Supersymmetric Flavour Universality in String Theory

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The MSSM Flavour Problem

The LHC experiments hope to discover supersymmetry.

But why should any new susy signatures occur at all?

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But why should any new susy signatures occur at all?

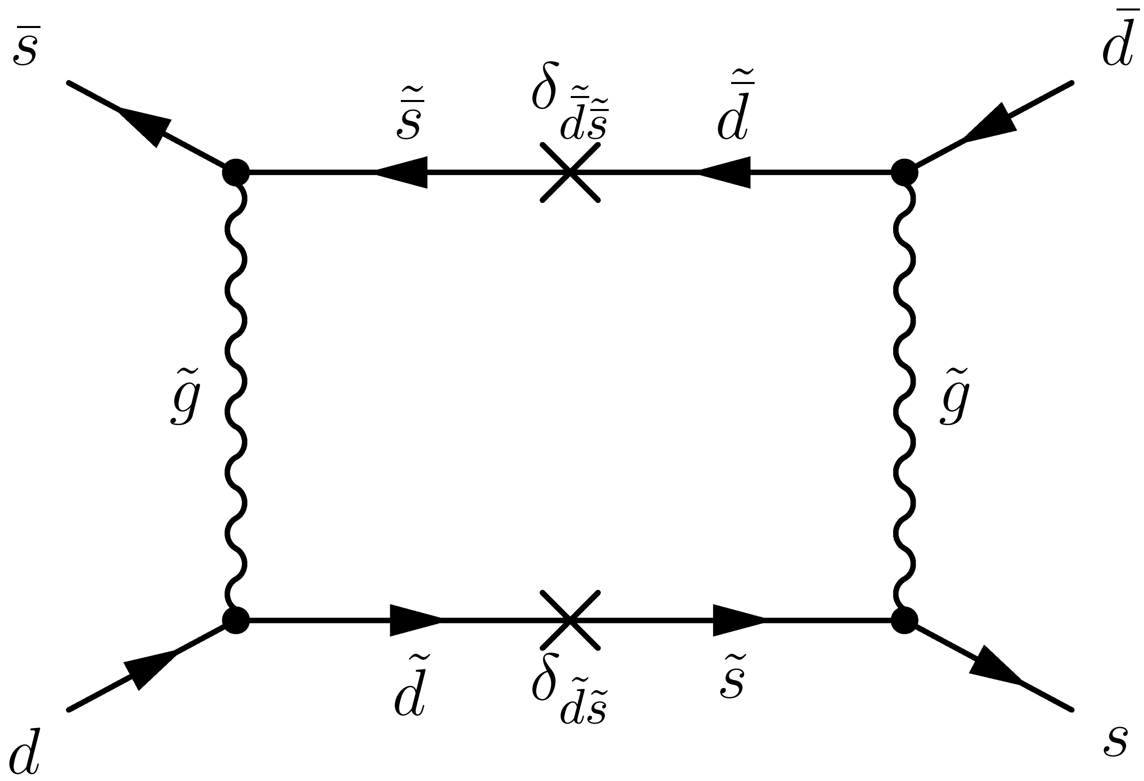
One major problem:

Why hasn't supersymmetry been discovered already?

Compare with

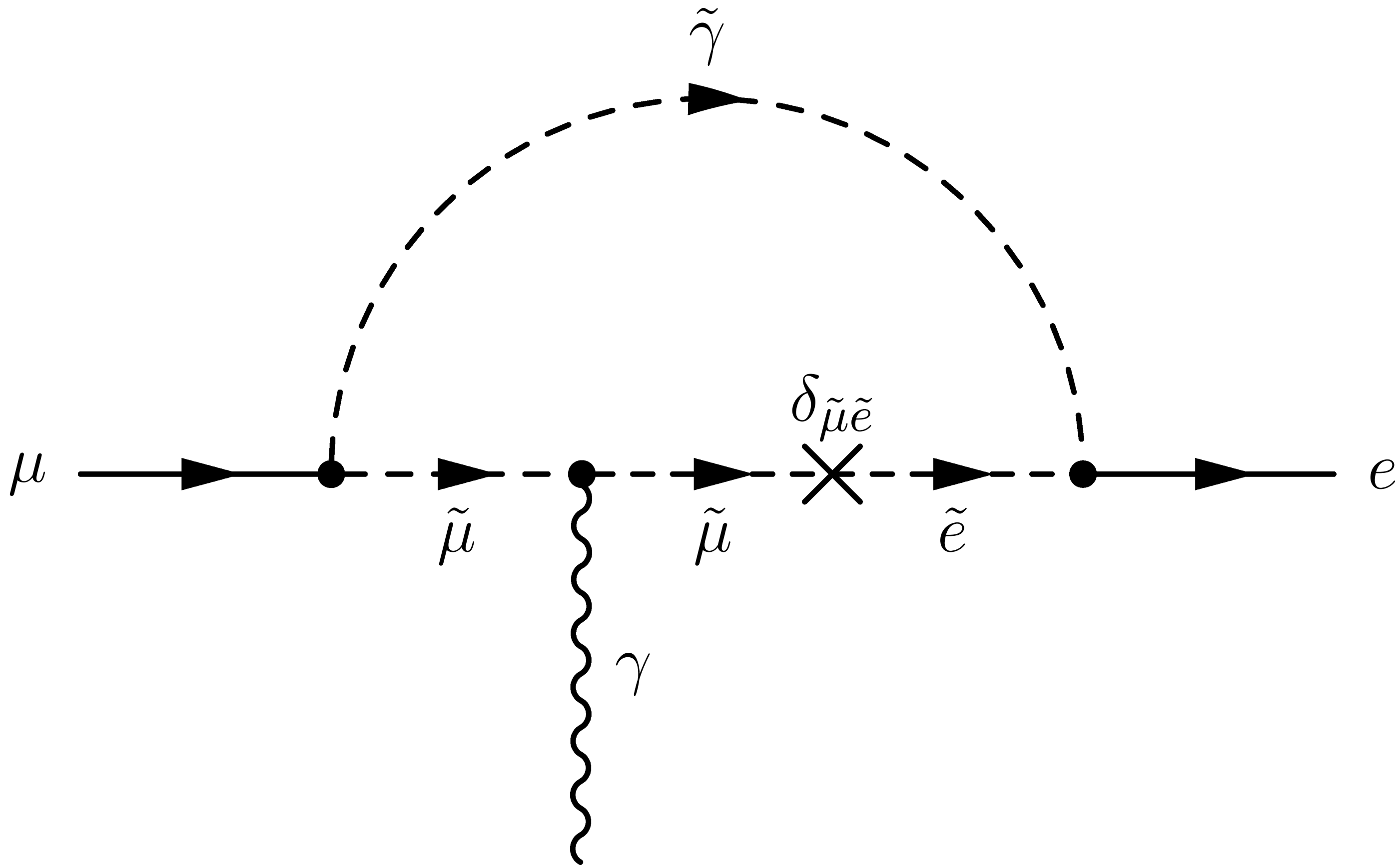
- The c quark - predicted before discovery through its contribution to FCNCs (the GIM mechanism)
- The t quark - mass predicted accurately through loop contributions at LEP I.

The MSSM Flavour Problem



The MSSM gives new contributions to $K_0 - \bar{K}_0$ mixing.

The MSSM Flavour Problem



The MSSM generates new contributions to $BR(\mu \to e\gamma)$.

Universality in Effective Field Theory

SUSY is a happy bunny if soft terms are flavour universal.

$$\begin{aligned} m^2_{Q,\alpha\bar{\beta}} &= m^2_Q, \\ A_{\alpha\beta\gamma} &= AY_{\alpha\beta\gamma}, \\ \phi_{M_1} &= \phi_{M_2} = \phi_{M_3} = \phi_A. \end{aligned}$$

Why should this be?

- Answer to flavour problem significantly affects all of susy phenomenology.

Soft Terms in Supergravity

- There exists lore that gravity(=string)-mediated susy breaking always suffers from large FCNCs.
- Soft terms come from expanding K and W in powers of matter fields C^α and moduli fields Φ,

$$\begin{aligned} W &= \hat{W}(\Phi) + \mu(\Phi) H_1 H_2 + \frac{1}{6} Y_{\alpha\beta\gamma}(\Phi) C^\alpha C^\beta C^\gamma + \ldots, \\ K &= \hat{K}(\Phi, \bar{\Phi}) + \tilde{K}_{\alpha\bar{\beta}}(\Phi, \bar{\Phi}) C^\alpha C^{\bar{\beta}} + [Z H_1 H_2 + h.c.] + \ldots, \\ f_a &= f_a(\Phi). \end{aligned}$$

- To compute soft terms, need to know $\tilde{K}_{\alpha\bar{\beta}}(\Phi, \bar{\Phi})$, $Y_{\alpha\beta\gamma}(\Phi)$ and $f_a(\Phi)$.

Soft Terms in Supergravity

Soft scalar masses $m^2_{i\bar{j}}$ and trilinears $A_{\alpha\beta\gamma}$ are given by

$$\begin{aligned}
\tilde{m}^2_{\alpha\bar{\beta}} &= (m^2_{3/2} + V_0)\tilde{K}_{\alpha\bar{\beta}} \\
&\quad -\bar{F}^{\bar{m}} F^n \left(\partial_{\bar{m}}\partial_n \tilde{K}_{\alpha\bar{\beta}} - (\partial_{\bar{m}}\tilde{K}_{\alpha\bar{\gamma}})\tilde{K}^{\bar{\gamma}\delta}(\partial_n \tilde{K}_{\delta\bar{\beta}})\right) \\
A'_{\alpha\beta\gamma} &= e^{\hat{K}/2} F^m \Big[\hat{K}_m Y_{\alpha\beta\gamma} + \partial_m Y_{\alpha\beta\gamma} \\
&\quad - \left((\partial_m \tilde{K}_{\alpha\bar{\rho}})\tilde{K}^{\bar{\rho}\delta} Y_{\delta\beta\gamma} + (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma)\right)\Big]. \\
M_a &= \frac{F^m \partial_m f_a}{\mathrm{Re}(f_a)}
\end{aligned}$$

Universality in Effective Field Theory

Sufficient conditions for flavour universality:

1. Hidden sector factorises

$$\Phi_{hidden} = \Psi_{susy} \oplus \chi_{flavour}.$$

2. The kinetic terms are decoupled

$$\mathcal{K}(\Phi, \bar{\Phi}) = \mathcal{K}_1(\Psi + \bar{\Psi}) + \mathcal{K}_2(\chi, \bar{\chi}),$$

3. The superpotential Yukawas depend only on $\chi_{flavour}$:

$$Y_{\alpha\beta\gamma}(\Psi, \chi) = Y_{\alpha\beta\gamma}(\chi).$$

4. The gauge couplings depend only on Ψ fields:

$$f_a(\Psi, \chi) = \sum_i \lambda_i \Psi_i.$$

Universality in Effective Field Theory

5. The visible metric factorises.

$$\mathcal{K}_{\alpha\bar{\beta}}(\Psi, \bar{\Psi}, \chi, \bar{\chi}) = h(\Psi + \bar{\Psi})k_{\alpha\bar{\beta}}(\chi, \bar{\chi})$$

6. Ψ breaks susy, χ does not:

$$D_{\Psi_i}W \neq 0, D_{\chi_j}W = 0.$$

If all these assumptions hold, susy breaking generates flavour universal soft terms.

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Totally *ad hoc* in effective field theory!

Universality in String Theory

String theory knows this structure!

- Calabi-Yau moduli space factorises in this fashion.
- Kähler (T) and complex structure (U) moduli have factorised moduli spaces
- $$IIB : \Psi_{susy} \rightarrow T, \chi_{flavour} \rightarrow U,$$
  $$IIA : \Psi_{susy} \rightarrow U, \chi_{flavour} \rightarrow T.$$
- In flux compactifications susy breaking factorises:

$$F^T \neq 0, \qquad F^U = 0.$$

Universality in String Theory

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$$\Phi_{hidden} = \Psi_{susy} \bigoplus \chi_{flavour}.$$

$$\Phi_{moduli} = \Psi_{\text{Kähler(T)}} \bigoplus \chi_{\text{complex structure(U)}}.$$

The moduli space of Calabi-Yau manifolds has two distinct, factorised parts: Kähler and complex structure moduli.

Universality in String Theory

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The IIB moduli space Kähler potential is

$$K = -2\ln(\mathcal{V}(T + \bar{T})) - \ln\left(\int \Omega \wedge \bar{\Omega}(U, \bar{U})\right) - \ln(S + \bar{S})$$

The kinetic terms factorise into T and U parts.

Universality in String Theory

The imaginary part of T is axionic.

T has a perturbative shift symmetry,

$$T \rightarrow T + i\epsilon.$$

This shift symmetry is unbroken in both world-sheet (α') and space-time (g_s) perturbation theory.

Perturbative quantities depend only on $(T + \bar{T})$.

Universality in String Theory

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Perturbativity would require $Y_{\alpha\beta\gamma}(T) \sim T^{\lambda}$.

The shift symmetry $T \to T + i\epsilon$ forbids this.

In perturbation theory, $Y_{\alpha\beta\gamma}(T, U) = Y_{\alpha\beta\gamma}(U)$.

Universality in String Theory

Example:

For toroidal compactifications, the superpotential Yukawas take the following form,

$$Y_{ijk}(U) = \vartheta \begin{bmatrix} \delta^r_{ijk} \\ 0 \end{bmatrix} (0; U^r I^r_{ab} I^r_{bc} I^r_{ca})$$

- No dependence on T
- A very complicated (exponential) dependence on U.

Universality in String Theory

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D7 brane gauge coupling:

$$f_a = \frac{T}{4\pi}.$$

- No U-dependence
- Linear dependence on T.

Universality in String Theory

5. The visible metric factorises.

$$\mathcal{K}_{\alpha\bar{\beta}}(\Psi, \bar{\Psi}, \chi, \bar{\chi}) = h(\Psi + \bar{\Psi}) k_{\alpha\bar{\beta}}(\chi, \bar{\chi})$$

Universality in String Theory

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$$\mathcal{K}_{\alpha\bar{\beta}}(\Psi, \bar{\Psi}, \chi, \bar{\chi}) = h(\Psi + \bar{\Psi}) k_{\alpha\bar{\beta}}(\chi, \bar{\chi})$$

This comes from the universal scaling property of physical Yukawa couplings

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha'\beta'\gamma'}}{(\tilde{K}_{\alpha\alpha'} \tilde{K}_{\beta\beta'} \tilde{K}_{\gamma\gamma'})^{\frac{1}{2}}}.$$

This arises from the origin of physical Yukawa couplings as due to wavefunction overlap.

Universality in String Theory

Example: For toroidal compactifications, the Kähler metric is

$$K_{ij}^{ab} = \delta_{ij} S^{-1/4}((T^1 + \bar{T}^1)(T^2 + \bar{T}^2)(T^3 + \bar{T}^3))^{-1/4} \times \prod_{I=1}^{3} U_I^{-\frac{1}{2}} \left(\frac{\Gamma(\theta^1_{ab})\Gamma(\theta^2_{ab})\Gamma(1-\theta^1_{ab}-\theta^2_{ab})}{\Gamma(1-\theta^1_{ab})\Gamma(1-\theta^2_{ab})\Gamma(\theta^1_{ab}+\theta^2_{ab})} \right)^{\frac{1}{2}}$$

This has

- At leading order a universal scaling dependence on $(T + \bar{T})$
- Subleading (universal) corrections at $\mathcal{O}(\alpha'^2)$

Universality in String Theory

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Universality in String Theory

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$$D_{\Psi_i} W \neq 0, D_{\chi_j} W = 0.$$

- This is a statement about the vacuum structure and is equivalent to $F^T \neq 0, F^U = 0$.
- The T fields break supersymmetry and the U fields do not.
- In IIB flux compactifications these conditions are satisfied.

Moduli Stabilisation: Fluxes

$$\hat{K} = -2\ln\left(\mathcal{V}(T+\bar{T})\right) - \ln\left(i\int \Omega\wedge\bar{\Omega}(U)\right) - \ln\left(S+\bar{S}\right),$$

$$W = \int G_3 \wedge \Omega\,(U).$$

$$V = e^{\hat{K}}\left(\sum_{U,S}\hat{K}^{\alpha\bar{\beta}}D_\alpha W D_{\bar{\beta}}\bar{W} + \sum_{T}\hat{K}^{i\bar{j}}D_i W D_{\bar{j}}\bar{W} - 3|W|^2\right)$$

$$= e^{\hat{K}}\left(\sum_{U,S}\hat{K}^{\alpha\bar{\beta}}D_\alpha W D_{\bar{\beta}}\bar{W}\right)$$

Stabilise S and U by solving $D_S W = D_U W = 0$.

Moduli Stabilisation: Fluxes

$$
\begin{aligned}
\hat{K} &= -2\ln\left(\mathcal{V}(T_i+\bar{T}_i)\right), \\
W &= W_0 . \\
V &= e^{\hat{K}}\left(\sum_T \hat{K}^{i\bar{j}} D_i W D_{\bar{j}}\bar{W} - 3|W|^2\right) \\
&= 0
\end{aligned}
$$

No-scale model :

- vanishing vacuum energy
- broken susy ($F^T \neq 0, F^U = 0$)
- T unstabilised
- Goldstino is breathing mode $g_{\mu\nu} \to \lambda^2 g_{\mu\nu}$

Moduli Stabilisation: Fluxes

- Susy breaking factorises: $F^T \neq 0, F^U = 0$.
- Goldstino is breathing mode and is manifestly flavour universal.

  The breathing mode modifies the normalisation but not the structure of Yukawa couplings.
- The factorisation of supersymmetry breaking persists when all moduli are stabilised (e.g. large volume models)

Corrections

- Factorisation holds at leading order.
- Factorisation is inherited from the underlying $\mathcal{N} = 2$ structure and holds in the large-radius limit.
- It is broken by e.g. loop corrections that are present in $\mathcal{N} = 1$ compactifications.
- Corrections are loop-suppressed, but very hard to compute explicitly.

Conclusions

- String flux compactifications give a natural solution to the MSSM flavour problems.
- Calabi-Yau moduli space factorises into Kähler and complex structure moduli.
- One sector generates flavour, the other generates susy breaking.
- IIB flux compactifications explicitly realise this factorisation.