How long is a piece of string? String Theory and Particle Physics

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The Problem of Quantum Gravity

General relativity action is

$$\begin{aligned} S_{EH} &= \frac{M_P^2}{16\pi^2} \int \sqrt{g}\mathcal{R} \\ &= \frac{M_P^2}{16\pi^2} \int \partial_\mu \left(\delta h_{\alpha\beta}\right) \partial^\mu \left(\delta h^{\alpha\beta}\right) + \partial_\alpha \left(\delta h_{\mu\nu}\right) \partial_\beta \left(\delta h_{\mu\nu}\right) \delta h^{\alpha\beta} + \end{aligned}$$

Canonical normalisation gives interactions of

$$\begin{aligned} S_{EH} &= \int d^4x \partial_\mu (M_P \delta h) \partial^\mu (M_P \delta h) + \frac{(M_P \delta h) \partial (M_P \delta h) \partial (M_P \delta h)}{M_P} + \ldots \\ &= \int d^4x \, \partial_\mu \bar{\delta} h \partial^\mu \bar{\delta} h + \frac{1}{M_P} (\bar{\delta} h)(\partial \bar{\delta} h)(\partial \bar{\delta} h) + \ldots \end{aligned}$$

This is a non-renormalisable theory.

The Problem of Quantum Gravity

There are an infinite number of potential higher derivative operators

$$S_{EH} = \frac{M_P^2}{16\pi^2}\int \sqrt{g}\left(\mathcal{R} + \left(\alpha_1 \mathcal{R}^2 + \alpha_2 \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \alpha_3 \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}\right) + \right.$$
$$\left. + \left(\beta_1 \mathcal{R}^3 + \ldots\right) + \left(\gamma_1 \mathcal{R}^4 + \ldots\right)\right)$$

A scattering amplitude takes the schematic form

$$\mathcal{A}(p^2) \sim \frac{p^2}{M_P^2}\left(1 + \underbrace{\alpha\frac{p^2}{M_P^2} + \beta\frac{p^4}{M_P^4} + \gamma\frac{p^6}{M_P^6} + \ldots}_{\text{higher dimension operators}}\right)$$

General relativity fails as an effective theory at energies $p^2 \sim M_P^2$ and needs a UV completion.

The Problem of Quantum Gravity

A scattering amplitude takes the schematic form

$$\mathcal{A}(p^2) \sim \frac{p^2}{M_P^2}\left(1 + \underbrace{\alpha\frac{p^2}{M_P^2} + \beta\frac{p^4}{M_P^4} + \gamma\frac{p^6}{M_P^6} + \dots}_{\text{higher dimension operators}}\right)$$

The problem of quantum gravity is the absence of a microscopic theory to

- Determine the coefficients α, β, γ and allow predictivity at energy scales $E \sim M_P$.
- Specify the new degrees of freedom that arise at energies $E \sim M_P$.

Other Non-Renormalisable Theories

The Euler-Heisenberg Lagrangian:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha^2}{90 m_e^4}\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2\right] \\ &= \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right) + \frac{2\alpha^2}{45 m_e^4}\left[\left(\mathbf{E}^2 - \mathbf{B}^2\right)^2 + 7\left(\mathbf{E}\cdot\mathbf{B}\right)^2\right] + \ldots\end{aligned}$$

For energies, $E \ll m_e$, this is the effective theory describing $\gamma\gamma \to \gamma\gamma$ scattering.

Microscopic completion for $E \gtrsim m_e$: quantum electrodynamics.

New degrees of freedom: spin-1/2 fermions electron and positron.

Other Non-Renormalisable Theories

The Fermi Lagrangian for muon decay $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$

$$\mathcal{L}_F = -\frac{G_F}{2} \left(\bar{\nu}_\mu \gamma^\alpha (1 - \gamma^5) \mu \right) \left(\bar{e} \gamma_\alpha (1 - \gamma^5) \nu_e \right) + \ldots .$$

For energies, $E \ll M_W$, this is the effective theory describing weak interactions.

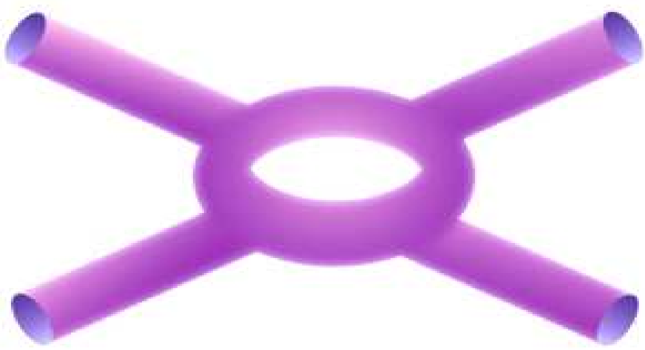
Microscopic completion for $E \gtrsim M_W$: electroweak gauge theory.

New degrees of freedom: electroweak vector bosons.

String Theory

Low energy effective theories require new microscopic degrees of freedom to be good high-energy theories.

In the case of gravity the best idea is strings...



String Theory

String theory is the theory of consistent, quantum-mechanical relativistic strings:

$$S_{\text{Nambu-Goto}} = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \left(\sqrt{g} + B_2 + \sqrt{g} R\right)$$

Action is determined by

- Area swept out by worldsheet ($\int \sqrt{g}$)
- Topology of target space embedding ($\int B_2$)
- Internal topology of worldsheet $\int \sqrt{g} R$

Quantum consistency of the theory (world-sheet conformal anomaly) requires ten space-time dimensions and Einstein's equations.

String theory then gives finite and well-defined answers for the scattering of particles with energies $E \sim M_P$.

String Theory

The ten-dimensional action of IIB string theory behaves as

$$\frac{1}{l_s^8} \int d^{10}x \sqrt{g} \frac{1}{g_s^2} \left(\mathcal{R} + \zeta(3) l_s^6 \mathcal{R}^4 + \ldots \right)$$

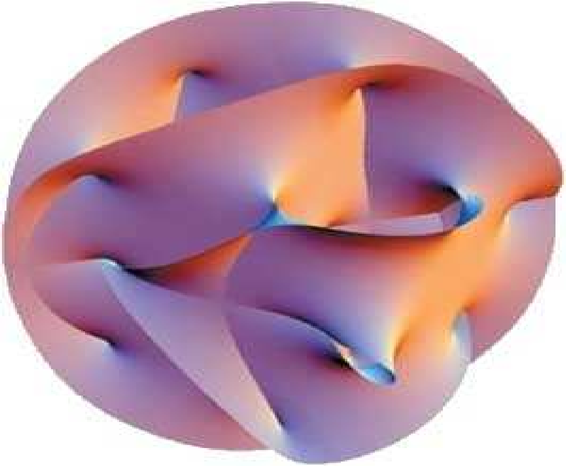
The coefficients of the non-renormalisable terms are fixed by the theory.

This makes the theory well-defined for scattering processes at energies near M_P, and provides a microscopic completion of (ten-dimensional) general relativity.

Connecting to four dimensional physics requires that six of the dimensions are compactified at a very small scale.

String Theory

The extra dimensions form a *Calabi-Yau manifold*.



String Theory

The ten-dimensional action of IIB string theory behaves as

$$\frac{1}{l_s^8}\int d^{10}x\sqrt{g}\,\frac{1}{g_s^2}\left(\mathcal{R}+\zeta(3)\mathcal{R}^4+\ldots\right)$$

Dimensionally reducing this action on a six-dimensional space of volume $\mathcal{V}_6$ gives an effective 4-dimensional action of

$$\frac{\mathcal{V}_6}{g_s^2 l_s^6}\frac{1}{l_s^2}\int d^4x\sqrt{g_4}\left(\mathcal{R}_4+\mathcal{R}_4^2+\ldots\right)\equiv M_P^2\int d^4x\sqrt{g_4}\mathcal{R}_4+\ldots$$

This gives a very important relation:

$$m_s = g_s M_P\sqrt{\frac{l_s^6}{\mathcal{V}_6}},\qquad l_s=\frac{l_P}{g_s}\sqrt{\frac{\mathcal{V}_6}{l_s^6}}$$

String Theory

In addition to the graviton, there are many new extra hidden sector degrees of freedom.

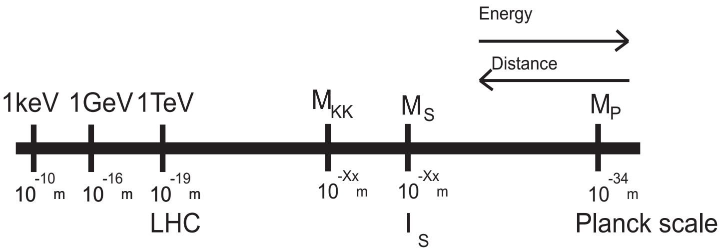
The simplest of these come from higher dimensional modes of the graviton.

They are associated with the geometric deformations of the extra dimensions: for example their size and shape. They are called moduli.

These are light degrees of freedom, weakly coupled to the Standard Model by gravitationally suppressed interactions.

String Theory

The distance scales involved:



The values of M_{KK} and M_S are determined by the string coupling and the extra-dimensional volume.

String Theory

Low-energy (gravitational) theory:

$$\frac{M_P^2}{16\pi^2}\int d^4x\sqrt{g}\left(\mathcal{R}+\mathcal{R}^2+\mathcal{R}^3+\ldots\right)$$

For energies $E \ll M_P$, this is the effective theory describing gravitational interactions.

Microscopic completion for energies $E \gtrsim M_s\left(\frac{l_s}{R}\right)$: 10-dimensional general relativity

Microscopic completion for energies $E \gtrsim M_s$: String theory

New degrees of freedom for energies $E \gtrsim M_s\left(\frac{l_s}{R}\right)$: 6-dimensional KK modes

New degrees of freedom for energies $E \gtrsim M_s$: excited string states.

String Theory

At low energies, the 'stringy corrections' appear as higher dimension non-renormalisable operators.

These are suppressed by scales from the fundamental theory - M_S, M_{KK} and M_P.

These corrections are essential for theoretical consistency at high energy scales near $M_S/M_{KK}/M_P$.

All such corrections are irrelevant operators and at low energies/long distances $E \ll M_S$ their effects are suppressed by powers of $\frac{E}{M_S}$.

String Theory Summary

- String theory is the microscopic completion of general relativity at length scales $l \lesssim l_s$.
- The new degrees of freedom it introduces are higher-dimensional Kaluza-Klein modes and excited string harmonics, at energy scales $M_s \left(\frac{l_s}{R}\right)$ and M_s.
- At low energies/long distances this short-distance physics generates irrelevant operators suppressed by powers of $\frac{1}{M_s}$ or $\frac{1}{M_P}$.
- Why care?

When are irrelevant operators relevant?

Irrelevant operators generally give very small effects at long distances:

- At loop level relevant and marginal operators generate all operators allowed by symmetry.
- At low energy these dominate irrelevant operators suppressed by high energy scales.

The principal exception is when irrelevant operators give the leading symmetry breaking effects in the low energy theory.

In this case the irrelevant operator *is* the leading physics.

When are irrelevant operators relevant?

Irrelevant operators matter when they break symmetries.

Example: Both strong and electromagnetic interactions preserve quark/lepton flavour type, but weak interactions do not.

$$\tau_{\text{neutron}} \sim 15\text{min}, \quad n \rightarrow p^+ + e^- + \bar{\nu}_e,$$
$$\tau_\mu \sim 2 \times 10^{-6}\text{s}, \quad \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu.$$

Weak interactions are both *irrelevant* and the *leading symmetry breaking effects* at low energy.

When are irrelevant operators relevant?

For Planck scale physics to matter at low energies we want a symmetry that is:

- Preserved by gauge and Yukawa interactions of the Standard Model and unbroken by renormalisable quantum field theory
- Broken by non-renormalisable gravitational strength interactions.

Analogy of Fermi theory of weak interactions, except going the other way.

Supersymmetry

This symmetry is supersymmetry.

Supersymmetry is the unique extension of the Lorentz symmetry.

Haag, Lopuskanski and Sohnius: *The most general symmetry of the S-matrix is a direct product of super-Poincare and internal symmetries.*

$$G = G_{super-Poincare} \times G_{internal}$$

Supersymmetry has *fermionic generators* and combines spacetime and internal symmetries:

$$\mathcal{Q}_\alpha |\text{boson} >= |\text{fermion} >, \qquad \mathcal{Q}_\alpha |\text{fermion} >= |\text{boson} > .$$

Supersymmetry

Supersymmetry can resolve the biggest problem of the Standard Model and for this reason is one of the principal targets of the LHC physics programme.

This problem is the hierarchy problem: the closeness of the Standard Model to a critical point.

The Standard Model Higgs potential is

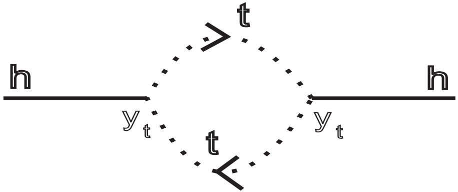
$$V_H = -m^2|\phi|^2 + \lambda|\phi|^4$$

Here $m^2 \sim \mathcal{O}(100\text{GeV})^2 \ll (10^{18}\text{GeV})^2 \equiv M_P^2$.

Why is the Standard Model so close to the critical point $m = 0$?

Supersymmetry

Quantum loops can renormalise the Higgs mass parameter:

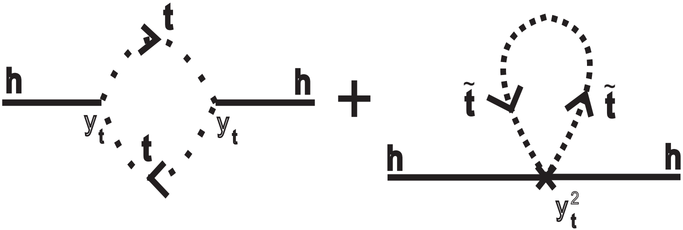


$$\delta m_H^2 \underset{k\to\infty}{\sim} \frac{y_t^2}{16\pi^2}\int \frac{d^4k}{k^2} \sim \frac{y_t^2}{16\pi^2}\int^{\Lambda} d|k||k| \sim \frac{y_t^2}{16\pi^2}\Lambda^2.$$

Microscopic quantum corrections drive the Higgs mass to the microscopic scale of the theory.

Supersymmetry

Supersymmetry gives extra contributions:



$$\delta m_H^2 \underset{k\to\infty}{\sim} \frac{y_t^2}{16\pi^2}\int \frac{d^4k}{k^2} - \frac{y_t^2}{16\pi^2}\int \frac{d^4k}{(k^2+m_{\tilde{t}}^2)} \sim \frac{y_t^2}{16\pi^2} \ln\left(\frac{\Lambda^2}{m_t^2}\right).$$

Supersymmetry at short distances eliminates the quadratic divergences and makes the Standard Model stable against radiative corrections.

Supersymmetry

Supersymmetry is not a long-distance symmetry and must be broken at energies below the weak scale.

The breaking is described by

$$\mathcal{L} = \mathcal{L}_{\text{Supersymmetric Standard Model}} + \mathcal{L}_{\text{soft terms}}$$

The soft terms are *relevant terms* in the Lagrangian that *explicitly break* supersymmetry but *do not reintroduce* quadratic divergences

They consist of:

Scalar masses $m^2|\phi^2|$, Trilinear scalar A-terms $A_{\alpha\beta\gamma}\phi^\alpha\phi^\beta\phi^\gamma$
Gaugino masses $M_a\lambda\lambda$, Bilinear scalar B-terms $B_{ab}\phi^a\phi^b$.

Supersymmetry

Soft terms consist of:

Scalar masses $m^2|\phi^2|$,	Trilinear scalar A-terms $A_{\alpha\beta\gamma}\phi^\alpha\phi^\beta\phi^\gamma$
Gaugino masses $M_a\lambda\lambda$,	Bilinear scalar B-terms $B_{ab}\phi^a\phi^b$.

The soft terms parametrise the breaking of supersymmetry and determine the phenomenology of supersymmetry at the LHC.

In a Panglossian universe, the LHC will measure all the soft terms of the MSSM.

Origin of Supersymmetry Breaking

$$\mathcal{L} = \underbrace{\mathcal{L}_{\text{hidden sector}}}_{\text{susy breaking}} + \underbrace{\mathcal{L}_{\text{visible sector}}}_{\text{Standard Model}} + \underbrace{\mathcal{L}_{\text{hidden-visible interactions}}}_{\text{Mediation of susy breaking}}$$

- Hidden sector dynamics in $\mathcal{L}_{\text{hidden}}$ break supersymmetry
- Supersymmetry breaking is communicated to visible sector by $\mathcal{L}_{\text{hidden-visible}}$
- Effects of supersymmetry breaking appear in $\mathcal{L}_{\text{visible}}$ as the soft terms.

Origin of Supersymmetry Breaking

Most generic and ubiquitous form of couplings for $\mathcal{L}_{\text{hidden-visible}}$ are those suppressed by the fundamental scale M_s/M_P.

$$\mathcal{L}_{\text{hidden-visible}} = \int d^4x \, \frac{\mathcal{O}_{\text{hidden}}\mathcal{O}_{\text{visible}}}{M_P^{\lambda}} + \ldots$$

Simple example (for Ψ a hidden sector field):

$$\mathcal{L}_{\text{hidden-visible}} \in \int d^4x \left(\frac{\lambda_1 \Psi F^{Y}_{\mu\nu} F^{Y,\mu\nu}}{M_P} + \frac{\lambda_2 \Psi F^{SU(2)}_{\mu\nu} F^{SU(2),\mu\nu}}{M_P} + \frac{\lambda_3 \Psi F^{SU(3)}_{\mu\nu} F^{SU(3),\mu\nu}}{M_P} \right)$$

Origin of Supersymmetry Breaking

The coupling

$$\mathcal{L}_{\text{hidden-visible}} \in \int d^4x \, \frac{\lambda_1 \Psi F^Y_{\mu\nu} F^{Y,\mu\nu}}{M_P}$$

describes the dependence of gauge couplings on the hidden sector field Ψ.

If Ψ is part of the supersymmetry breaking sector, this coupling induces gaugino masses for the $U(1)_Y$ gauginos.

Dependence of gauge couplings on Ψ generates gaugino masses.

Dependence of kinetic terms on Ψ generates scalar masses.

Dependence of Yukawa couplings on Ψ generates A-terms.

Origin of Supersymmetry Breaking

$$\text{Gauge couplings} \int d^4x\, f_a\left(\frac{\Psi}{M_P}\right) F_{\mu\nu}F^{\mu\nu} \longrightarrow M_a\lambda\lambda(\text{Gaugino masses})$$

$$\text{Kinetic terms:} \int d^4x\, K\left(\frac{\Psi}{M_P}, \frac{\bar{\Psi}}{M_P}\right) \bar{\psi}\gamma^\mu\partial_\mu\psi \longrightarrow m^2\phi^*\phi(\text{Scalar masses})$$

$$\text{Yukawa couplings:} \int d^4x\, Y_{\alpha\beta\gamma}\left(\frac{\Psi}{M_P}\right) \psi_\alpha\psi_\beta\phi^\gamma \longrightarrow A_{\alpha\beta\gamma}\phi^\alpha\phi^\beta\phi^\gamma$$

The soft terms that the LHC measure come directly from Planck suppressed non-renormalisable operators.

(In gravity mediation) Planck scale physics is the leading physics in generating these terms and necessary for understanding their structure.

The Weak Hierarchy Problem

Supersymmetry at distances below 10^{-18}m stabilises the weak scale against quantum corrections.

However why is supersymmetry broken at a distance scale 10^{15} times larger than the 'natural' value M_P?

Any dynamics breaking supersymmetry at distances smaller than 10^{-18}m will destabilise this hierarchy.

More generally, what is the origin of the hierarchy in scales between the weak scale $M_Z \sim 90$GeV and the Planck scale $M_P \sim 10^{18}$GeV?

The Weak Hierarchy Problem

In string theory questions about 4-dimensional physics can be translated into questions about the extra-dimensional geometry.

Recall the relationship between the string scale and the extra-dimensional volume:

$$M_s = \frac{M_P}{\sqrt{\mathcal{V}/l_s^6}}$$

There is a similar scaling for the supersymmetry breaking scale

$$M_{susy} \propto \frac{M_P}{\mathcal{V}/l_s^6}$$

If the extra-dimensional volume is exponentially large in string units, the scale of supersymmetry breaking is exponentially smaller than the Planck scale.

The Weak Hierarchy Problem

The difficulty is ensuring the system is stable against runaway to $\mathcal{V} \to \infty$.

This requires a controlled potential for the field controlling the extra-dimensional volume.

I am one of the discoverers of the LARGE volume models.

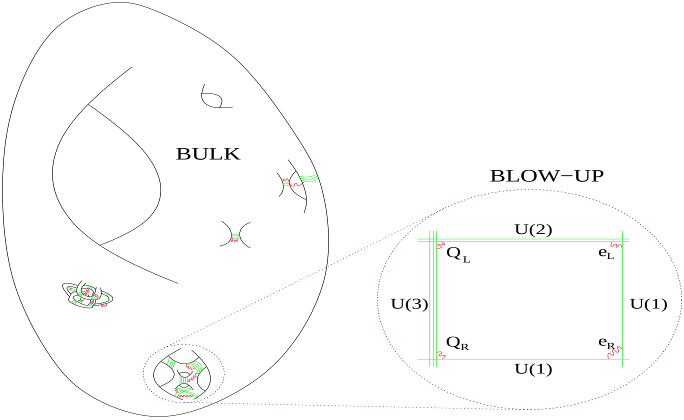
These stabilise the extra-dimensional volume at exponentially large values, with supersymmetry broken at exponentially small scales. For $\mathcal{V} \sim 10^{15} l_s^6$, then

$$M_{susy} \sim \frac{M_P}{\mathcal{V}/l_s^6} \sim 1\text{TeV},$$

$$M_s \sim \frac{M_P}{\sqrt{\mathcal{V}/l_s^6}} \sim 10^{11}\text{GeV} \longrightarrow l_s \sim l_P \left(\sqrt{\mathcal{V}/l_s^6}\right) \sim 10^{-26}\text{m}.$$

The Weak Hierarchy Problem

Large Volume Models



The MSSM Flavour Problem

Supersymmetry is fantastic in many ways.

It has only one major problem:

Why hasn't supersymmetry shown itself already?

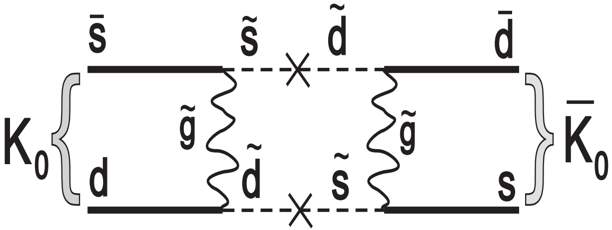
Compare with

- The c quark - predicted before discovery through its contribution to FCNCs (the GIM mechanism)
- The t quark - mass predicted accurately through loop contributions at LEP I.

Physics is sensitive to virtual particles:

$$\Delta E \Delta t \gtrsim \hbar.$$

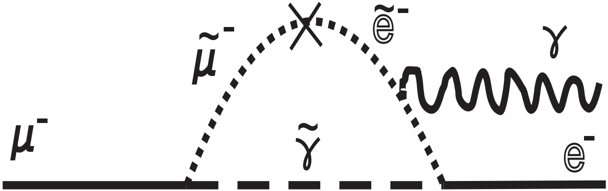
The MSSM Flavour Problem



'Generic' models of supersymmetry give large, additional, unobserved contributions to $K_0 - \bar{K}_0$ mixing.

The $K_L - K_S$ mass difference measured via $K_0 - \bar{K}_0$ oscillation frequency is entirely consistent with the Standard Model.

The MSSM Flavour Problem



'Generic' models of supersymmetry allow the decay mode $\mu \to e\gamma$.

This is not an observed decay mode of the muon.

The MSSM Flavour Problem

The flavour and CP problems highly constrains the supersymmetric spectrum. The simplest resolution is universal MSSM soft terms (fixes approx 100 from 120 of the MSSM parameters)

$$\begin{aligned} m^2_{Q,\alpha\bar{\beta}} &= m^2_Q, \\ A_{\alpha\beta\gamma} &= AY_{\alpha\beta\gamma}. \\ \phi_A &= \phi_{\lambda_1} = \phi_{\lambda_2} = \phi_{\lambda_3}. \end{aligned}$$

Why should this be?

- The flavour problem cannot be solved within the MSSM alone - it needs more fundamental physics.
- The solution to the flavour problem affects all of supersymmetric phenomenology!

The MSSM Flavour Problem

- The flavour problem says that supersymmetry breaking cannot know about flavour physics.
- *A priori*, this is unnatural - why should the susy breaking sector be entirely ignorant of flavour?
- String theory can give a natural answer.

The size and shape moduli fields of the Calabi-Yau factorise:

$$\begin{aligned} \Phi_{all} &= T_{size} \oplus U_{shape}, \\ \mathcal{K}_{total} &= \mathcal{K}_1(T_{size} + \bar{T}_{size}) + \mathcal{K}_2(U_{shape}, \bar{U}_{shape}). \end{aligned}$$

T and U represent two decoupled sectors.

This is a *mathematical* statement about Calabi-Yaus.

The MSSM Flavour Problem

Decoupling means that

$$\mathcal{L} = \underbrace{K(T_i, \bar{T}_j)\partial_\mu T_i \partial^\mu \bar{T}_j}_{\text{T sector}} + \underbrace{K(U_i, \bar{U}_j)\partial_\mu U_i \partial^\mu \bar{U}_j}_{\text{U sector}} + \underbrace{0 \times K(T_i, \bar{U}_j)\partial_\mu T_i \partial^\mu \bar{U}_j}_{\text{cross terms}}$$

There are no cross terms between the T and U sectors.

This factorisation is unnatural from the perspective of low energy effective theory, but natural within the context of the string compactification.

The MSSM Flavour Problem

We have

$$\Phi_{all,hidden} = \Psi_{Kahler} \oplus \chi_{complex}$$

with Ψ and χ representing two distinct and decoupled sectors.

It turns out that in flux compactifications

1. Ψ_{Kahler} gives rise to supersymmetry breaking.
2. $\chi_{complex}$ generates the flavour structure: $Y_{\alpha\beta\gamma} = Y_{\alpha\beta\gamma}(\chi)$.

Supersymmetry breaking and flavour physics decouple and soft terms are flavour universal!

The origin of this decoupling is the geometric properties of Calabi-Yaus.

Conclusions

- String theory offers a consistent short distance completion of general relativity.
- The new degrees of freedom it introduces are extra geometric dimensions and extended string harmonics.
- String theory requires supersymmetry for consistency, which is also one of the leading LHC search targets.
- The leading effects of supersymmetry breaking can occur via Planck-scale physics, in which case the supersymmetric spectrum directly reflects Planck scale physics.
- This provides insights into some of the open phenomenological questions of Beyond-the-Standard-Model physics.