# Holomorphy Constraints on Local GUTs

Joseph Conlon (Oxford University)

Strings and GUTs meeting, Munich, February 10, 2010
Based on work with Daniel Cremades and Eran Palti

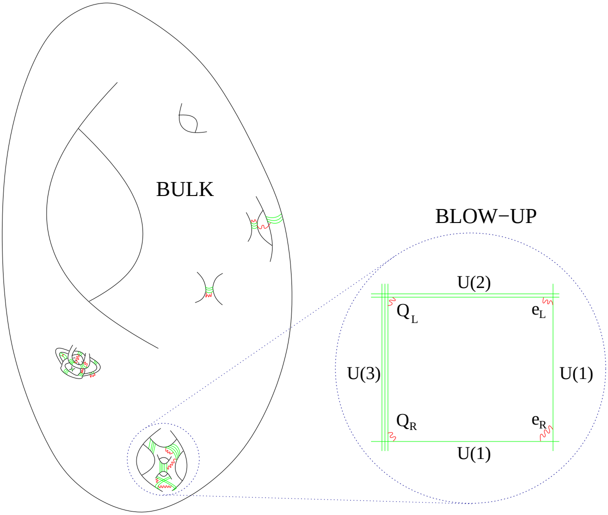
# Locality

There are many different proposals to realise Standard Model in string theory.

What information comes from locality?

- Examples branes at singularity, F-theory/IIB GUTs
- Definition: Standard Model gauge and Yukawa couplings remain finite in the limit $\mathcal{V} \to \infty$.
- Model specified by local and not global geometry
- Hierarchy between string ($\frac{M_P}{\sqrt{\mathcal{V}}}$) and Planck ($M_P$) scales.

# Locality



# Holomorphy

This talk will make repeated use of one simple point:

- Kähler moduli $T_i$ have a perturbative shift symmetry $T_i \rightarrow T_i + i\epsilon$ and by holomorphy cannot appear in the perturbative superpotential.

This plus locality plus GUT gives a surprising amount of information.

Will focus on IIB/F-theory cases.

# Matter Kinetic Terms

Physical Yukawa couplings are given by

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{Z_\alpha Z_\beta Z_\gamma}}$$

- Overall Kähler potential is

$$\hat{K} = -2 \ln \mathcal{V}(T_i + \bar{T}_i) + \ldots$$

- Holomorphic Yukawa couplings do not depend on $T_i$.
- Physical Yukawa couplings (by locality) do not depend on the overall volume.
- Volume dependence of matter metrics is given by $Z_\alpha \sim \mathcal{V}^{-2/3}$.

# Non-Renormalisable Operators

Supergravity action is

$$Z_a \left( \partial_\mu \Phi_a \partial^\mu \Phi_a + \bar{\psi}_a \sigma^\mu \partial_\mu \psi_a \right) + e^{\hat{K}/2} \partial_\alpha \partial_\beta W \psi^\alpha \psi^\beta$$

Consider a nonrenormalisable operator (e.g. $HHLL, QQQL$) in the superpotential:

$$W = \frac{1}{M_P} C_{abcd} \Phi_a \Phi_b \Phi_c \Phi_d.$$

$C_{abcd}$ cannot depend on $\mathcal{V}$. Physical operator is

$$\begin{aligned}\hat{C}_{abcd} \psi_a \psi_b \Phi_c \Phi_d &= \frac{e^{\hat{K}/2}}{M_P} \frac{C_{abcd} \psi_a \psi_b \Phi_c \Phi_d}{\sqrt{Z_a Z_b Z_c Z_d}} \\ &\sim \left( \frac{1}{R M_s} \right) C_{abcd}.\end{aligned}$$

# Non-Renormalisable Operators

Suppresion scale is neither $M_s$ nor $M_P$ but in fact $RM_s$.

It's easy to see that this follows more generally: a dimension-(3+n) superpotential operator has a physical suppression scale of $\left(\frac{1}{RM_s}\right)^n$.

Consequence for local GUTs:

Higher dimension terms in the superpotential are suppressed by a scale parametrically above the string scale.

Why is this and what is the origin of this scale?

# Super-renormalisable Operators

Likewise consider a $\mu$-term:

$$W = M_P H_u H_d$$

Locality and $Z \sim \mathcal{V}^{-2/3}$ now gives a mass term

$$m_H \sim (RM_s)$$

This is parametrically above the string scale in the limit $\mathcal{V} \to \infty$:

hard to see how such a term can be present.

# Threshold Corrections

In supergravity, physical and holomorphic gauge couplings are related by Kaplunovsky-Louis formula:

$$
\begin{aligned}
g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = \quad & \mathrm{Re}(f_a(\Phi)) & \text{(Holomorphic coupling)} \\
& + \frac{b_a}{16\pi^2} \ln\left(\frac{M_P^2}{\mu^2}\right) & \text{($\beta$-function running)} \\
& + \frac{T(G)}{8\pi^2} \ln g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) & \text{(NSVZ term)} \\
& + \frac{(\sum_r n_r T_a(r) - T(G))}{16\pi^2} \hat{K}(\Phi, \bar{\Phi}) & \text{(Kähler-Weyl anomaly)} \\
& - \sum_r \frac{T_a(r)}{8\pi^2} \ln \det Z^r(\Phi, \bar{\Phi}, \mu). & \text{(Konishi anomaly)}
\end{aligned}
$$

Relates *measurable* couplings and *holomorphic* couplings.

Focus on volume dependence in local models:

- GUTs - assume universal holomorphic gauge kinetic function.
- Kähler potential $\hat{K}$ is given by

$$\hat{K} = -2 \ln \mathcal{V} + \dots$$

- Matter kinetic terms $\hat{Z}$ are given by

$$\hat{Z} \sim \frac{1}{\mathcal{V}^{2/3}}$$

These are the *only* non-universal ways the volume can enter the Kaplunovsky-Louis formula.

Plug in $\hat{K} = -2\ln\mathcal{V}$ and $\hat{Z} = \frac{1}{\mathcal{V}^{2/3}}$ into Kaplunovsky-Louis formula.

We restrict to terms enhanced by $\ln\mathcal{V}$ and obtain:

$$\begin{aligned} g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) &= \mathrm{Re}(f_a(\Phi)) + \frac{(\sum_r n_r T_a(r) - 3T_a(G))}{8\pi^2} \ln\left(\frac{M_P}{\mathcal{V}^{1/3}\mu}\right) \\ &= \mathrm{Re}(f_a(\Phi)) + \beta_a \ln\left(\frac{(RM_s)^2}{\mu^2}\right). \end{aligned}$$

- Gauge couplings start running from an effective scale $RM_s$ rather than $M_s$.
- Universal $\mathrm{Re}(f_a(\Phi))$ implies unification occurs at a super-stringy scale $RM_s$ rather than $M_s$.

- Argument implies inferred low-energy unification scale is systematically above the string scale and has only relied on model-independent $\mathcal{V}$ factors.
- Unification scale is a mirage scale - new string states already occur at $M_s = M_{GUT}/R < M_{GUT}$.
- What is the string interpretation?

Running gauge couplings are the 1-loop coefficient of

$$\frac{1}{4g^2}\int d^4x\sqrt{g}F^a_{\mu\nu}F^{a,\mu\nu}$$

- Turn on background magnetic field $F_{23} = B$.
- Compute the quantised string spectrum.
- Use the string partition function to compute the 1-loop vacuum energy

$$\Lambda = \Lambda_0 + \frac{1}{2}\left(\frac{B}{2\pi^2}\right)^2 \Lambda_2 + \frac{1}{4!}\left(\frac{B}{2\pi^2}\right)^4 \Lambda_4 + \ldots$$

- From $\Lambda_2$ term we can extract beta function running and threshold corrections.

String theory 1-loop vacuum function given by partition function

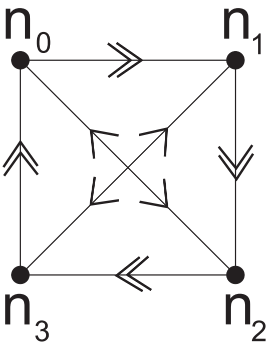
$$\Lambda_{1-loop} = \frac{1}{2}(T + KB + A(B) + MS(B)).$$

- Require $\mathcal{O}(B^2)$ term of this expansion.
- Background magnetic field only shifts moding of open string states.
- Torus and Klein Bottle amplitudes do not couple to open strings.
- Only annulus and Möbius strip amplitudes contribute at $\mathcal{O}(B^2)$.

We want examples of calculable local models with non-zero beta functions.

- The simplest such examples are (fractional) D3 branes at orbifold singularities.
- String can be exactly quantised and all calculations can be performed explicitly.
- Orbifold singularities only involve annulus amplitude further simplifying the computations.
- Have studied D3 branes on $\mathbb{C}^3/\mathbb{Z}_4$, $\mathbb{C}^3/\mathbb{Z}_6$, $\mathbb{C}^3/\mathbb{Z}_6'$, $\mathbb{C}^3/\Delta_{27}$.
- Will focus here on D-branes at $\mathbb{C}^3/\mathbb{Z}_4$ (reuslts all generalise).

- The quiver for $\mathbb{C}^3/\mathbb{Z}_4$ is:




- Anomaly cancellation requires $n_0 = n_2$, $n_1 = n_3$.

- Orbifold action generated by $z_i \to \exp(2\pi i \theta_i)$ with $\theta = (1/4, 1/4, -1/2)$.
- We only need to compute the annulus diagram

$$\mathcal{A}(B) = \int_0^\infty \frac{dt}{2t} \,\mathrm{STr} \left( \frac{(1+\theta+\theta^2+\theta^3)}{4} \frac{1+(-1)^F}{2} q^{(p^\mu p_\mu + m^2)/2} \right)$$

Here

$$q = e^{-\pi t}, \qquad \mathrm{STr} = \sum_{bosons} - \sum_{fermions} \equiv \sum_{NS} - \sum_{R}, \qquad \alpha' = 1/2$$

.

- $\beta$-function running and threshold corrections are encoded in the $\mathcal{O}(B^2)$ term.

We separately evaluate each amplitude in the $\theta^N$ sector.

$$\mathcal{A}(B) = \int_0^\infty \frac{dt}{2t} \mathrm{STr} \left( \frac{(1 + \theta + \theta^2 + \theta^3)}{4} \frac{1 + (-1)^F}{2} q^{(p^\mu p_\mu + m^2)/2} \right)$$

- $\theta^0 = (1, 1, 1)$ is an '$\mathcal{N} = 4$' sector.
- $\theta^1 = (1/4, 1/4, -1/2)$ and $\theta^3 = (-1/4, -1/4, 1/2)$ are '$\mathcal{N} = 1$' sectors.
- $\theta^2 = (1/2, 1/2, 0)$ is an '$\mathcal{N} = 2$' sector.

The amplitudes reduce to products of Jacobi $\vartheta$-functions with different prefactors.

$$\mathcal{A}_{untwisted} = \int \frac{dt}{2t}\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2 \times 0 = 0. \quad (\mathcal{N} = 4 \text{ susy})$$

- The untwisted sector has effective $\mathcal{N} = 4$ supersymmetry and cannot contribute to the running gauge coupling.

$$\mathcal{A}_{\theta} = \mathcal{A}_{\theta^3} = \int \frac{dt}{2t}\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2 \times \frac{(n_0 - n_2)}{2}\Big(\vartheta - \text{functions}\Big)$$

- The contribution of $\mathcal{N} = 1$ sectors to gauge coupling running has a prefactor $(n_0 - n_2)$.
- This necessarily vanishes once non-abelian anomaly cancellation is imposed.

$$\mathcal{A}_{\theta^2} = \int \frac{dt}{2t}\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2 \times (-3n_0 + n_1 + n_2 + n_3)\Big(\vartheta - \text{function}\Big).$$

Here $\Big(\vartheta - \text{function}\Big)$ is

$$\frac{-1}{4\pi^2}\sum \eta_{\alpha\beta}(-1)^{2\alpha}\frac{\vartheta''\begin{bmatrix}\alpha\\ \beta\end{bmatrix}}{\eta^3}\frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}}{\eta^3}\frac{\vartheta\begin{bmatrix}\alpha\\ \beta+\theta_1\end{bmatrix}}{\vartheta\begin{bmatrix}1/2\\ 1/2+\theta_1\end{bmatrix}}\frac{\vartheta\begin{bmatrix}\alpha\\ \beta+\theta_2\end{bmatrix}}{\vartheta\begin{bmatrix}1/2\\ 1/2+\theta_2\end{bmatrix}} = 1.$$

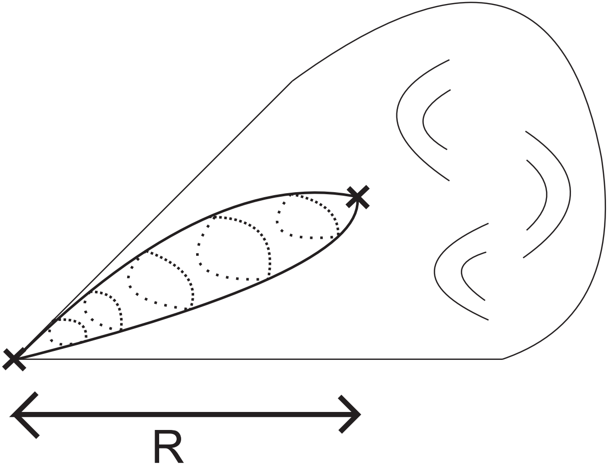
We obtain

$$\mathcal{A}_{\theta^2} = \int \frac{dt}{2t}\frac{1}{2}\left(\frac{B}{2\pi^2}\right)^2 \times (-3n_0 + n_1 + n_2 + n_3).$$

$$\mathcal{A} = \mathcal{A}_{\theta^2} \quad = \quad \int_0^\infty \frac{dt}{2t} \frac{1}{2} \left( \frac{B}{2\pi^2} \right)^2 \times \underbrace{(-3n_0 + n_1 + n_2 + n_3)}_{b_0}.$$

- Reduction of $\vartheta$-functions to a constant is a consequence of $\mathcal{N} = 2$ supersymmetry.
- Only BPS multiplets can affect gauge coupling running and excited string states are non-BPS.
- Resultant amplitude is non-zero and gives field theory $\beta$-function running in both IR and UV limits.

The purely local computation omits the following worldsheets:



- The purely local string computation includes all open string states for $t > 1/(RM_s)^2$, i.e. $M < RM_s$.
- However for $t < 1/(RM_s)^2$ we must include new winding states in the partition function.
- These are essential for global consistency but are omitted by a purely local computation.
- These enter the computation for $t < 1/(RM_s)^2$ and enforce finiteness (RR tapdole cancellation).

- The bulk worldsheets enforce global RR tadpole cancellation and effectively cut off the integral at $t = \frac{1}{(RM_s)^2}$.
- Threshold corrections become finite

$$\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a \rightarrow \int_{1/(RM_s)^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a$$

- Effective UV cutoff is actually $RM_s$ and *not* $M_s$.!

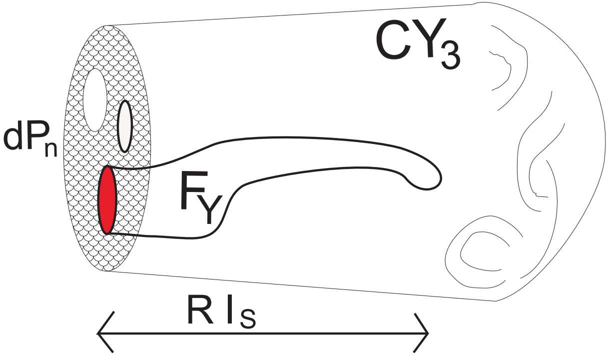
Physics of high scale $RM_s$:

Local model generates a tadpole which is uncancelled locally and cancelled globally.

Divergences cannot be regulated until finiteness is ensured at bulk scale $RM_S$.

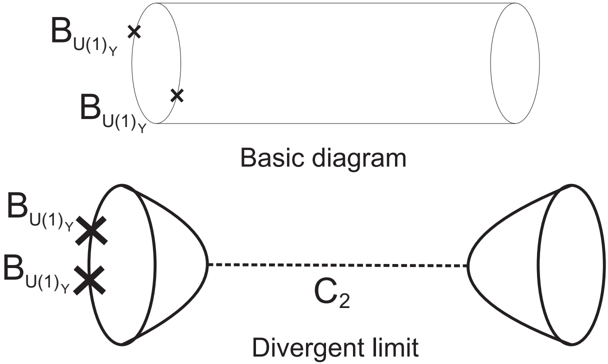
# F-theory GUTs

Same physics should apply:



# F-theory GUTs

$U(1)_Y$ induces a local tadpole that vanishes globally




- induces running from an enhanced scale $RM_s$.

# Flavour

In string theory gauge couplings are real parts of chiral superfields:

$$\alpha_{GUT} = \frac{4\pi}{g^2} = \mathrm{Re}(T)$$

where $T$ has shift symmetry $T \to T + i\epsilon$.

Can $\alpha_{GUT} \sim 1/25$ be used as an expansion parameter for the Yukawa couplings?

$Y_{\alpha\beta\gamma}(\alpha_{GUT})$ is not compatible with holomorphy and the shift symmetry of $T$.

# Flavour

Physical Yukawas are given by

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{Z_\alpha Z_\beta Z_\gamma}}$$

What about using different behaviour in $Z_\alpha$?

An expansion $1 : \alpha_{GUT} : \alpha^2_{GUT}$ in the physical Yukawa couplings requires kinetic terms $Z_\alpha$ to behave as

$$Z_1 : Z_2 : Z_3 \text{ as } \frac{(T_{SM} + \bar{T}_{SM})^4}{\mathcal{V}^{2/3}} : \frac{(T_{SM} + \bar{T}_{SM})^2}{\mathcal{V}^{2/3}} : \frac{1}{\mathcal{V}^{2/3}}$$

Requires kinetic terms that diverge in large-volume weak-coupling limit $g_{\mu\nu} \to \lambda g_{\mu\nu}$: seems implausible.

# Conclusions

Locality and holomorphy give surprising amounts of information about effective action.

- Non-local suppression scale for higher dimension operators.
- Significant contributions to threshold corrections.
- Restrictions on scenarios for expansion of Yukawa and CKM matrices.

One final message...

Come to Cambridge!

Mathematics and Applications of Branes in String and M-Theory

Isaac Newton Institute program January - June 2012

Organisers: David Berman, Joseph Conlon, Neil Lambert, Sunil Mukhi, Fernando Quevedo