Kähler Potentials for Chiral Matter in Calabi-Yau String Compactifications

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This talk is based on

hep-th/0609180 (JC, D.Cremades, F. Quevedo)

hep-th/0610129 (JC, S. Abdussalam, F. Quevedo, K. Suruliz)

hep-ph/061xxxx (JC, D. Cremades)

and also uses results from

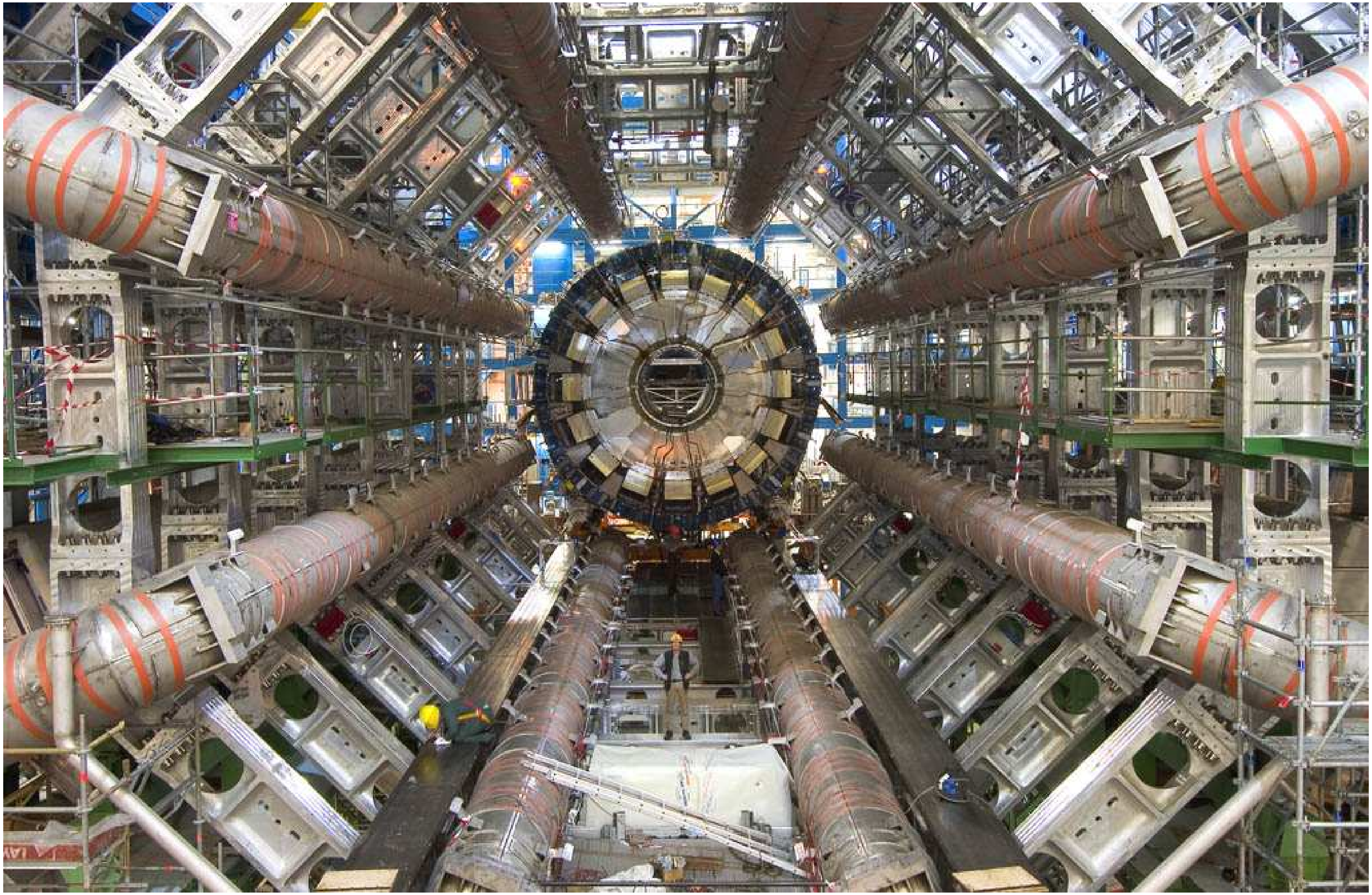
hep-th/0502058 (V. Balasubramanian, P. Berglund, JC, F. Quevedo)

hep-th/0505076 (JC, F. Quevedo, K. Suruliz)

Talk Structure

- Motivation
- Introduction and Review of Supersymmetry Breaking
- Computing Matter Metrics from Yukawa Couplings
- Results for Particular Models
- Applications: Soft Terms and Neutrino Masses
- Conclusions

Motivation!



Motivation

- The LHC!
- This will probe TeV-scale physics in unprecedented detail. If supersymmetry exists, it will (probably) be discovered at the LHC.
- Low-energy supersymmetry represents one of the best possibilities for connecting string theory (or any high-scale theory) to data.
- Understanding supersymmetry breaking and predicting the pattern of superpartners is one of the most important tasks of string phenomenology.

MSSM Basics

The MSSM is specified by its matter content (that of the supersymmetric Standard Model) plus soft breaking terms. These are

- Soft scalar masses, $m_i^2 \phi_i^2$
- Gaugino masses, $M_a \lambda^a \lambda^a$,
- Trilinear scalar A-terms, $A_{\alpha\beta\gamma}\phi^\alpha \phi^\beta \phi^\gamma$
- B-terms, $B H_1 H_2$.

The LHC will (hopefully) give experimental information about these soft breaking terms.

Our task as theorists is to beat the experimentalists to the spectrum!

SUSY Breaking basics

The soft terms are generated by the mechanism of supersymmetry breaking and how this is transmitted to the observable sector. Examples are

- Gravity mediation - hidden sector supersymmetry breaking
- Gauge mediation - visible sector supersymmetry breaking
- Anomaly mediaton - susy breaking through loop effects

These all have characteristic features and scales.

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- communicated to the observable sector through non-renormalisable supergravity contact interactions
- which are suppressed by M_P.

Naively,

$$m_{susy} \sim \frac{F^2}{M_P}$$

This requires $F \sim 10^{11}$GeV for TeV-scale soft terms.

Gravity Mediation II

- The computation of soft terms starts by expanding the supergravity K and W in terms of the matter fields C^α,

$$
\begin{aligned}
W &= \hat{W}(\Phi) + \mu(\Phi) H_1 H_2 + \frac{1}{6} Y_{\alpha\beta\gamma}(\Phi) C^\alpha C^\beta C^\gamma + \ldots, \\
K &= \hat{K}(\Phi, \bar{\Phi}) + \tilde{K}_{\alpha\bar{\beta}}(\Phi, \bar{\Phi}) C^\alpha C^{\bar{\beta}} + [Z H_1 H_2 + h.c.] + \ldots, \\
f_a &= f_a(\Phi).
\end{aligned}
$$

- The function $\tilde{K}_{\alpha\bar{\beta}}(\Phi)$ is crucial in computing soft terms, as it determines the normalisation of the matter fields.
- Given this expansion, the computation of the physical soft terms is straightforward.

Gravity Mediation III

- Soft scalar masses $m^2_{i\bar{j}}$ and trilinears $A_{\alpha\beta\gamma}$ are given by

$$\begin{aligned}
\tilde{m}^2_{\alpha\bar{\beta}} &= (m^2_{3/2} + V_0)\tilde{K}_{\alpha\bar{\beta}} \\
&\quad -\bar{F}^{\bar{m}}F^n\left(\partial_{\bar{m}}\partial_n\tilde{K}_{\alpha\bar{\beta}} - (\partial_{\bar{m}}\tilde{K}_{\alpha\bar{\gamma}})\tilde{K}^{\bar{\gamma}\delta}(\partial_n\tilde{K}_{\delta\bar{\beta}})\right) \\
A'_{\alpha\beta\gamma} &= e^{\hat{K}/2}F^m\Big[\hat{K}_m Y_{\alpha\beta\gamma} + \partial_m Y_{\alpha\beta\gamma} \\
&\quad -\Big((\partial_m\tilde{K}_{\alpha\bar{\rho}})\tilde{K}^{\bar{\rho}\delta}Y_{\delta\beta\gamma} + (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma)\Big)\Big].
\end{aligned}$$

- *Any* physical prediction for the soft terms requires a knowledge of $\tilde{K}_{\alpha\bar{\beta}}$ for chiral matter fields.
- However, $\tilde{K}_{\alpha\bar{\beta}}$ is non-holomorphic and thus hard to compute.

What is known?

- In Calabi-Yau backgrounds, the Kähler metrics for (non-chiral) D3 and D7 position moduli and D7 Wilson line moduli have been computed by dimensional reduction.

$$\tilde{K}_{D7} \sim \frac{1}{S+\bar{S}} \qquad \tilde{K}_{D3} \sim \frac{1}{T+\bar{T}}$$

- Using explicit string scattering computations, matter metrics for chiral D7/D7 matter have been computed in IIB toroidal backgrounds.

$$\tilde{K}_{D7_i D7_j} \sim \frac{1}{\sqrt{T_k+\bar{T}_k}}$$

This talk

- I will describe new techniques for computing the matter metric $\tilde{K}_{\alpha\bar{\beta}}$ for chiral matter fields on a Calabi-Yau.
- These techniques will apply to chiral D7/D7 matter in IIB compactifications.
- I will compute the modular dependence of the matter metrics from the modular dependence of the (physical) Yukawa couplings.

I start by reviewing how Yukawa couplings arise in supergravity.

Yukawa Couplings in Supergravity

- In supergravity, the Yukawa couplings arise from the Lagrangian terms (Wess & Bagger)

$$\begin{aligned}\mathcal{L}_{kin} + \mathcal{L}_{yukawa} &= \tilde{K}_\alpha \partial_\mu C^\alpha \partial^\mu \bar{C}^{\bar{\alpha}} + e^{\hat{K}/2}\, \partial_\beta \partial_\gamma W\, \psi^\beta \psi^\gamma \\ &= \tilde{K}_\alpha \partial_\mu C^\alpha \partial^\mu \bar{C}^{\bar{\alpha}} + e^{\hat{K}/2} Y_{\alpha\beta\gamma} C^\alpha \psi^\beta \psi^\gamma\end{aligned}$$

(This assumes diagonal matter metrics, but we can relax this)

- The matter fields are not canonically normalised.
- The *physical* Yukawa couplings are given by

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}}.$$

Computing $\tilde{K}_{\alpha\bar{\beta}}$ (I)

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}}.$$

- We know the modular dependence of $\hat{K}$:

$$\hat{K} = -2\ln(\mathcal{V}) - \ln\left(i\int \Omega \wedge \bar{\Omega}\right) - \ln(S + \bar{S})$$

- We compute the modular dependence of $\tilde{K}_\alpha$ from the modular dependence of $\hat{Y}_{\alpha\beta\gamma}$. We work in a power series expansion and determine the leading power λ,

$$\tilde{K}_\alpha \sim (T + \bar{T})^\lambda k_\alpha(\phi) + (T + \bar{T})^{\lambda - 1} k'_\alpha(\phi) + \dots$$

λ is the modular weight of the field T.

Computing $\tilde{K}_{\alpha\bar{\beta}}$ (II)

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}}.$$

- We are unable to compute $Y_{\alpha\beta\gamma}$. If $Y_{\alpha\beta\gamma}$ depends on a modulus T, knowledge of $\hat{Y}_{\alpha\beta\gamma}$ gives no information about the dependence $\tilde{K}_{\alpha\bar{\beta}}(T)$.
- Our results will be restricted to those moduli that do not appear in the superpotential.
- The main example will be the T-moduli (Kähler moduli) in IIB compactifications. In perturbation theory,

$$\partial_{T_i} Y_{\alpha\beta\gamma} \equiv 0.$$

Computing $\tilde{K}_{\alpha\bar{\beta}}$ (III)

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}}.$$

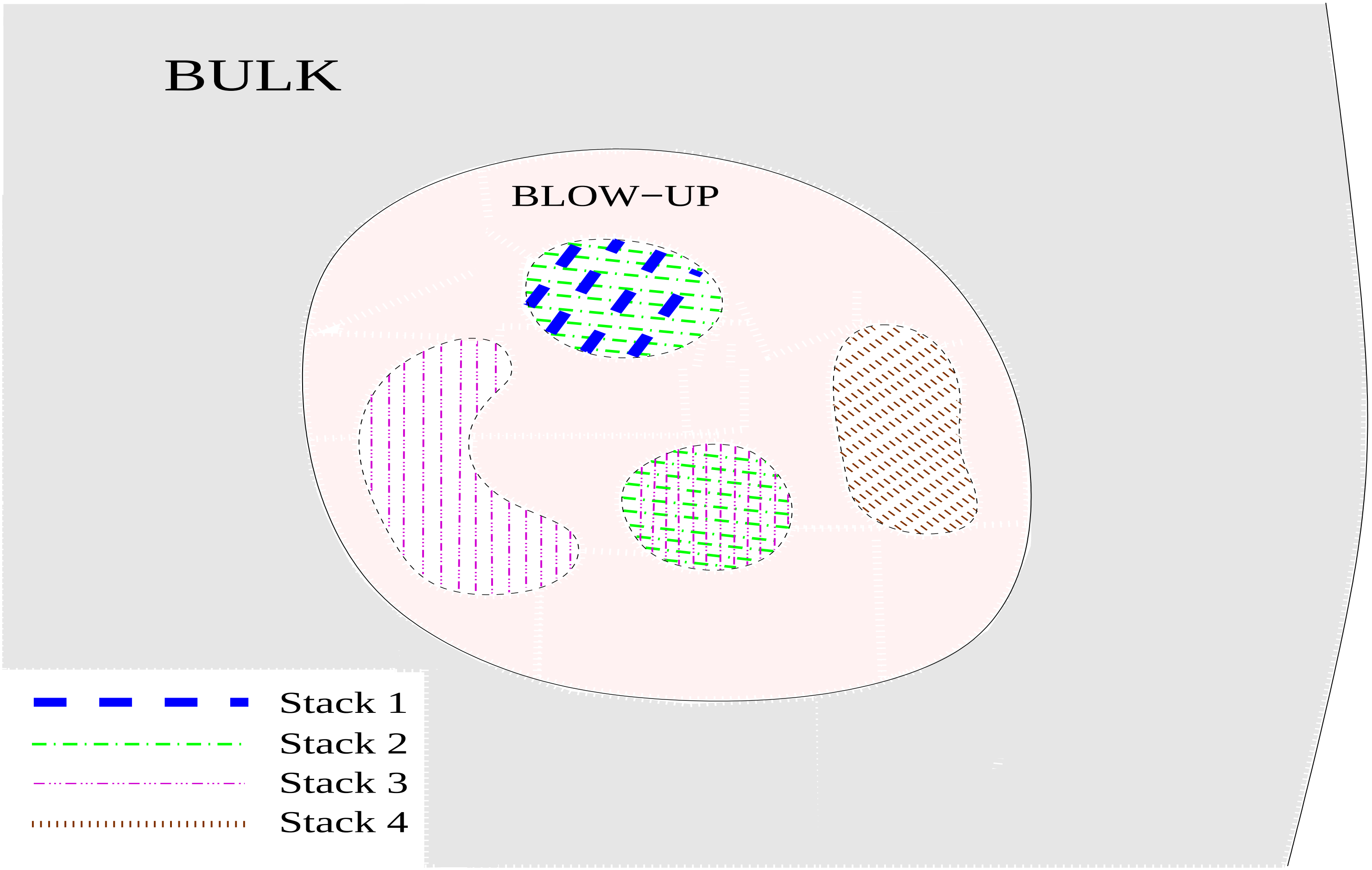
- We know $\hat{K}(T)$.
- If we can compute $\hat{Y}_{\alpha\beta\gamma}(T)$, we can then deduce $\tilde{K}_\alpha(T)$.
- Computing $\hat{Y}_{\alpha\beta\gamma}(T)$ is not as hard as it sounds!
- In IIB compactifications, this can be carried out through wavefunction overlap.

We now describe the computation of $\hat{Y}_{\alpha\beta\gamma}$ for chiral matter on a stack of magnetised D7-branes.

The brane geometry

- We consider a stack of (magnetised) branes all wrapping an identical cycle.
- If the branes are magnetised, chiral fermions can stretch between differently magnetised branes.
- This is a typical geometry in ‘branes at singularities’ models.

The brane geometry



Computing $\hat{Y}_{\alpha\beta\gamma}$

- We use a simple computational technique:

- Physical Yukawa couplings are determined by the triple overlap of normalised wavefunctions.

- These wavefunctions can be computed (in principle) by dimensional reduction of the brane action.

Dimensional reduction

Consider a stack of D7 branes wrapping a 4-cycle Σ in a Calabi-Yau X. The low-energy limit of the DBI action reduces to super Yang-Mills,

$$S_{SYM} = \int_{\mathbb{M}_4 \times \Sigma} d^n x \sqrt{g} \left(F_{\mu\nu} F^{\mu\nu} + \bar{\lambda} \Gamma^i (A_i + \partial_i) \lambda \right)$$

- Magnetic flux on the brane gives chiral fermions ψ_α in the low energy spectrum.
- These fermions come from dimensional reduction of the gaugino λ_i. They are counted topologically by the number of solutions of the Dirac equation

$$\Gamma^i D_i \psi = 0.$$

Comments

- The full action to be reduced is the DBI action rather than that of Super Yang-Mills.
- In the limit of large cycle volume, magnetic fluxes are diluted in the cycle, and the DBI action reduces to that of super Yang-Mills.
- Our results will hold within this large cycle volume, dilute flux approximation.

Computing the physical Yukawas $\hat{Y}_{\alpha\beta\gamma}$

Consider the higher-dimensional term

$$\mathcal{L} \sim \int_{\mathbb{M}_4 \times \Sigma} d^n x \sqrt{g}\, \bar{\lambda} \Gamma^i (A_i + \partial_i) \lambda$$

Dimensional reduction of this gives the kinetic terms

$$\bar{\psi} \partial \psi$$

and the Yukawa couplings

$$\bar{\psi} \phi \psi$$

- The physical Yukawa couplings are set by the combination of the above!

Kinetic Terms

$$\mathcal{L} \sim \int_{\mathbb{M}_4 \times \Sigma} d^n x \sqrt{g}\, \bar{\lambda} \Gamma^i (A_i + \partial_i) \lambda$$

- Dimensional reduction gives

$$\lambda = \psi_4 \otimes \psi_6, \qquad A_i = \phi_4 \otimes \phi_6.$$

and the reduced kinetic terms are

$$\left(\int_\Sigma \psi_6^\dagger \psi_6 \right) \int_{\mathbb{M}_4} \bar{\psi}_4 \Gamma^\mu (A_\mu + \partial_\mu) \psi_4$$

- Canonically normalised kinetic terms require

$$\int_\Sigma \psi_6^\dagger \psi_6 = 1.$$

Yukawa Couplings

$$\mathcal{L} \sim \int_{\mathbb{M}_4 \times \Sigma} d^n x \, \bar{\lambda} \Gamma^i (A_i + \partial_i) \lambda$$

- Dimensional reduction gives the four-dimensional interaction

$$\left(\int_\Sigma \bar{\psi}_6 \Gamma^I \phi_{I,6} \psi_6 \right) \int_{\mathbb{M}_4} d^4 x \, \phi_4 \bar{\psi}_4 \psi_4 .$$

- The physical Yukawa couplings are determined by the (normalised) overlap integral

$$\hat{Y}_{\alpha\beta\gamma} = \left(\int_\Sigma \bar{\psi}_6 \Gamma^I \phi_{I,6} \psi_6 \right) .$$

The Result

$$\int_\Sigma \psi_6^\dagger \psi_6 = 1, \qquad \hat{Y}_{\alpha\beta\gamma} = \left(\int_\Sigma \bar{\psi}_6 \Gamma^I \phi_{I,6} \psi_6 \right).$$

- For canonical kinetic terms,

$$\psi_6(y) \sim \frac{1}{\sqrt{\mathrm{Vol}(\Sigma)}}, \qquad \Gamma^I \phi_{I,6} \sim \frac{1}{\sqrt{\mathrm{Vol}(\Sigma)}},$$

and so

$$\hat{Y}_{\alpha\beta\gamma} \sim \mathrm{Vol}(\Sigma) \cdot \frac{1}{\sqrt{\mathrm{Vol}(\Sigma)}} \cdot \frac{1}{\sqrt{\mathrm{Vol}(\Sigma)}} \cdot \frac{1}{\sqrt{\mathrm{Vol}(\Sigma)}} \sim \frac{1}{\sqrt{\mathrm{Vol}(\Sigma)}}$$

- This gives the scaling of $\hat{Y}_{\alpha\beta\gamma}(T)$.

Comments

- Q. Under the cycle rescaling, why should $\psi(y)$ scale simply as

$$\psi(y) \to \frac{\psi(y)}{\sqrt{\text{Re}(T)}}?$$

  A. The T-moduli do not appear in $Y_{\alpha\beta\gamma}$ and do not see flavour.

  Any more complicated behaviour would alter the form of the triple overlap integral and $Y_{\alpha\beta\gamma}$ - but this would require altering the complex structure moduli.

- The result for the scaling of $\hat{Y}_{\alpha\beta\gamma}$ holds in the classical limit of large cycle volume.

Application: One-modulus KKLT

We can now compute $\tilde{K}_\alpha(T)$. In a 1-modulus KKLT model, all chiral matter is supported on D7 branes wrapping the single 4-cycle T. From above,

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}} \sim \frac{1}{\sqrt{T+\bar{T}}}$$

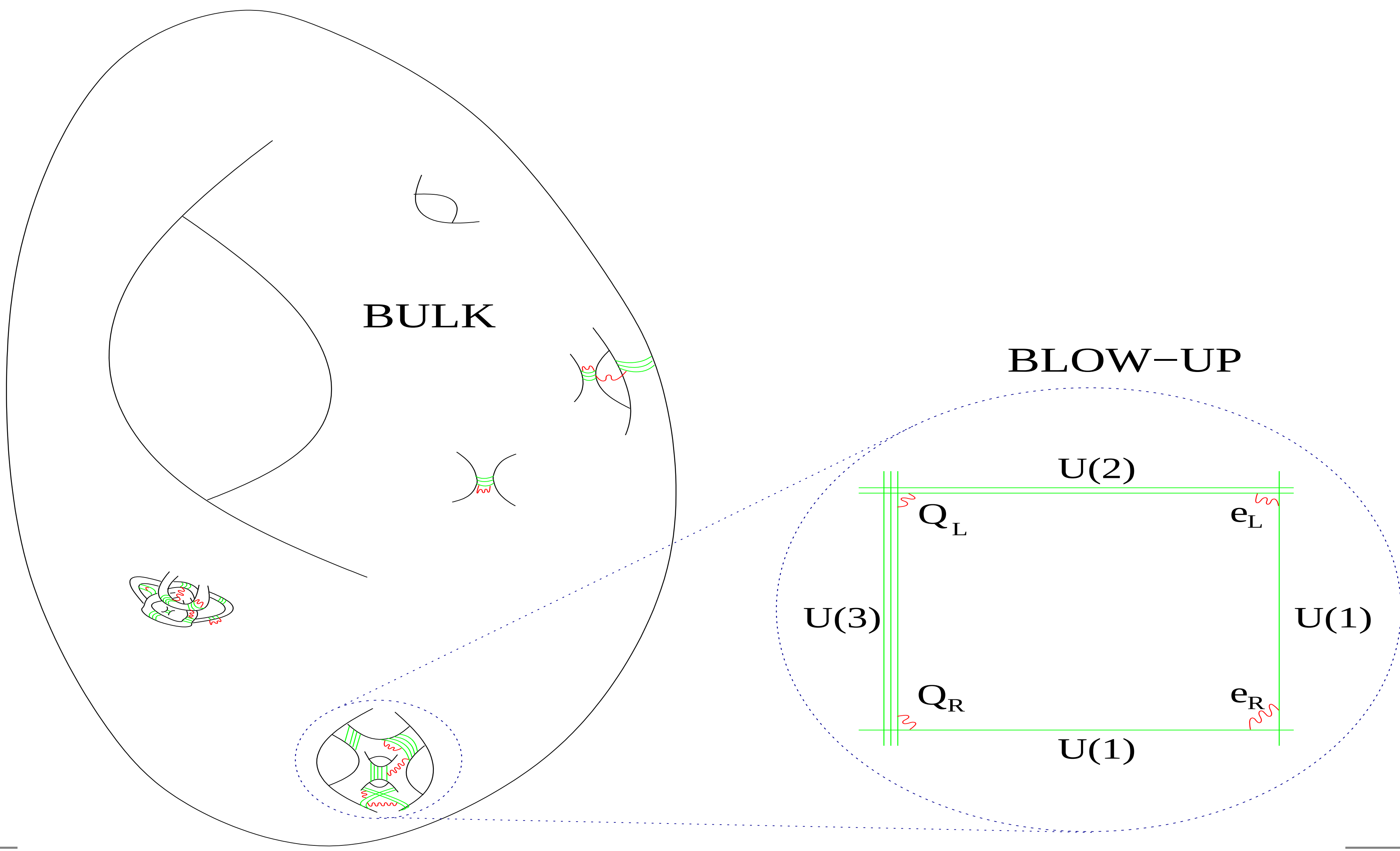
We know the moduli Kähler potential,

$$\hat{K} = -3\ln(T+\bar{T})$$

and so the matter Kähler potential must scale as

$$\tilde{K}_\alpha \sim \frac{1}{(T+\bar{T})^{2/3}}.$$

Application: Large-Volume Models



Application: Large-Volume Models

- The overall volume is very large, $\mathcal{V} \sim 10^{15}\, l_s^6$, with small cycles $\tau_s \sim 10\, l_s^4$.
- For the simplest model $\mathbb{P}^4_{[1,1,1,6,9]}$, the Kähler potential is

$$\hat{K} = -2\ln\mathcal{V} = -2\ln\left((T_b+\bar{T}_b)^{3/2} - (T_s+\bar{T}_s)^{3/2}\right).$$

- We can interpret the T_s cycle as a local, 'blow-up' cycle.
- We want $\tilde{K}_\alpha(T)$ for chiral matter on branes wrapping this cycle.
- The gauge theory supported on a brane wrapping T_s is determined by local geometry.

Application: Large-Volume Models

- The physical Yukawa coupling

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}}$$

is local and thus independent of $\mathcal{V}$.

- As $\hat{K} = -2 \ln \mathcal{V}$, we can deduce *simply from locality* that

$$\tilde{K}_\alpha \sim \frac{1}{\mathcal{V}^{2/3}}.$$

- As this is for a Calabi-Yau background, this is already non-trivial!

Application: Large-Volume Models

- We can go further. With all branes wrapping the same 4-cycle,

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}} \sim \frac{1}{\sqrt{\tau_s}}.$$

- We can then deduce that

$$\tilde{K}_\alpha \sim \frac{\tau_s{}^{1/3}}{\mathcal{V}^{2/3}}.$$

- We also have the dependence on τ_s!

Application: Large-Volume Models

- The dependence

$$\tilde{K}_\alpha \sim \frac{1}{\mathcal{V}^{2/3}}$$

follows purely from the requirement that physical Yukawa couplings are local.

- The dependence on τ_s,

$$\tilde{K}_\alpha \sim \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}}$$

follows from the specific brane configuration (all D7 branes wrapping the same small cycle).

Application: Large-Volume Models

- If we wrap branes on the large cycle, as for the one-modulus KKLT model we find

$$\tilde{K} \sim \frac{1}{(T_b + \bar{T}_b)^{2/3}}.$$

- All the above results hold for both diagonal and non-diagonal matter metrics.
- The reason is that the T moduli do not see flavour, and so the T-dependence is flavour-universal.

Soft Terms

- We can use the above matter metric to compute the soft terms for the large-volume models.
- We get

$$\begin{aligned}
M_i &= \frac{F^s}{2\tau_s} \equiv M, \\
m_{\alpha\bar{\beta}} &= \frac{M}{\sqrt{3}} \tilde{K}_{\alpha\bar{\beta}}, \\
A_{\alpha\beta\gamma} &= -M\,\hat{Y}_{\alpha\beta\gamma}, \\
B &= -\frac{4M}{3}.
\end{aligned}$$

Soft Terms

- These soft terms are the same as in the dilaton-dominated heterotic scenario.
- They are also flavour-universal.
- This is surprising - there is a naive expectation that gravity mediation will give non-universal soft terms.
- Why? Flavour physics is Planck-scale physics, and in gravity mediation supersymmetry breaking is also Planck-scale physics.
- Naively, we expect the susy-breaking sector to ‘see’ flavour and thus give non-universal soft terms.

Soft Terms: Flavour Universality

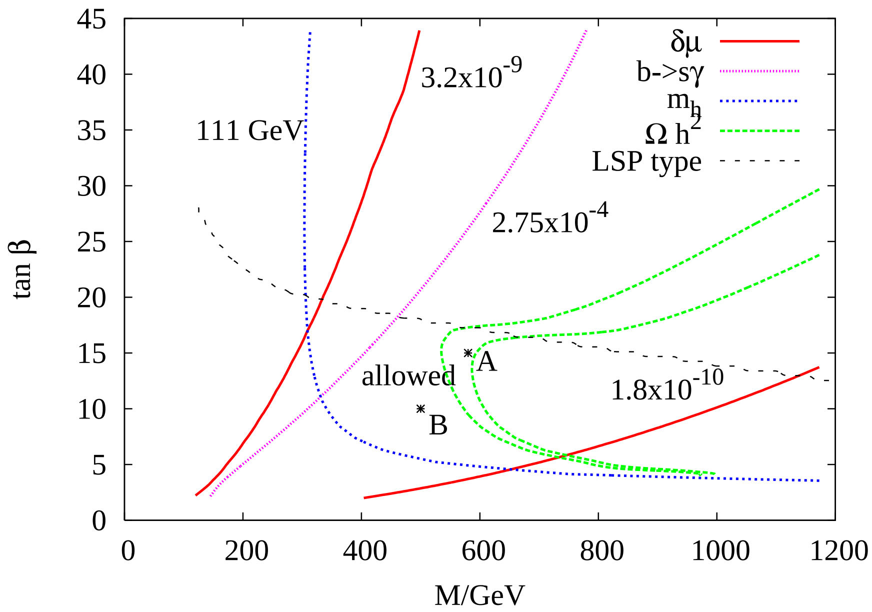
- These expectations are EFT expectations
- In string theory, we have Kähler (T) and complex structure (U) moduli.
- These are decoupled at leading order.

$$\mathcal{K} = -2\ln\left(\mathcal{V}(T)\right) - \ln\left(i\int \Omega \wedge \bar{\Omega}(U)\right) - \ln(S+\bar{S}).$$

- Here, U sources flavour and T breaks supersymmetry.
- At leading order, susy-breaking and flavour decouple.
- The origin of universality is the decoupling of Kähler and complex structure moduli.

Soft Terms: Phenomenology

- We run the soft terms to low energy using SoftSUSY and study the spectrum.
- The constraints are given by



Neutrino Masses

- The theoretical origin of neutrino masses is a mystery. Experimentally

$$0.05\text{eV} \lesssim m_\nu^H \lesssim 0.3\text{eV}.$$

- This corresponds to a Majorana mass scale

$$M_{\nu_R} \sim 3 \times 10^{14}\text{GeV}.$$

- Equivalently, this is the suppression scale Λ of the dimension five Standard Model operator

$$\mathcal{O} = \frac{1}{\Lambda} HHLL.$$

Neutrino Masses

- In the supergravity MSSM, consider the superpotential operator
$$\frac{\lambda}{M_P} H_2 H_2 L L \in W,$$
where λ is dimensionless.
- This corresponds to the *physical* coupling
$$e^{\hat{K}/2} \frac{\lambda}{M_P} \frac{\langle H_2 H_2 \rangle L L}{(\tilde{K}_{H_2} \tilde{K}_{H_2} \tilde{K}_L \tilde{K}_L)^{\frac{1}{2}}}.$$
- Once the Higgs receives a vev, this generates neutrino masses.

Neutrino Masses

- Using the large-volume result $\tilde{K}_\alpha \sim \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}}$, this becomes

$$\frac{\lambda \mathcal{V}^{1/3}}{\tau_s^{2/3} M_P} \langle H_2 H_2 \rangle LL.$$

- Use $\mathcal{V} \sim 10^{15}$ (to get $m_{3/2} \sim 1$TeV) and $\tau_s \sim 10$:

$$\frac{\lambda}{10^{14}\text{GeV}} \langle H_2 H_2 \rangle LL$$

- With $\langle H_2 \rangle = \frac{v}{\sqrt{2}} = 174$GeV, this gives

$$m_\nu = \lambda (0.3\,\text{eV}).$$

Large Volumes are Powerful

- In large-volume models, an exponentially large volume appears naturally ($\mathcal{V} \sim e^{\frac{c}{g_s}}$).
- A volume $\mathcal{V} \sim 10^{15} l_s^6$, required for TeV-scale supersymmetry, automatically gives the correct scale for neutrino masses.

- This same volume also gives an axion decay constant in the allowed window (JC, hep-th/0602233),

$$10^9 \text{GeV} \lesssim f_a \lesssim 10^{12} \text{GeV}.$$

- This yoking of three distinct scales is very attractive.
- The origin of all three hierarchies is the exponentially large volume.

Conclusions

- Kähler metrics for chiral matter enter crucially in the computation of MSSM soft terms.
- I have described techniques to compute these in IIB Calabi-Yau string compactifications.
- We computed the modular weights both for one-modulus KKLT models and for the multi-modulus large-volume models.
- The soft terms are flavour-universal, which comes from the decoupling of Kähler and complex structure moduli.
- For the large-volume models, TeV-scale supersymmetry naturally gives the correct scale for neutrino masses.


"That's all Folks!"