LARGE Volume and Gauge Unification

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LARGE Volume Scenario

- Moduli stabilisation is essential for string phenomenology.
- Plays an important role in supersymmetry breaking, cosmology, flavour physics, axions physics....
- Start by describing one promising approach - LARGE Volume Scenario (JC, Quevedo 2005-)

LARGE Volume Scenario

$$\hat{K} = -2\ln\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right), \qquad \left(\xi = \frac{\chi(\mathcal{M})\zeta(3)}{2(2\pi)^3 g_s^{3/2}}\right)$$

$$W = W_0 + \sum_i A_i e^{-a_i T_i}.$$

- IIB flux compactifications including both perturbative and non-perturbative corrections to the scalar potential.
- Include α' corrections to the Kähler potential and instanton corrections to the superpotential.
- This leads to dramatic changes in the large-volume vacuum structure.

LARGE Volume Scenario

The simplest model $\mathbb{P}^4_{[1,1,1,6,9]}$ has two Kähler moduli.

$$\mathcal{V} = \left(\frac{T_b + \bar{T}_b}{2}\right)^{3/2} - \left(\frac{T_s + \bar{T}_s}{2}\right)^{3/2} \equiv \left(\tau_b^{3/2} - \tau_s^{3/2}\right).$$

If we compute the scalar potential, we get for $\mathcal{V} \gg 1$,

$$V = \frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s |A_s W_0| \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\xi |W_0|^2}{g_s^{3/2}\mathcal{V}^3}.$$

LARGE Volume Scenario

$$V = \underbrace{\frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2 a_s \tau_s}}{\mathcal{V}} - \frac{a_s |A_s W_0| \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2}}_{\text{Integrate out } \tau_s} + \frac{\xi |W_0|^2}{g_s^{3/2} \mathcal{V}^3}.$$

$$V = -\frac{|W_0|^2 (\ln \mathcal{V})^{3/2}}{\mathcal{V}^3} + \frac{\xi |W_0|^2}{g_s^{3/2} \mathcal{V}^3}.$$

A minimum exists at

$$\mathcal{V} \sim |W_0| e^{a_s \tau_s}, \qquad \tau_s \sim \frac{\xi^{2/3}}{g_s}.$$

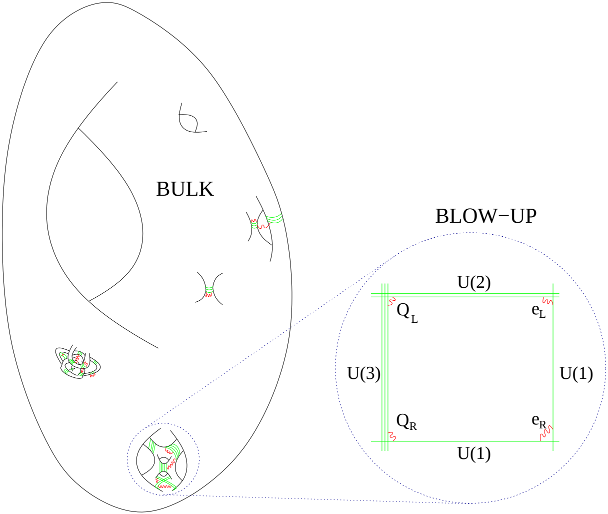
This minimum is non-supersymmetric AdS and at exponentially large volume.

LARGE Volume Scenario

- Calabi-Yau has 'Swiss Cheese' structure (LARGE bulk, small blow-up 4-cycles)
- Standard Model must be realised on D7 branes wrapping shrinkable 4-cycle.
- Gravitino mass and string scale are

$$m_{3/2} = \frac{M_P}{\mathcal{V}}, \qquad M_s = \frac{M_P}{\sqrt{\mathcal{V}}}.$$

- TeV gravitino mass requires $\mathcal{V} \sim 10^{15} l_s^6$, $M_s \sim 10^{11}\text{GeV}$.


BULK
BLOW−UP
U(2)
Q_L
e_L
U(3)
U(1)
Q_R
e_R
U(1)

LARGE Volume Scenario

LARGE volume models automatically requires local realisation of Standard Model.

- Gravity is decoupled at leading order.
- Variety of local models: D3 branes at singularities, F-theory/IIB GUTs, M-theory local models...
- $M_s = 10^{11}$GeV is good for the hierarchy problem, but seems to make gauge unification impossible.
- I will now discuss threshold corrections for local model building.

Threshold Corrections in Supergravity

In supergravity, physical and holomorphic gauge couplings are related by Kaplunovsky-Louis formula:

$$
\begin{aligned}
g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = \quad & \mathrm{Re}(f_a(\Phi)) && \text{(Holomorphic coupling)} \\
& + \frac{b_a}{16\pi^2} \ln\left(\frac{M_P^2}{\mu^2}\right) && \text{(β-function running)} \\
& + \frac{T(G)}{8\pi^2} \ln g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) && \text{(NSVZ term)} \\
& + \frac{(\sum_r n_r T_a(r) - T(G))}{16\pi^2} \hat{K}(\Phi, \bar{\Phi}) && \text{(Kähler-Weyl anomaly)} \\
& - \sum_r \frac{T_a(r)}{8\pi^2} \ln \det Z^r(\Phi, \bar{\Phi}, \mu). && \text{(Konishi anomaly)}
\end{aligned}
$$

Relates *measurable* couplings and *calculable* couplings.

For local models in IIB

- Kähler potential $\hat{K}$ is given by

$$\hat{K} = -2 \ln \mathcal{V} + \ldots$$

- Matter kinetic terms $\hat{Z}$ are given by

$$\hat{Z} = \frac{f(\tau_s)}{\mathcal{V}^{2/3}}$$

Why? When we decouple gravity the bulk physical couplings

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{\hat{Z}_\alpha \hat{Z}_\beta \hat{Z}_\gamma}}$$

should be independent of $\mathcal{V}$.

$$\hat{K} = -2 \ln \mathcal{V}, \qquad \hat{Z} = \frac{f(\tau_s)}{\mathcal{V}^{2/3}}$$

- Local models require a LARGE bulk volume ($\mathcal{V} \sim 10^4$ for $M_s \sim M_{GUT}$, $\mathcal{V} \sim 10^{15}$ for $M_s \sim 10^{11}\mathrm{GeV}$).
- If volume is LARGE, both Kähler and Konishi anomalies are enhanced by $\ln \mathcal{V}$ factors.
- Big, significant anomalous contributions to *physical* gauge couplings!

Plug in $\hat{K} = -2 \ln \mathcal{V}$ and $\hat{Z} = \frac{1}{\mathcal{V}^{2/3}}$ into Kaplunovsky-Louis formula.

Focus on terms enhanced by $\ln \mathcal{V}$ to obtain:

$$
\begin{aligned}
g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) &= \mathrm{Re}(f_a(\Phi)) + \frac{\left(\sum_r n_r T_a(r) - 3T_a(G)\right)}{8\pi^2} \ln\left(\frac{M_P}{\mathcal{V}^{1/3}\mu}\right) \\
&= \mathrm{Re}(f_a(\Phi)) + \beta_a \ln\left(\frac{(RM_s)^2}{\mu^2}\right).
\end{aligned}
$$

- Gauge couplings start running from an effective scale RM_s rather than M_s.
- Universal $\mathrm{Re}(f_a(\Phi))$ implies unification occurs at a super-stringy scale RM_s rather than M_s.

- Argument implies inferred low-energy unification scale is systematically above the string scale.
- Argument has only relied on model-independent $\mathcal{V}$ factors - result should hold for any local model (D3 at singularities, IIB GUTs, F-theory GUTs, local M-theory models)
- Unification scale is a mirage scale - new string states already occur at $M_s = M_{GUT}/R \ll M_{GUT}$.
- We want to investigate this question directly in string theory.

Threshold Corrections in String Theory

- In string theory gauge couplings are

$$\frac{1}{g_a^2(\mu)} = \frac{1}{g_{0,a}^2} + \frac{b_a}{16\pi^2} \ln\left(\frac{M_s^2}{\mu^2}\right) + \Delta_a(M, \bar{M})$$

- $\Delta_a(M, \bar{M})$ are the threshold corrections induced by massive string/KK states.
- Study of threshold corrections pioneered by Kaplunovsky and Louis for weakly coupled heterotic string.
- We shall use the background field method.

Running gauge couplings are the 1-loop coefficient of

$$\frac{1}{4g^2}\int d^4x\sqrt{g}F^a_{\mu\nu}F^{a,\mu\nu}$$

- Turn on background magnetic field $F_{23} = B$.
- Compute the quantised string spectrum.
- Use the string partition function to compute the 1-loop vacuum energy

$$\Lambda = \Lambda_0 + \frac{1}{2}\left(\frac{B}{2\pi^2}\right)^2\Lambda_2 + \frac{1}{4!}\left(\frac{B}{2\pi^2}\right)^4\Lambda_4 + \dots$$

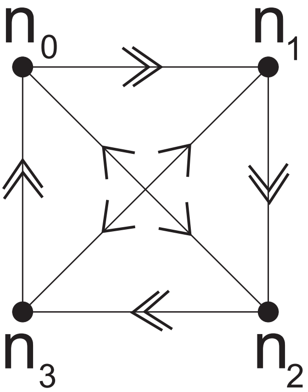
- From Λ_2 term we extract threshold corrections and beta function running.

String theory 1-loop vacuum function given by partition function

$$\Lambda_{1-loop} = \frac{1}{2}(T + KB + A(B) + MS(B)).$$

- Torus and Klein Bottle amplitudes do not couple to open strings.
- Only annulus and Möbius strip amplitudes contribute at $\mathcal{O}(B^2)$.

- The simplest local models are D3 branes at singularities.
- Focus on D-branes at $\mathbb{C}^3/\mathbb{Z}_4$ (can generalise to other singularities). Quiver is:



- No orientifold action implies Möbius string does not contribute.
- We only need to compute the annulus diagram

$$\mathcal{A}(B) = \int_0^\infty \frac{dt}{2t} \, \mathrm{STr} \left(\frac{(1+\theta+\theta^2+\theta^3)}{4} \frac{1+(-1)^F}{2} \, q^{(p^\mu p_\mu + m^2)/2} \right)$$

Here

$$q = e^{-\pi t}, \qquad \mathrm{STr} = \sum_{bosons} - \sum_{fermions} \equiv \sum_{NS} - \sum_{R}, \qquad \alpha' = 1/2$$

.

- Extract $\mathcal{O}(B^2)$ term to obtain the threshold corrections

The various annulus amplitudes reduce to

$$
\begin{aligned}
\mathcal{A}_{untwisted} &= \int \frac{dt}{2t}\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2 \times 0 = 0. \quad (\mathcal{N} = 4 \text{ susy}) \\
\mathcal{A}_{\theta} = \mathcal{A}_{\theta^3} &= \int \frac{dt}{2t}\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2 \times \frac{(n_0 - n_2)}{2}\Big(\vartheta - \text{function}\Big) \\
&= \int \frac{dt}{2t}\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2 \times 0 = 0 \quad \text{(due to anomaly cancellation)} \\
\mathcal{A}_{\theta^2} &= \int \frac{dt}{2t}\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2 \times (-3n_0 + n_1 + n_2 + n_3)\Big(\vartheta - \text{function}\Big) \\
&= \int \frac{dt}{2t}\frac{1}{2}\left(\frac{B}{2\pi^2}\right)^2 \times (-3n_0 + n_1 + n_2 + n_3).
\end{aligned}
$$

Summary:

- Untwisted sector has $\mathcal{N} = 4$ susy and gives no contribution to running of gauge couplings.
- θ and θ^3 twisted sectors have $\mathcal{N} = 1$ susy. Contributions vanish when anomaly cancellation is imposed.
- $\mathcal{N} = 2$ θ^2 sectors gives non-vanishing contribution

$$\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left(\frac{B}{2\pi^2} \right)^2 b_a$$

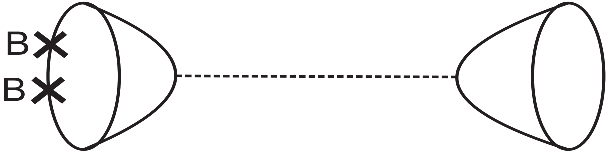
- How should we interpret this?

$$\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t}\frac{1}{4}\left(\frac{B}{2\pi^2}\right)^2 b_a$$

- Divergence in the $t \to \infty$ limit is physical: this is the IR limit and we recover ordinary β-function running.
- Divergence in $t \to 0$ limit is unphysical: this is the open string UV limit and this amplitude must be finite in a consistent string theory.
- Physical understanding of the divergence is best understood from closed string channel.

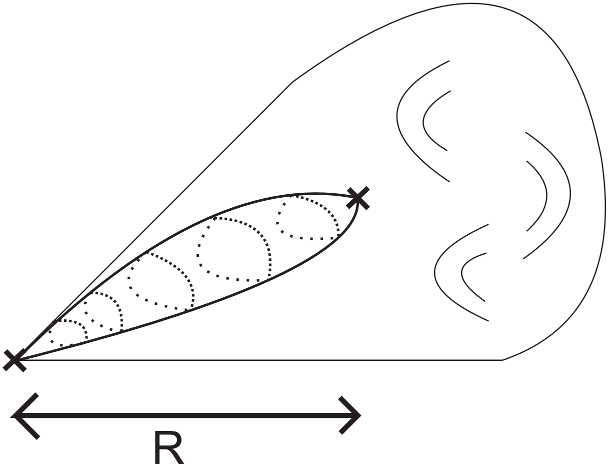
Annulus amplitude:

Annulus amplitude in $t \rightarrow 0$ limit:



- $t \longrightarrow 0$ divergence corresponds to a source for a partially twisted RR 2-form.
- In the local model this propagates into the bulk of the Calabi-Yau.
- Logarithmic divergence is divergence for a 2-dimensional source.
- In a compact model this becomes a physical divergence and tadpole must be cancelled by bulk sink branes/O-planes.

The purely local computation omits the following worldsheets:



- The purely local string computation includes all states for $t > 1/(RM_s^2)$.
- For $t < 1/(RM_s)^2$ we must include new winding states in the partition function.
- These are essential for global consistency but are omitted by a purely local computation.
- These modify the computation for $t < 1/(RM_s)^2$.

- The bulk worldsheets enforce global RR tadpole cancellation and effectively cut off the integral at $t = \frac{1}{(RM_s)^2}$.
- Threshold corrections become finite

$$\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left(\frac{B}{2\pi^2} \right)^2 b_a \to \int_{1/(RM_s)^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left(\frac{B}{2\pi^2} \right)^2 b_a$$

- Effective UV cutoff is actually RM_s and *not* M_s.!

Result Summary

- String computation reproduces result of supergravity analysis.
- Effective unification scale is $RM_s \gg M_s$.
- In string theory, presence of radius arises from an RR tadpole sourced by the local model but which is cancelled by the bulk.
- In open string channel, model does not 'know' its self-consistency until an energy scale RM_s.

Result Summary

- Main result: for local models, both supergravity and string theory imply gauge couplings start running from RM_s and not M_s.
- This should hold for all local models: D3 branes at singularities, F-theory GUTs, IIB GUTs...
- Large effect: for $M_s \sim 10^{12}$GeV changes Λ_{UV} by a factor of 100 and for $M_s \sim 10^{15}$GeV changes Λ_{UV} by a factor of 10.

What should the string scale be?

- $M_s = 10^{11} - 10^{12}$GeV is good for the hierarchy problem, TeV supersymmetry and axions.
  Threshold corrections shift the unification scale to $10^{13} \rightarrow 10^{14}$GeV.
- If we want unification, then threshold corrections shift the required string scale from 10^{16}GeV to 10^{15}GeV.

Tension between hierarchy problem and gauge unification is ameliorated but not solved by threshold corrections.