5 \langle \prod_i e^{ik_i \cdot X(z_i)} \rangle \frac{\vartheta_1(z_2 - z_5)\vartheta_1(-z_1 + z_2 - z_4 + z_5)\vartheta_1(-z_1 + z_4)\eta^3 Z(t)}{\vartheta_1(z_1 - z_5)\vartheta_1(z_4 - z_5)\vartheta_1(-z_1 + z_2)\vartheta_1(z_2 - z_4)}.$$

Need zero momentum limit: one pole in $k_4 \cdot k_5$ as $z_4 \longrightarrow z_5$, simplifies to

$$k_2 \cdot k_3 \int \frac{dt}{t^3} \int dz_1 dz_2 dz_3 dz_4 Z(t) \langle \prod_i e^{ik \cdot X} \rangle$$

This has an additional pole as $t \longrightarrow 0$ to give

$$\frac{k_2 \cdot k_3}{R^6} \int \frac{dt}{t^2} e^{-\frac{\pi k_1 \cdot k_2}{t}} \longrightarrow \frac{k_2 \cdot k_3}{R^6 k_1 \cdot k_2}.$$

Summing over picture-changing combinatorics gives

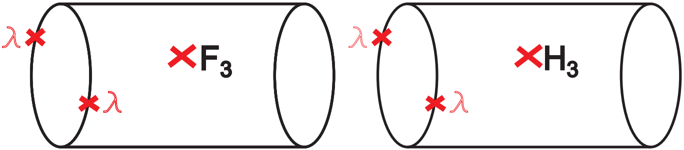
$$\frac{k_2 \cdot (k_3 + k_4 + k_5)}{k_1 \cdot k_2} \frac{1}{R^6} = \frac{k_2 \cdot (-k_1 - k_2)}{k_1 \cdot k_2} \frac{1}{R^6} \longrightarrow \frac{1}{R^6}$$

This corresponds to dilaton-mediated susy breaking.

Flux-Induced Gaugino Masses

Tree-level flux induced gaugino masses studied on the disk by Billo, Ferro, Frau, Fucito, Lerda, Morales

Now come to target computation:



What are we looking for and how to interpret it?

Supergravity Expectations

$$m_{1/2,tree} = \frac{F^I \partial_I f_a}{2\mathrm{Re}(f_a)}$$

$$m_{1/2,anomaly} = -\frac{g^2}{16\pi^2}\left[\delta_{AM}\left(3T_G - T_R\right)m_{3/2} - \left(T_G - T_R\right)K_i F^i - \frac{2T_R}{d_R}F^i\partial_i\left(\ln\det Z\right) + 2T_G F^I \partial_I \ln\left(\frac{1}{g_0^2}\right)\right].$$

Consider this formula for 3-form flux backgrounds in the case of NS-NS and RR flux.

$$W = \int G_3 \wedge \Omega$$

for G_3 pure NS-NS or pure RR (G_3 is $(0,3) \pm (3,0)$).

Note that in worldsheet CFT

$$G_3 = F_3 - iSH_3 \rightarrow F_3 - \frac{i}{g_s}H_3$$

NS-NS flux

$$W = -iS \int H_3 \wedge \Omega$$

Tree mass $$M_{\lambda,tree} = \frac{F^S}{2\text{Re}(S)} = \bar{m}_{3/2},$$

Running couplings: $$\frac{1}{g^2(\mu)} = \frac{1}{g^2_{tree}} \left(1 + \frac{g^2 b_a}{16\pi^2} \ln\left(\frac{M_W^2}{\mu^2}\right)\right) .$$

Running masses $$M_{\lambda,running} = \bar{m}_{3/2} \left(1 - \left(\frac{g^2 b_a}{16\pi^2}\right) \ln\left(\frac{M_W^2}{\mu^2}\right)\right) .$$

Anomalous masses $$M_{\lambda,anomaly} = \frac{g^2 b_a}{16\pi^2} \delta_{AM} \bar{m}_{3/2} .$$

R-R flux

$$W = \int F_3 \wedge \Omega$$

Tree mass $$M_{\lambda,tree} = \frac{F^S}{2\mathrm{Re}(S)} = -\bar{m}_{3/2},$$

Running couplings: $$\frac{1}{g^2(\mu)} = \frac{1}{g^2_{tree}}\left(1 + \frac{g^2 b_a}{16\pi^2}\ln\left(\frac{M_W^2}{\mu^2}\right)\right).$$

Running masses $$M_{\lambda,running} = -\bar{m}_{3/2}\left(1 - \left(\frac{g^2 b_a}{16\pi^2}\right)\ln\left(\frac{M_W^2}{\mu^2}\right)\right).$$

Anomalous masses $$M_{\lambda,anomaly} = \frac{g^2 b_a}{16\pi^2}(\delta_{AM} - 2)\bar{m}_{3/2}.$$

String computation of $\langle H_3 \lambda\lambda \rangle$ or $\langle F_3 \lambda\lambda \rangle$ can give both running and anomalous mass terms.

Test formula by looking at ratio of anomalous mass and running mass.

$$\frac{M_{\lambda,running}}{M_{\lambda,anomaly}} = \begin{cases} -\frac{1}{\delta_{AM}} \ln\left(\frac{M_W^2}{\mu^2}\right) & \text{NSNS} \\ \frac{1}{\delta_{AM}-2} \ln\left(\frac{M_W^2}{\mu^2}\right) & \text{RR} \end{cases}$$

Can check results independently via computations with NS-NS and RR flux vertex operators.

Want

$$\int \frac{dt}{t^3} \int dz_1 dz_2 d^2 w \langle \mathcal{V}_{\lambda}^{-1/2}(z_1) V_{\lambda}^{1/2}(z_2) \mathcal{V}_{H_3}^{0,0}(w) \rangle$$

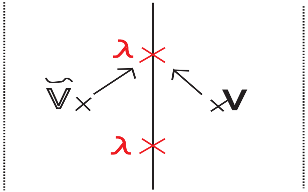
Flux vertex operator in $(-1,-1)$ picture is

$$\mathcal{V}_B^{(-1,-1)} = H_{123} e^{-\phi(w) - \tilde{\phi}(\bar{w})} \overline{X}^3(w,\bar{w}) \left(\overline{\psi^1}(w) \overline{\tilde{\psi}^2}(\bar{w}) - \overline{\psi^2}(w) \overline{\tilde{\psi}^1}(\bar{w}) \right) e^{ik \cdot X(w,\bar{w})}.$$

Flux vertex operator in $(0,0)$ picture is

$$\begin{aligned}
\mathcal{V}_B^{(0,0)} &= \frac{1}{4} H_{123} e^{ik\cdot X} \left[-\alpha' \overline{\psi^3} \left(\overline{\psi^1} \left(\overline{\partial} \overline{\tilde{X}^2} - \frac{i\alpha'}{2} \left(k \cdot \tilde{\psi} \right) \overline{\tilde{\psi}^2} \right) - \overline{\psi^2} \left(\overline{\partial} \overline{\tilde{X}^1} - \frac{i\alpha'}{2} \left(k \cdot \tilde{\psi} \right) \overline{\tilde{\psi}^1} \right) \right) \right. \\
&- \alpha' \overline{\tilde{\psi}^3} \left(\left(\partial \overline{X}^1 - \frac{i\alpha'}{2} (k \cdot \psi) \overline{\psi^1} \right) \overline{\tilde{\psi}^2} - \left(\partial \overline{X}^2 - \frac{i\alpha'}{2} (k \cdot \psi) \overline{\psi^2} \right) \overline{\tilde{\psi}^1} \right) \\
&+ \overline{X}^3 \left(\left(\partial \overline{X}^1 - \frac{i\alpha'}{2} (k \cdot \psi) \overline{\psi^1} \right) \left(\partial \overline{\tilde{X}}^2 - \frac{i\alpha'}{2} \left(k \cdot \tilde{\psi} \right) \overline{\tilde{\psi}^2} \right) \right. \\
&- \left. \left. \left(\partial \overline{X}^2 - \frac{i\alpha'}{2} (k \cdot \psi) \overline{\psi^2} \right) \left(\partial \overline{\tilde{X}}^1 - \frac{i\alpha'}{2} \left(k \cdot \tilde{\psi} \right) \overline{\tilde{\psi}^1} \right) \right) \right] .
\end{aligned}$$

Two kinds of pole: factorise flux vertex operators onto gaugini



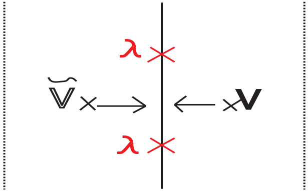
Factorise flux vertex operator onto gaugino to generate a momentum pole. There are two residual vertex operator integrals

$$\int \frac{dt}{t^3} \int dz_1 dz_2 d^2 w \to \int \frac{dt}{t^3} \int dz_1 dz_2 \to \int \frac{dt}{t}.$$

This gives a running mass

$$\mathcal{A} \sim \int \frac{dt}{t} \frac{-2k_2 \cdot k_3 - k_3 \cdot k_3}{2k_2 \cdot k_3 + k_3 \cdot k_3} Z(t)$$

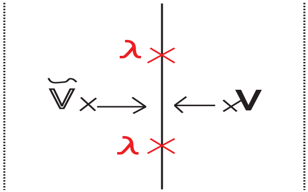
Two kinds of pole: factorise flux onto boundary



$$\mathcal{A} \sim \int \frac{dt}{t^3} \int dz_1 dz_2 d\mathrm{Im}(w) \frac{k_3 \cdot k_3}{k_3 \cdot k_3} \langle \partial_n X^{\bar{3}}(z_2) \left(X^3(iw_2) - X^3(1/2 + iw_2) \right) \rangle.$$

$$\langle \partial_n X^{\bar{3}}(z_2) \left(X^3(iw_2) - X^3(1/2 + iw_2) \right) \rangle \quad = \quad \frac{dZ_{winding}}{dt}$$

$$= \sum_{n,m} (n^2 + m^2) R^2 e^{-(n^2+m^2)R^2)\alpha' t} - \sum_{n,m} ((n + \frac{1}{2})^2 + (m + \frac{1}{2})^2) R^2) e^{-((n+\frac{1}{2})^2 + (m+\frac{1}{2})^2) R^2)\alpha' t}$$



$$\mathcal{A} \sim \frac{k_3 \cdot k_3}{k_3 \cdot k_3} \int \frac{dt}{t^3} \int dz_1 dz_2 d\text{Im}(w) \frac{dZ_{winding}}{dt}$$
$$\sim \int_{t=0}^{\infty} \frac{dZ}{dt}$$
$$= Z(\infty) - Z(0) = Z(\infty).$$

This is an anomalous mass term.

Note it arises from a sum purely over ultraviolet states (only non-zero winding number contributes to the sum), is only well-defined for a tadpole-cancelling theory, but is then determined entirely by the massless spectrum.

Overall amplitude in low-momentum limit is

$$\int \frac{dt}{t}\left(\frac{2k_2\cdot k_3 + k_3\cdot k_3}{-2k_2\cdot k_3 - k_3\cdot k_3}Z(t) + t\frac{k_3\cdot k_3}{k_3\cdot k_3}\frac{d}{dt}Z(t)\right)$$

Poles and zeros cancel and there is a definite answer in the on-shell limit $k_i \cdot k_j = 0$, $k_i^2 = 0$.

Anomalous and mass terms are opposite sign and same magnitude - precisely consistent with $\delta_{AM} = 1$.

RR Flux

Can do the same thing for RR flux. Vertex operator is

$$V_F^{(-1/2,-1/2)} = N_F g_s e^{-\phi/2-\tilde{\phi}/2} F_{mnp} \Theta(z) C \Gamma^{mnp} \tilde{\Theta}(\tilde{z}) e^{ik\cdot X},$$

Can compute

$$\int \frac{dt}{t^3} \int dz_1 dz_2 d^2w \langle \mathcal{V}_\lambda^{1/2}(z_1) \mathcal{V}_\lambda^{1/2}(z_2) \mathcal{V}_F^{-1/2,-1/2}(w,\bar{w}) \rangle$$

$$\int \frac{dt}{t^3} \int dz_1 dz_2 d^2w \langle \mathcal{V}_\lambda^{1/2}(z_1) \mathcal{V}_\lambda^{-1/2}(z_2) \mathcal{V}_F^{1/2,-1/2}(w,\bar{w}) \rangle$$

$$\int \frac{dt}{t^3} \int dz_1 dz_2 d^2w \langle \mathcal{V}_\lambda^{-1/2}(z_1) \mathcal{V}_\lambda^{-1/2}(z_2) \mathcal{V}_F^{1/2,1/2}(w,\bar{w}) \rangle$$

and evaluate correlators in zero momentum limit.

Open Issue I

- Currently not clear how different pictures agree for RR flux computation
- Pictures

$$\langle \mathcal{V}_{\lambda}^{1/2}(z_1)\mathcal{V}_{\lambda}^{-1/2}(z_2)\mathcal{V}_{F}^{1/2,-1/2}(w,\bar{w})\rangle$$

  has an anomalous mass term similar to NS-NS flux case.
- For pictures

$$\langle \mathcal{V}_{\lambda}^{1/2}(z_1)\mathcal{V}_{\lambda}^{1/2}(z_2)\mathcal{V}_{F}^{-1/2,-1/2}(w,\bar{w})\rangle$$

  an anomalous mass term appears to be absent/takes a different form.
- Work in progress to understand this.

Open Issue II

Encounter integrals

$$\int \frac{dt}{t^3} \int dz_1 dz_2 dz_3 \, k_2 \cdot k_3 \frac{\vartheta_1(z_1 - z_2 + \theta_1)\eta^3}{\vartheta_1(z_1 - z_2)\vartheta_1(\theta_1)} \langle \prod_i e^{ik \cdot X(z_i)} \rangle$$

with $k_2 \cdot k_3 = 0$ on-shell.

As $t \to \infty$, $\langle e^{ik \cdot X} \rangle \sim e^{-\alpha' k_i . k_j (z_{ij} - \frac{z_{ij}^2}{t})}$ This has a momentum pole as $z_{ij}, t \to \infty$ (infinite IR limit) which may cancel $k_2 \cdot k_3$.

Is this physical? These appear in the 1-loop Yukawa computation and would give a finite vertex correction from the strict infinite IR limit of the loop parameter.

Non-renormalisation suggests this should be absent, but is there a rigorous argument?

Conclusions

- We are studying 1-loop flux-induced gaugino masses on the worldsheet.
- Anomalous mass terms exist and take the form

$$M_a \sim \frac{m_{3/2}}{16\pi^2}\int dt \frac{d}{dt} Z(t) = \frac{m_{3/2}}{16\pi^2}\left(Z(\infty) - Z(0)\right) = \frac{m_{3/2} b_a}{16\pi^2}.$$

  These appear as an explicit sum over UV states, but the result depends only on the massless spectrum.
- For NS-NS flux sign and magnitude of anomalous mass term agrees with BMP formula.
- RR flux case not fully understood yet.