

# Gauge Threshold Corrections for Local String Models

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## Local vs Global

There are many different proposals to realise Standard Model in string theory:

- ▶ Weakly coupled heterotic string / heterotic M-theory
- ▶ M-theory on  $G_2$  manifolds
- ▶ Intersecting/magnetised brane worlds in IIA/IIB string theory
- ▶ Branes at singularities
- ▶ F-theory GUTs

## Local vs Global

These approaches are usefully classified as either **local** or **global**.

Global models:

- ▶ Canonical example is weakly coupled heterotic string.
- ▶ Model specification requires global consistency conditions.
- ▶ Relies on geometry of entire compact space
- ▶ Limit  $\mathcal{V} \rightarrow \infty$  also gives  $\alpha_{SM} \rightarrow 0$ : cannot decouple string and Planck scales.
- ▶ Other examples: IIA/IIB intersecting brane worlds, M-theory on  $G_2$  manifolds

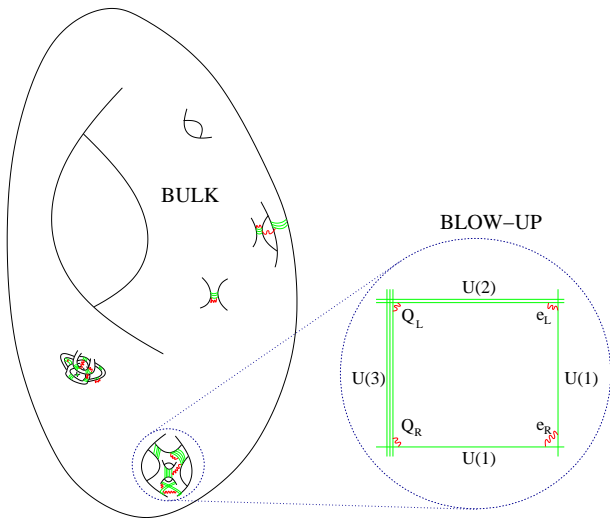
## Local vs Global

Local models:

- ▶ Canonical example branes at singularity
- ▶ Model specification only requires knowledge of local geometry and local tadpole cancellation.
- ▶ Full consistency depends on existence of a compact embedding of the local geometry.
- ▶ Standard Model gauge and Yukawa couplings remain finite in the limit  $\mathcal{V} \rightarrow \infty$ .

It is possible to have  $M_P \gg M_s$  by taking  $\mathcal{V} \rightarrow \infty$ .

- ▶ Examples: branes at singularities, local F-theory GUTs.



## Local vs Global

Local models have various promising features:

- ▶ Easier to construct than fully global models.
- ▶ Typically have small numbers of families.
- ▶ Combine easily with moduli stabilisation, supersymmetry breaking and hierarchy generation (LARGE volume construction)
- ▶ Promising recent constructions of local stringy GUTs.

## Local vs Global

One of the most phenomenologically important quantities in local models is the bulk volume.

This determines

- ▶ String scale  $M_s = \frac{M_P}{\sqrt{\mathcal{V}}}$
- ▶ Gravitino mass through the flux superpotential

$$m_{3/2} \sim \frac{\langle \int G_3 \wedge \Omega \rangle}{\mathcal{V}}$$

- ▶ The unification scale in models where gauge couplings naturally unify.

The purpose of this talk is to study this question precisely.

## Threshold Corrections

- ▶ If gauge coupling unification is non-accidental, it is important to understand the significance of  $M_{GUT} \sim 3 \times 10^{16} \text{GeV}$ .
- ▶ In particular, we want to understand the relationship of  $M_{GUT}$  to the string scale  $M_s$  and the Planck scale  $M_P = 2.4 \times 10^{18} \text{GeV}$ .
- ▶ Is  $M_{GUT}$  an actual scale or a mirage scale?
- ▶ I will discuss this first using supergravity arguments and subsequently directly in string theory.



## Threshold Corrections in Supergravity I

In supergravity, physical and holomorphic gauge couplings are related by Kaplunovsky-Louis formula:

$$\begin{aligned}
 g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = & \quad \text{Re}(f_a(\Phi)) && \text{(Holomorphic coupling)} \\
 & + \frac{b_a}{16\pi^2} \ln\left(\frac{M_P^2}{\mu^2}\right) && (\beta\text{-function running}) \\
 & + \frac{T(G)}{8\pi^2} \ln g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) && \text{(NSVZ term)} \\
 & + \frac{(\sum_r n_r T_a(r) - T(G))}{16\pi^2} \hat{K}(\Phi, \bar{\Phi}) && \text{(Kähler-Weyl anomaly)} \\
 & - \sum_r \frac{T_a(r)}{8\pi^2} \ln \det Z^r(\Phi, \bar{\Phi}, \mu). && \text{(Konishi anomaly)}
 \end{aligned}$$

Relates *measurable* couplings and *holomorphic* couplings.

For local models in IIB

- ▶ Kähler potential  $\hat{K}$  is given by

$$\hat{K} = -2 \ln \mathcal{V} + \dots$$

- ▶ Matter kinetic terms  $\hat{Z}$  are given by

$$\hat{Z} = \frac{f(\tau_s)}{\mathcal{V}^{2/3}}$$

Why? When we decouple gravity the physical couplings

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{\hat{Z}_\alpha \hat{Z}_\beta \hat{Z}_\gamma}}$$

should remain finite and be  $\mathcal{V}$ -independent.

$$\hat{K} = -2 \ln \mathcal{V}, \quad \hat{Z} = \frac{f(\tau_s)}{\mathcal{V}^{2/3}}$$

- ▶ Local models require a LARGE bulk volume ( $\mathcal{V} \sim 10^4$  for  $M_s \sim M_{GUT}$ ,  $\mathcal{V} \sim 10^{15}$  for  $M_s \sim 10^{11} \text{ GeV}$ ).
- ▶ Kähler and Konishi anomalies are formally one-loop suppressed.  
However if volume is LARGE, both anomalies are enhanced by  $\ln \mathcal{V}$  factors.
- ▶ This implies the existence of large anomalous contributions to *physical* gauge couplings!

Plug in  $\hat{K} = -2 \ln \mathcal{V}$  and  $\hat{Z} = \frac{1}{\mathcal{V}^{2/3}}$  into Kaplunovsky-Louis formula.

We restrict to terms enhanced by  $\ln \mathcal{V}$  and obtain:

$$\begin{aligned} g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) &= \text{Re}(f_a(\Phi)) + \frac{(\sum_r n_r T_a(r) - 3T_a(G))}{8\pi^2} \ln \left( \frac{M_P}{\mathcal{V}^{1/3} \mu} \right) \\ &= \text{Re}(f_a(\Phi)) + \beta_a \ln \left( \frac{(RM_s)^2}{\mu^2} \right). \end{aligned}$$

- ▶ Gauge couplings start running from an effective scale  $RM_s$  rather than  $M_s$ .
- ▶ Universal  $\text{Re}(f_a(\Phi))$  implies unification occurs at a super-stringy scale  $RM_s$  rather than  $M_s$ .

- ▶ Argument implies inferred low-energy unification scale is systematically above the string scale.
- ▶ Argument has only relied on model-independent  $\mathcal{V}$  factors - result should hold for any local model (D3 at singularities, IIB GUTs, F-theory GUTs, local M-theory models)
- ▶ Unification scale is a mirage scale - new string states already occur at  $M_s = M_{GUT}/R \ll M_{GUT}$ .

## Threshold Corrections in String Theory

- ▶ We now want to investigate this directly in string theory.
- ▶ In string theory gauge couplings are

$$\frac{1}{g_a^2(\mu)} = \frac{1}{g_{0,a}^2} + \frac{b_a}{16\pi^2} \ln \left( \frac{M_s^2}{\mu^2} \right) + \Delta_a(M, \bar{M})$$

- ▶  $\Delta_a(M, \bar{M})$  are the **threshold corrections** induced by massive string/KK states.
- ▶ Study of threshold corrections pioneered by Kaplunovsky and Louis for weakly coupled heterotic string.
- ▶ For our calculations we use the **background field method**.

Running gauge couplings are the 1-loop coefficient of

$$\frac{1}{4g^2} \int d^4x \sqrt{g} F_{\mu\nu}^a F^{a,\mu\nu}$$

- ▶ Turn on background magnetic field  $F_{23} = B$ .
- ▶ Compute the quantised string spectrum.
- ▶ Use the string partition function to compute the 1-loop vacuum energy

$$\Lambda = \Lambda_0 + \frac{1}{2} \left( \frac{B}{2\pi^2} \right)^2 \Lambda_2 + \frac{1}{4!} \left( \frac{B}{2\pi^2} \right)^4 \Lambda_4 + \dots$$

- ▶ From  $\Lambda_2$  term we can extract beta function running and threshold corrections.

String theory 1-loop vacuum function given by partition function

$$\Lambda_{1-loop} = \frac{1}{2}(T + KB + A(B) + MS(B)).$$

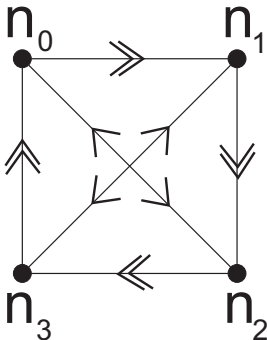
- ▶ Require  $\mathcal{O}(B^2)$  term of this expansion.
- ▶ Background magnetic field only shifts moding of open string states.
- ▶ Torus and Klein Bottle amplitudes do not couple to open strings.
- ▶ Only annulus and Möbius strip amplitudes contribute at  $\mathcal{O}(B^2)$ .



We want examples of calculable local models with non-zero beta functions.

- ▶ The simplest such examples are (fractional) D3 branes at orbifold singularities.
- ▶ String can be exactly quantised and all calculations can be performed explicitly.
- ▶ Orbifold singularities only involve annulus amplitude further simplifying the computations.
- ▶ Have studied D3 branes on  $\mathbb{C}^3/\mathbb{Z}_4$ ,  $\mathbb{C}^3/\mathbb{Z}_6$ ,  $\mathbb{C}^3/\mathbb{Z}'_6$ ,  $\mathbb{C}^3/\Delta_{27}$ .
- ▶ Will focus here on D-branes at  $\mathbb{C}^3/\mathbb{Z}_4$  (results all generalise).

- ▶ The quiver for  $\mathbb{C}^3/\mathbb{Z}_4$  is:



- ▶ Anomaly cancellation requires  $n_0 = n_2$ ,  $n_1 = n_3$ .

- ▶ Orbifold action generated by  $z_i \rightarrow \exp(2\pi i\theta_i)$  with  $\theta = (1/4, 1/4, -1/2)$ .
- ▶ We only need to compute the annulus diagram

$$\mathcal{A}(B) = \int_0^\infty \frac{dt}{2t} \text{STr} \left( \frac{(1 + \theta + \theta^2 + \theta^3)}{4} \frac{1 + (-1)^F}{2} q^{(p^\mu p_\mu + m^2)/2} \right)$$

Here

$$q = e^{-\pi t}, \quad \text{STr} = \sum_{\text{bosons}} - \sum_{\text{fermions}} \equiv \sum_{NS} - \sum_R, \quad \alpha' = 1/2$$

- ▶  $\beta$ -function running and threshold corrections are encoded in the  $\mathcal{O}(B^2)$  term.

We separately evaluate each amplitude in the  $\theta^N$  sector.

$$\mathcal{A}(B) = \int_0^\infty \frac{dt}{2t} \text{STr} \left( \frac{(1 + \theta + \theta^2 + \theta^3)}{4} \frac{1 + (-1)^F}{2} q^{(p^\mu p_\mu + m^2)/2} \right)$$

- ▶  $\theta^0 = (1, 1, 1)$  is an ' $\mathcal{N} = 4$ ' sector.
- ▶  $\theta^1 = (1/4, 1/4, -1/2)$  and  $\theta^3 = (-1/4, -1/4, 1/2)$  are ' $\mathcal{N} = 1$ ' sectors.
- ▶  $\theta^2 = (1/2, 1/2, 0)$  is an ' $\mathcal{N} = 2$ ' sector.

The amplitudes reduce to products of Jacobi  $\vartheta$ -functions with different prefactors.

$$\mathcal{A}_{untwisted} = \int \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \times 0 = 0. \quad (\mathcal{N} = 4 \text{ susy})$$

- ▶ The untwisted sector has effective  $\mathcal{N} = 4$  supersymmetry and cannot contribute to the running gauge coupling.

$$\mathcal{A}_\theta = \mathcal{A}_{\theta^3} = \int \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \times \frac{(n_0 - n_2)}{2} (\vartheta - \text{functions})$$

- ▶ The contribution of  $\mathcal{N} = 1$  sectors to gauge coupling running has a prefactor  $(n_0 - n_2)$ .
- ▶ This necessarily vanishes once non-abelian anomaly cancellation is imposed.

$$\mathcal{A}_{\theta^2} = \int \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \times (-3n_0 + n_1 + n_2 + n_3) (\vartheta - \text{function}).$$

Here  $(\vartheta - \text{function})$  is

$$\frac{-1}{4\pi^2} \sum \eta_{\alpha\beta} (-1)^{2\alpha} \frac{\vartheta'' \left[ \begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right]}{\eta^3} \frac{\vartheta \left[ \begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right]}{\eta^3} \frac{\vartheta \left[ \begin{smallmatrix} \alpha \\ \beta + \theta_1 \end{smallmatrix} \right]}{\vartheta \left[ \begin{smallmatrix} 1/2 \\ 1/2 + \theta_1 \end{smallmatrix} \right]} \frac{\vartheta \left[ \begin{smallmatrix} \alpha \\ \beta + \theta_2 \end{smallmatrix} \right]}{\vartheta \left[ \begin{smallmatrix} 1/2 \\ 1/2 + \theta_2 \end{smallmatrix} \right]} = 1.$$

We obtain

$$\mathcal{A}_{\theta^2} = \int \frac{dt}{2t} \frac{1}{2} \left( \frac{B}{2\pi^2} \right)^2 \times (-3n_0 + n_1 + n_2 + n_3).$$

$$\mathcal{A}_{\theta^2} = \int \frac{dt}{2t} \frac{1}{2} \left( \frac{B}{2\pi^2} \right)^2 \times \underbrace{(-3n_0 + n_1 + n_2 + n_3)}_{b_0}.$$

- ▶ Reduction of  $\vartheta$ -functions to a constant is a consequence of  $\mathcal{N} = 2$  supersymmetry.
- ▶ Only BPS multiplets can affect gauge coupling running and excited string states are non-BPS.
- ▶ Resultant amplitude is non-zero and gives field theory  $\beta$ -function running in both IR and UV limits.

## Summary:

- ▶ Untwisted sector has  $\mathcal{N} = 4$  susy and gives no contribution to running of gauge couplings.
- ▶  $\theta$  and  $\theta^3$  twisted sectors have  $\mathcal{N} = 1$  susy. Contributions vanish when anomaly cancellation is imposed.
- ▶  $\mathcal{N} = 2$   $\theta^2$  sectors gives non-vanishing contribution

$$\left[ \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \right] \times \int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} b_a$$

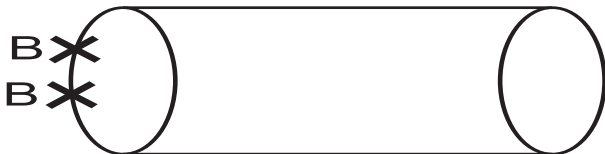
- ▶ How should we interpret this?



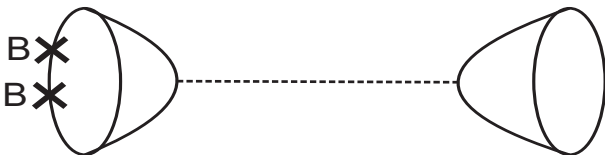
$$\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a$$

- ▶ Divergence in the  $t \rightarrow \infty$  limit is physical: this is the IR limit and we recover ordinary  $\beta$ -function running.
- ▶ Divergence in  $t \rightarrow 0$  limit is unphysical: this is the open string UV limit and this amplitude must be finite in a consistent string theory.
- ▶ Physical understanding of the divergence is best understood from closed string channel.

Annulus amplitude:

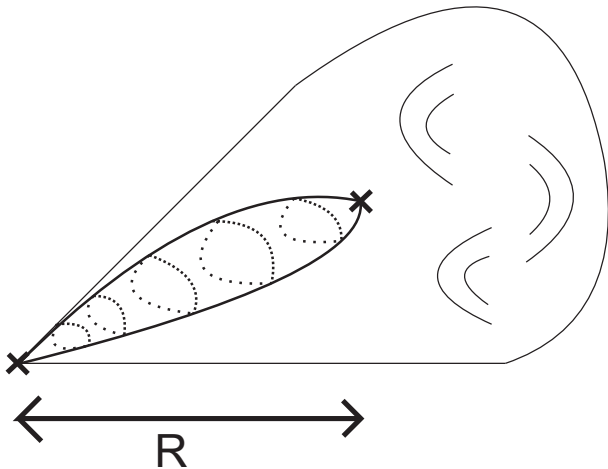


Annulus amplitude in  $t \rightarrow 0$  limit:



- ▶  $t \rightarrow 0$  divergence corresponds to a source for a partially twisted RR 2-form.
- ▶ In the local model this propagates into the bulk of the Calabi-Yau.
- ▶ Logarithmic divergence is divergence for a 2-dimensional source.
- ▶ In a compact model this becomes a physical divergence and tadpole must be cancelled by bulk sink branes/O-planes.
- ▶ Tadpole is sourced locally but must be cancelled globally

The purely local computation omits the following worksheets:



- ▶ The purely local string computation includes all open string states for  $t > 1/(RM_s)^2$ , i.e.  $M < RM_s$ .
- ▶ However for  $t < 1/(RM_s)^2$  we must include new winding states in the partition function.
- ▶ These are essential for global consistency but are omitted by a purely local computation.
- ▶ These enter the computation for  $t < 1/(RM_s)^2$  and enforce finiteness (RR tadpole cancellation).

- ▶ The bulk worldsheets enforce global RR tadpole cancellation and effectively cut off the integral at  $t = \frac{1}{(RM_s)^2}$ .

- ▶ Threshold corrections become finite

$$\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a \rightarrow \int_{1/(RM_s)^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a$$

- ▶ Effective UV cutoff is actually  $RM_s$  and *not*  $M_s$ !

## Result Summary

- ▶ For all cases studied string computation reproduces result of supergravity analysis.
- ▶ Effective unification scale is  $RM_s \gg M_s$ .
- ▶ In string theory, presence of radius arises from an RR tadpole sourced by the local model but which is cancelled by the bulk.
- ▶ In open string channel, model does not 'know' its self-consistency until an energy scale  $RM_s$ .

## Result Summary

- ▶ Main result: for local models, both supergravity and string theory imply gauge couplings start running from  $RM_s$  and not  $M_s$ .
- ▶ This should hold for all local models: D3 branes at singularities, F-theory GUTs, IIB GUTs...

Note the hypercharge flux in F-theory/IIB GUTs has necessary properties for relevant physics to apply.

- ▶ Large effect: for  $M_s \sim 10^{12}\text{GeV}$  changes  $\Lambda_{UV}$  by a factor of 100 and for  $M_s \sim 10^{15}\text{GeV}$  changes  $\Lambda_{UV}$  by a factor of 10.



## What should the string scale be?

- ▶  $M_s = 10^{11} - 10^{12} \text{GeV}$  is good for moduli stabilisation, the hierarchy problem, TeV supersymmetry and axions. Threshold corrections shift the unification scale to  $10^{13} \rightarrow 10^{14} \text{GeV}$ .
- ▶ If we want unification, then threshold corrections shift the required string scale from  $10^{16} \text{GeV}$  to  $10^{15} \text{GeV}$ .