Gauge Threshold Corrections for Local String Models

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Local vs Global

There are many different proposals to realise Standard Model in string theory:

- Weakly coupled heterotic string / heterotic M-theory
- M-theory on G2 manifolds
- Intersecting/magnetised brane worlds in IIA/IIB string theory
- Branes at singularities
- F-theory GUTs

Local vs Global

These approaches are usefully classified as either local or global.

Global models:

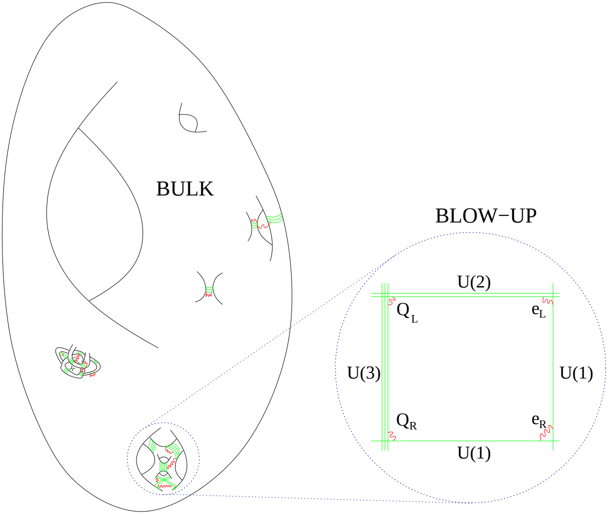
- Canonical example is weakly coupled heterotic string.
- Model specification requires global consistency conditions.
- Relies on geometry of entire compact space
- Limit $\mathcal{V} \to \infty$ also gives $\alpha_{SM} \to 0$: cannot decouple string and Planck scales.
- Other examples: IIA/IIB intersecting brane worlds, M-theory on G2 manifolds

Local vs Global

Local models:

- Canonical example branes at singularity
- Model specification only requires knowledge of local geometry and local tadpole cancellation.
- Full consistency depends on existence of a compact embedding of the local geometry.
- Standard Model gauge and Yukawa couplings remain finite in the limit $\mathcal{V} \to \infty$.

  It is possible to have $M_P \gg M_s$ by taking $\mathcal{V} \to \infty$.
- Examples: branes at singularities, local F-theory GUTs.


BULK
BLOW−UP
U(2)
Q_L
e_L
U(3)
U(1)
Q_R
e_R
U(1)

Local vs Global

Local models have various promising features:

- Easier to construct than fully global models.
- Typically have small numbers of families.
- Combine easily with moduli stabilisation, supersymmetry breaking and hierarchy generation (LARGE volume construction)
- Promising recent constructions of local stringy GUTs.

Local vs Global

One of the most phenomenologically important quantities in local models is the bulk volume.

This determines

- String scale $M_s = \frac{M_P}{\sqrt{\mathcal{V}}}$
- Gravitino mass through the flux superpotential

$$m_{3/2} \sim \frac{\langle \int G_3 \wedge \Omega \rangle}{\mathcal{V}}$$

- The unification scale in models where gauge couplings naturally unify.

The purpose of this talk is to study this question precisely.

Threshold Corrections

- If gauge coupling unification is non-accidental, it is important to understand the significance of $M_{GUT} \sim 3 \times 10^{16}\text{GeV}$.
- In particular, we want to understand the relationship of M_{GUT} to the string scale M_s and the Planck scale $M_P = 2.4 \times 10^{18}\text{GeV}$.
- Is M_{GUT} an actual scale or a mirage scale?
- I will discuss this first using supergravity arguments and subsequently directly in string theory.

Threshold Corrections in Supergravity I

In supergravity, physical and holomorphic gauge couplings are related by Kaplunovsky-Louis formula:

$$
\begin{aligned}
g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = \quad & \mathrm{Re}(f_a(\Phi)) && \text{(Holomorphic coupling)} \\
& + \frac{b_a}{16\pi^2} \ln \left(\frac{M_P^2}{\mu^2} \right) && \text{(β-function running)} \\
& + \frac{T(G)}{8\pi^2} \ln g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) && \text{(NSVZ term)} \\
& + \frac{(\sum_r n_r T_a(r) - T(G))}{16\pi^2} \hat{K}(\Phi, \bar{\Phi}) && \text{(Kähler-Weyl anomaly)} \\
& - \sum_r \frac{T_a(r)}{8\pi^2} \ln \det Z^r(\Phi, \bar{\Phi}, \mu). && \text{(Konishi anomaly)}
\end{aligned}
$$

Relates *measurable* couplings and *holomorphic* couplings.

For local models in IIB

- Kähler potential $\hat{K}$ is given by

$$\hat{K} = -2 \ln \mathcal{V} + \dots$$

- Matter kinetic terms $\hat{Z}$ are given by

$$\hat{Z} = \frac{f(\tau_s)}{\mathcal{V}^{2/3}}$$

Why? When we decouple gravity the physical couplings

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{\hat{Z}_\alpha \hat{Z}_\beta \hat{Z}_\gamma}}$$

should remain finite and be $\mathcal{V}$-independent.

$$\hat{K} = -2 \ln \mathcal{V}, \qquad \hat{Z} = \frac{f(\tau_s)}{\mathcal{V}^{2/3}}$$

- Local models require a LARGE bulk volume ($\mathcal{V} \sim 10^4$ for $M_s \sim M_{GUT}$, $\mathcal{V} \sim 10^{15}$ for $M_s \sim 10^{11}$GeV).
- Kähler and Konishi anomalies are formally one-loop suppressed.
  However if volume is LARGE, both anomalies are enhanced by $\ln \mathcal{V}$ factors.
- This implies the existence of large anomalous contributions to *physical* gauge couplings!

Plug in $\hat{K} = -2 \ln \mathcal{V}$ and $\hat{Z} = \frac{1}{\mathcal{V}^{2/3}}$ into Kaplunovsky-Louis formula.

We restrict to terms enhanced by $\ln \mathcal{V}$ and obtain:

$$\begin{aligned} g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) &= \text{Re}(f_a(\Phi)) + \frac{\left(\sum_r n_r T_a(r) - 3T_a(G)\right)}{8\pi^2} \ln \left(\frac{M_P}{\mathcal{V}^{1/3}\mu} \right) \\ &= \text{Re}(f_a(\Phi)) + \beta_a \ln \left(\frac{(RM_s)^2}{\mu^2} \right). \end{aligned}$$

- Gauge couplings start running from an effective scale RM_s rather than M_s.
- Universal $\text{Re}(f_a(\Phi))$ implies unification occurs at a super-stringy scale RM_s rather than M_s.

- Argument implies inferred low-energy unification scale is systematically above the string scale.
- Argument has only relied on model-independent $\mathcal{V}$ factors - result should hold for any local model (D3 at singularities, IIB GUTs, F-theory GUTs, local M-theory models)
- Unification scale is a mirage scale - new string states already occur at $M_s = M_{GUT}/R \ll M_{GUT}$.

Threshold Corrections in String Theory

- We now want to investigate this directly in string theory.
- In string theory gauge couplings are

$$\frac{1}{g_a^2(\mu)} = \frac{1}{g_{0,a}^2} + \frac{b_a}{16\pi^2} \ln\left(\frac{M_s^2}{\mu^2}\right) + \Delta_a(M, \bar{M})$$

- $\Delta_a(M, \bar{M})$ are the threshold corrections induced by massive string/KK states.
- Study of threshold corrections pioneered by Kaplunovsky and Louis for weakly coupled heterotic string.
- For our calculations we use the background field method.

Running gauge couplings are the 1-loop coefficient of

$$\frac{1}{4g^2}\int d^4x\sqrt{g}F^a_{\mu\nu}F^{a,\mu\nu}$$

- Turn on background magnetic field $F_{23} = B$.
- Compute the quantised string spectrum.
- Use the string partition function to compute the 1-loop vacuum energy

$$\Lambda = \Lambda_0 + \frac{1}{2}\left(\frac{B}{2\pi^2}\right)^2\Lambda_2 + \frac{1}{4!}\left(\frac{B}{2\pi^2}\right)^4\Lambda_4 + \dots$$

- From Λ_2 term we can extract beta function running and threshold corrections.

String theory 1-loop vacuum function given by partition function

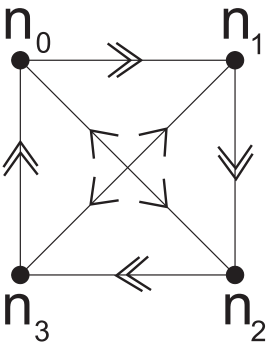
$$\Lambda_{1-loop} = \frac{1}{2}(T + KB + A(B) + MS(B)).$$

- Require $\mathcal{O}(B^2)$ term of this expansion.
- Background magnetic field only shifts moding of open string states.
- Torus and Klein Bottle amplitudes do not couple to open strings.
- Only annulus and Möbius strip amplitudes contribute at $\mathcal{O}(B^2)$.

We want examples of calculable local models with non-zero beta functions.

- The simplest such examples are (fractional) D3 branes at orbifold singularities.
- String can be exactly quantised and all calculations can be performed explicitly.
- Orbifold singularities only involve annulus amplitude further simplifying the computations.
- Have studied D3 branes on $\mathbb{C}^3/\mathbb{Z}_4$, $\mathbb{C}^3/\mathbb{Z}_6$, $\mathbb{C}^3/\mathbb{Z}_6'$, $\mathbb{C}^3/\Delta_{27}$.
- Will focus here on D-branes at $\mathbb{C}^3/\mathbb{Z}_4$ (reuslts all generalise).

- The quiver for $\mathbb{C}^3/\mathbb{Z}_4$ is:



- Anomaly cancellation requires $n_0 = n_2$, $n_1 = n_3$.

- Orbifold action generated by $z_i \to \exp(2\pi i \theta_i)$ with $\theta = (1/4, 1/4, -1/2)$.
- We only need to compute the annulus diagram

$$\mathcal{A}(B) = \int_0^\infty \frac{dt}{2t} \mathrm{STr} \left(\frac{(1 + \theta + \theta^2 + \theta^3)}{4} \frac{1 + (-1)^F}{2} q^{(p^\mu p_\mu + m^2)/2} \right)$$

Here

$$q = e^{-\pi t}, \qquad \mathrm{STr} = \sum_{bosons} - \sum_{fermions} \equiv \sum_{NS} - \sum_{R}, \qquad \alpha' = 1/2$$

.

- β-function running and threshold corrections are encoded in the $\mathcal{O}(B^2)$ term.

We separately evaluate each amplitude in the θ^N sector.

$$\mathcal{A}(B) = \int_0^\infty \frac{dt}{2t} \,\mathrm{STr}\left(\frac{(1 + \theta + \theta^2 + \theta^3)}{4} \frac{1 + (-1)^F}{2} \, q^{(p^\mu p_\mu + m^2)/2} \right)$$

- $\theta^0 = (1, 1, 1)$ is an '$\mathcal{N} = 4$' sector.
- $\theta^1 = (1/4, 1/4, -1/2)$ and $\theta^3 = (-1/4, -1/4, 1/2)$ are '$\mathcal{N} = 1$' sectors.
- $\theta^2 = (1/2, 1/2, 0)$ is an '$\mathcal{N} = 2$' sector.

The amplitudes reduce to products of Jacobi ϑ-functions with different prefactors.

$$\mathcal{A}_{untwisted} = \int \frac{dt}{2t}\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2 \times 0 = 0. \quad (\mathcal{N} = 4 \text{ susy})$$

- The untwisted sector has effective $\mathcal{N} = 4$ supersymmetry and cannot contribute to the running gauge coupling.

$$\mathcal{A}_{\theta} = \mathcal{A}_{\theta^3} = \int \frac{dt}{2t}\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2 \times \frac{(n_0 - n_2)}{2}\Big(\vartheta - \text{functions}\Big)$$

- The contribution of $\mathcal{N} = 1$ sectors to gauge coupling running has a prefactor $(n_0 - n_2)$.
- This necessarily vanishes once non-abelian anomaly cancellation is imposed.

$$\mathcal{A}_{\theta^2} = \int \frac{dt}{2t}\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2 \times (-3n_0 + n_1 + n_2 + n_3)\Big(\vartheta - \text{function}\Big).$$

Here $\Big(\vartheta - \text{function}\Big)$ is

$$\frac{-1}{4\pi^2}\sum \eta_{\alpha\beta}(-1)^{2\alpha}\frac{\vartheta''\begin{bmatrix}\alpha\\ \beta\end{bmatrix}}{\eta^3}\frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}}{\eta^3}\frac{\vartheta\begin{bmatrix}\alpha\\ \beta+\theta_1\end{bmatrix}}{\vartheta\begin{bmatrix}1/2\\ 1/2+\theta_1\end{bmatrix}}\frac{\vartheta\begin{bmatrix}\alpha\\ \beta+\theta_2\end{bmatrix}}{\vartheta\begin{bmatrix}1/2\\ 1/2+\theta_2\end{bmatrix}} = 1.$$

We obtain

$$\mathcal{A}_{\theta^2} = \int \frac{dt}{2t}\frac{1}{2}\left(\frac{B}{2\pi^2}\right)^2 \times (-3n_0 + n_1 + n_2 + n_3).$$

$$\mathcal{A}_{\theta^2} = \int \frac{dt}{2t}\frac{1}{2}\left(\frac{B}{2\pi^2}\right)^2 \times \underbrace{(-3n_0 + n_1 + n_2 + n_3)}_{b_0}.$$

- Reduction of ϑ-functions to a constant is a consequence of $\mathcal{N} = 2$ supersymmetry.
- Only BPS multiplets can affect gauge coupling running and excited string states are non-BPS.
- Resultant amplitude is non-zero and gives field theory β-function running in both IR and UV limits.

Summary:

- Untwisted sector has $\mathcal{N} = 4$ susy and gives no contribution to running of gauge couplings.
- θ and θ^3 twisted sectors have $\mathcal{N} = 1$ susy. Contributions vanish when anomaly cancellation is imposed.
- $\mathcal{N} = 2$ θ^2 sectors gives non-vanishing contribution

$$\left[\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2\right] \times \int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} b_a$$

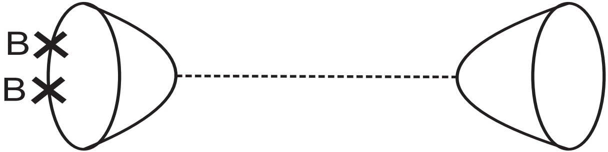
- How should we interpret this?

$$\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left(\frac{B}{2\pi^2} \right)^2 b_a$$

- Divergence in the $t \to \infty$ limit is physical: this is the IR limit and we recover ordinary β-function running.
- Divergence in $t \to 0$ limit is unphysical: this is the open string UV limit and this amplitude must be finite in a consistent string theory.
- Physical understanding of the divergence is best understood from closed string channel.

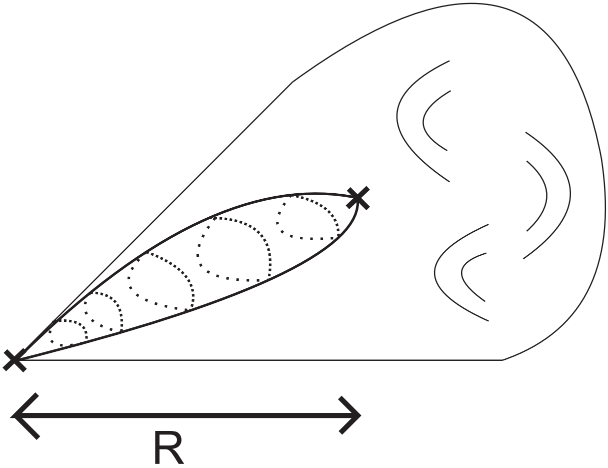
Annulus amplitude:

Annulus amplitude in $t \to 0$ limit:



- $t \longrightarrow 0$ divergence corresponds to a source for a partially twisted RR 2-form.
- In the local model this propagates into the bulk of the Calabi-Yau.
- Logarithmic divergence is divergence for a 2-dimensional source.
- In a compact model this becomes a physical divergence and tadpole must be cancelled by bulk sink branes/O-planes.
- Tadpole is sourced locally but must be cancelled globally

The purely local computation omits the following worldsheets:



- The purely local string computation includes all open string states for $t > 1/(RM_s)^2$, i.e. $M < RM_s$.
- However for $t < 1/(RM_s)^2$ we must include new winding states in the partition function.
- These are essential for global consistency but are omitted by a purely local computation.
- These enter the computation for $t < 1/(RM_s)^2$ and enforce finiteness (RR tapdole cancellation).

- The bulk worldsheets enforce global RR tadpole cancellation and effectively cut off the integral at $t = \frac{1}{(RM_s)^2}$.
- Threshold corrections become finite

$$\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left(\frac{B}{2\pi^2} \right)^2 b_a \rightarrow \int_{1/(RM_s)^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left(\frac{B}{2\pi^2} \right)^2 b_a$$

- Effective UV cutoff is actually RM_s and *not* M_s.!

Result Summary

- For all cases studied string computation reproduces result of supergravity analysis.
- Effective unification scale is $RM_s \gg M_s$.
- In string theory, presence of radius arises from an RR tadpole sourced by the local model but which is cancelled by the bulk.
- In open string channel, model does not 'know' its self-consistency until an energy scale RM_s.

Result Summary

- Main result: for local models, both supergravity and string theory imply gauge couplings start running from RM_s and not M_s.

- This should hold for all local models: D3 branes at singularities, F-theory GUTs, IIB GUTs...

  Note the hypercharge flux in F-theory/IIB GUTs has necessary properties for relevant physics to apply.

- Large effect: for $M_s \sim 10^{12}$GeV changes Λ_{UV} by a factor of 100 and for $M_s \sim 10^{15}$GeV changes Λ_{UV} by a factor of 10.

What should the string scale be?

- $M_s = 10^{11} - 10^{12}$GeV is good for moduli stabilisation, the hierarchy problem, TeV supersymmetry and axions. Threshold corrections shift the unification scale to $10^{13} \rightarrow 10^{14}$GeV.
- If we want unification, then threshold corrections shift the required string scale from 10^{16}GeV to 10^{15}GeV.