

# Hierarchy Problems in String Theory : an Overview of the LARGE Volume Scenario

Joseph P. Conlon (Oxford)

Institute for Physics and Mathematics of the Universe

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This talk is based on work carried out from 2005 - present

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Shehu Abdussalam, Ben Allanach, Vijay Balasubramanian, Per Berglund, Cliff Burgess, Michele Cicoli, Daniel Cremades, Ling-Han Hung, Chun-Hay (Steve) Kom, Anshuman Maharana, Fernando Quevedo, Kerim Suruliz

# Maladies of Particle Physics

Nature likes hierarchies:

- The Planck scale,  $M_P = 2.4 \times 10^{18} \text{GeV}$ .
- The inflationary scale,  $M \sim 10^{13} \rightarrow 10^{16} \text{GeV}$ .
- The axion scale,  $10^9 \text{GeV} \lesssim f_a \lesssim 10^{12} \text{GeV}$
- The weak scale :  $M_W \sim 100 \text{GeV}$
- The fermion masses,  $m_e \sim 0.5 \text{MeV} \rightarrow m_t \sim 170 \text{GeV}$ .
- The neutrino mass scale,  $0.05 \text{eV} \lesssim m_\nu \lesssim 0.3 \text{eV}$ .
- The cosmological constant,  $\Lambda \sim (10^{-3} \text{eV})^4$

These demand an explanation!

# Maladies of Particle Physics

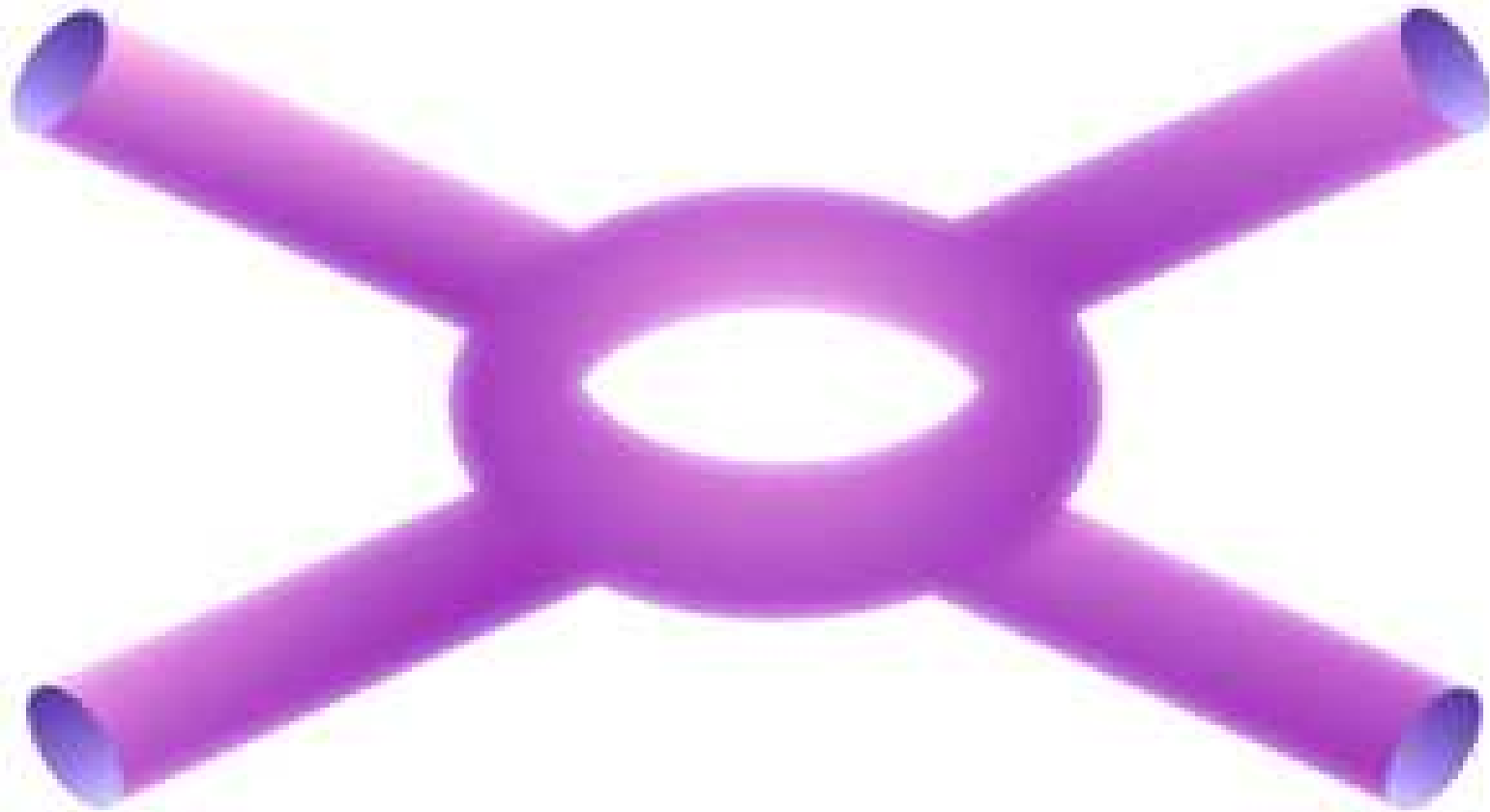
We can also ask

1. Why is the Standard Model gauge group  $SU(3) \times SU(2) \times U(1)$ ?
2. Why are there three generations?
3. What sets  $\alpha_{EW}$ ,  $\alpha_{strong}$ ,  $\alpha_Y$ ?
4. Where does the flavour structure come from?
5. Why four dimensions?

None of these questions can be answered within the Standard Model.

To address them, a more fundamental approach is needed.

# String Phenomenology



# String Phenomenology

String theory represents a candidate fundamental theory.

String theory is famous as a theory of quantum gravity.

It has also given important insights into algebraic geometry, black holes and quantum field theory.

Can string theory do the same for particle physics?

If nature is stringy, string theory should give insights into all the fundamental problems mentioned previously.

String phenomenology aims to use string theory to address these fundamental problems of particle physics.

# Supersymmetry

There are many difficult problems in particle physics.

In some ways the most important problem is the origin of the broken electroweak symmetry,

$$M_Z = 91.2\text{GeV}, \quad M_W = 80.4\text{GeV}.$$

- What breaks electroweak symmetry? Why?
- If symmetry is broken by a Higgs field, why is the weak scale stable against radiative corrections?
- Fortunately experiment will soon offer guidance.

The main assumption of my talk is that supersymmetry is relevant to these questions.

# Supersymmetry

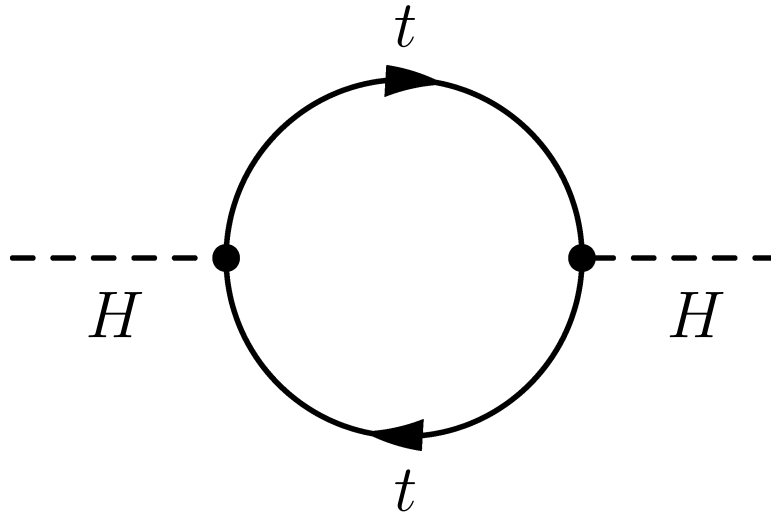
Supersymmetry adds different spin partners to all the Standard Model fields:

- Gluino  $\tilde{g}$ , squarks  $\tilde{q}$ ,
- Sleptons  $\tilde{e}, \tilde{\mu}, \tilde{\tau}$ , sneutrinos  $\tilde{\nu}$
- Neutralinos  $\tilde{\chi}_1, \tilde{\chi}_2, \tilde{\chi}_3, \tilde{\chi}_4$  ( $\tilde{B}, \tilde{Z}, \tilde{H}_1, \tilde{H}_2$ )
- Charginos  $\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$
- Higgs fields:  $h_0, H_0, A, H^\pm$

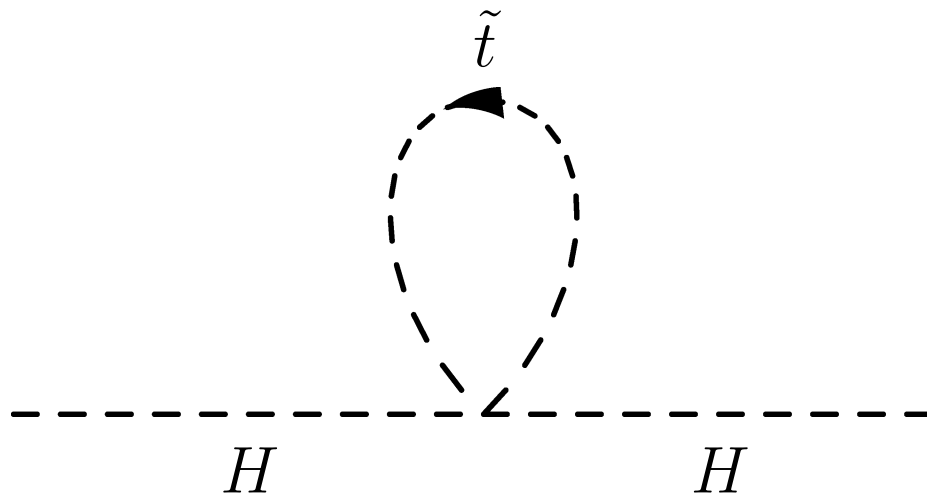


# Supersymmetry

Supersymmetry gives a solution to the gauge hierarchy problem:



$$\sim \Lambda^2$$



$$\sim -\Lambda^2$$

# Supersymmetry

## TeV supersymmetry

- stabilises the Higgs mass against radiative corrections
- gives a good dark matter candidate
- explains gauge symmetry breaking through radiative electroweak symmetry breaking
- is compatible with LEP I precision electroweak data
- is the prime candidate for new physics discovered at the LHC

# Supersymmetry

Susy partners the graviton with a spin 3/2 gravitino,  $\psi_{3/2}$ .

Supergravity is specified by

- The gauge groups  $G_a$ .
- The chiral superfields  $\Phi_i$  and their representations  $R_i(G_a)$ .
- A Kähler potential  $K(\Phi, \bar{\Phi})$ .
- A holomorphic superpotential  $W(\Phi)$ .
- Holomorphic gauge kinetic functions  $f_a(\Phi)$ .

The gravitino mass is given by

$$m_{3/2} = e^{K/2} W.$$

# Supersymmetry

All flavours of low-energy supersymmetry require a light gravitino mass

$$m_{3/2} = e^{K/2}W \lesssim 100\text{TeV} \lesssim 10^{-13}M_P.$$

The natural scale for  $m_{3/2}$  is  $M_P$  but moduli stabilisation must end up giving  $m_{3/2} \ll M_P$ .

There are precisely two ways of achieving this.

- $|W| \ll 1$ ,  $e^{K/2} \sim \mathcal{O}(1)$  and  $W$  approximately vanishes.
- $e^{K/2} \ll 1$ ,  $W \sim \mathcal{O}(1)$  and  $e^{K/2}$  approximately vanishes.

# Supersymmetry

- $|W| \ll 1, e^{K/2} \sim \mathcal{O}(1)$  and  $W$  approximately vanishes.

$W$  vanishes perturbatively and is generated through small non-perturbative effects.

Examples are gaugino condensation in the heterotic string/ racetrack stabilisation.

Here  $m_s \sim M_P$ .

- (This talk)

$e^{K/2} \ll 1, W \sim \mathcal{O}(1)$  and  $e^{K/2}$  approximately vanishes.

$W$  is perturbatively non-zero.

Hierarchies come from the lowering of all the scales in the problem ( $m_s \ll M_P$ ).

# String Phenomenology

My overall aim is to describe the LARGE volume models:

- an intermediate string scale  $m_s \sim 10^{11} \text{ GeV}$
- stabilised exponentially large extra dimensions ( $\mathcal{V} \sim 10^{15} l_s^6$ ).

and the insights they provide into the axionic and weak hierarchies as well as new suggestions for particle physics.

Different hierarchies come as different powers of the (LARGE) volume.

# String Phenomenology

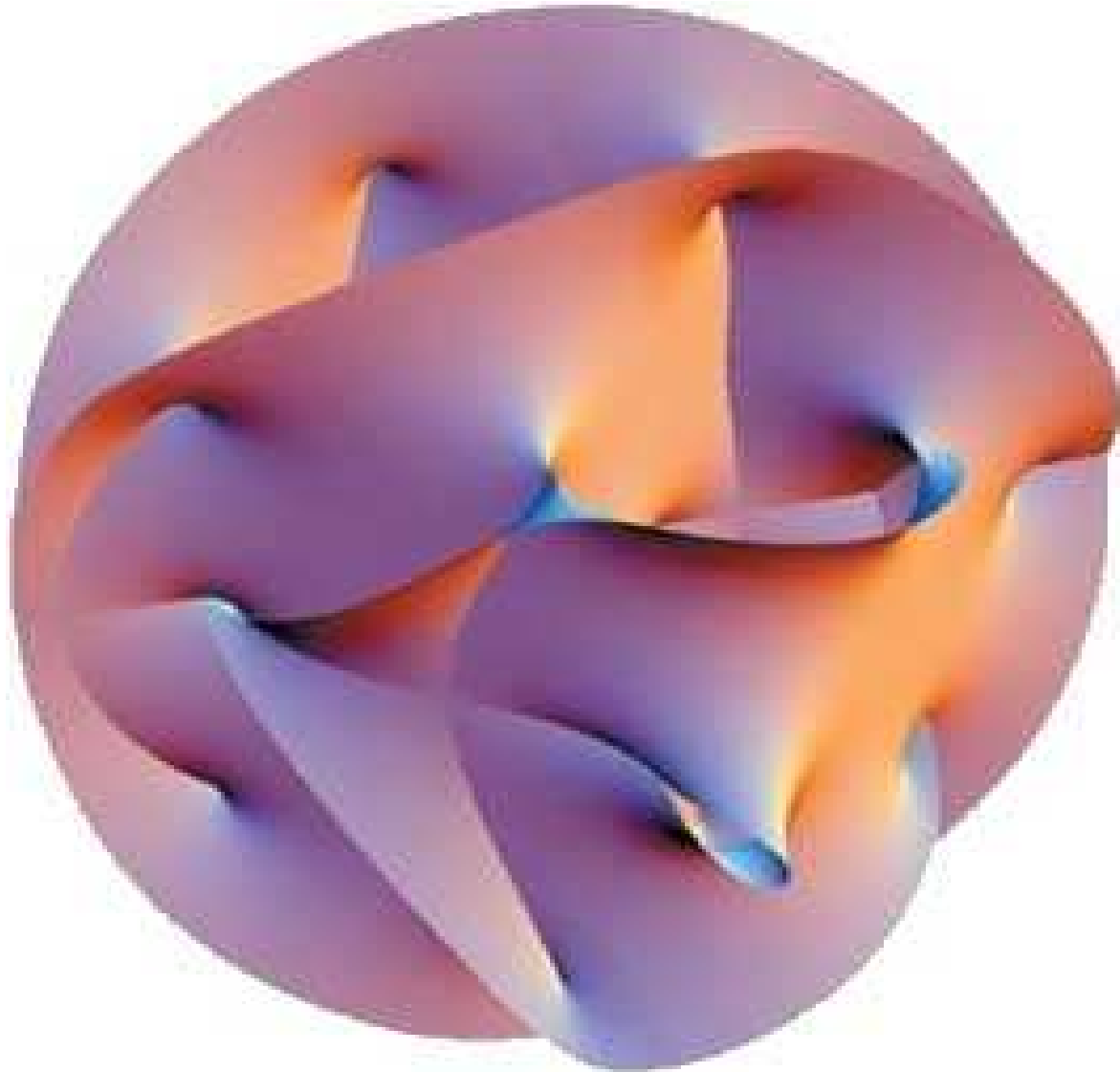
- String theory lives in ten dimensions.
- The world is four-dimensional, so ten-dimensional string theory needs to be compactified.
- To preserve  $\mathcal{N} = 1$  supersymmetry, we compactify on a six-dimensional Calabi-Yau manifold - Ricci-flat and Kähler.
- All scales, matter, particle spectra and couplings come from the geometry of the extra dimensions.

# String Phenomenology

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# String Phenomenology



# String Phenomenology

- The spectrum of light particles is determined by higher-dimensional topology.
- There are very many Calabi-Yaus - at least 30,000 distinct pairs of Hodge numbers and 240,000,000 different toric Calabi-Yaus.
- However many compactification properties are model-independent.
- In particular, string compactifications generically produce many uncharged scalar particles.
- These **moduli** parametrise the size and shape of the extra dimensions (Kähler and complex moduli).

# String Phenomenology

Recall:

- A Calabi-Yau manifold is a Kähler manifold with vanishing first Chern class.
- The Ricci-flat Kähler metric is uniquely determined by the complex structure and the Kähler class.
- The geometric moduli are  $h^{1,1}$  Kähler moduli ( $T_i$ ) and  $h^{2,1}$  complex structure moduli ( $U_j$ ).
- The moduli values entirely specify the Calabi-Yau metric.
- In string theory the Kähler moduli are **complexified**:  
$$T_i = \tau_i + ic_i.$$
- There is also the dilaton modulus  $S$  (the string coupling).

# String Phenomenology

- Moduli are naively massless scalar fields which may take large classical vevs.
- They are uncharged and interact gravitationally.
- Such massless scalars generate long-range, unphysical fifth forces.

To avoid fifth forces moduli must be given masses.

- It is essential to generate potentials for moduli and stabilise them.
- The LARGE volume models are an appealing scenario of moduli stabilisation.

# String Phenomenology

Furthermore:

- The moduli determine the geometry of the Calabi-Yau
- All scales, interactions and couplings come from the geometry of the extra dimensions.

example:

$$M_{string} = \frac{g_s M_P}{\sqrt{\mathcal{V}}}$$

- To say **anything** about the scales or couplings in particle physics, we need to stabilise the moduli.
- Moduli stabilisation is a **prerequisite** to string phenomenology.

# LARGE Volume Models

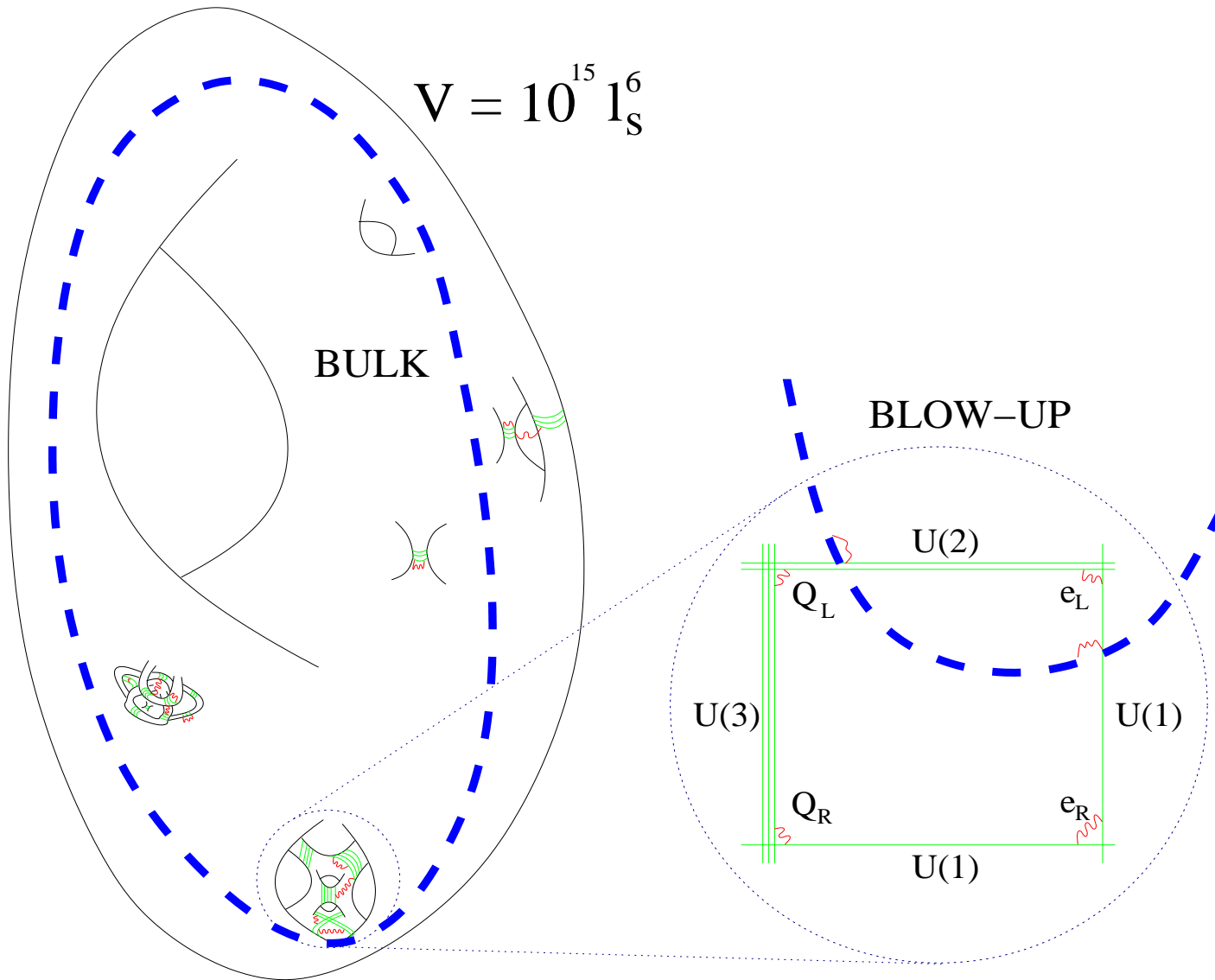
The main talk will describe in detail

- How to stabilise the volume at exponentially large values.

The general phenomenological features of these models:

- Low-scale supersymmetry  $M_{3/2} = \frac{M_P}{\mathcal{V}} \sim 1\text{TeV} \ll M_P$ .
- An intermediate string and axion scale  
 $M_s \sim f_a \sim (m_{3/2}M_P)^{\frac{1}{2}} \sim 10^{11}\text{GeV}$ .
- Quasi-realistic model-building with branes at del Pezzo singularities.
- New hyper-weakly coupled gauge forces with  
 $\frac{\alpha}{\alpha_{SM}} \sim 10^{-9}$ .

# LARGE Volume Models



# Break Time





# Welcome Back

## Talk Structure:

1. Moduli Stabilisation and LARGE Volume
2. Axions and the Strong CP Problem
3. Stringy Model Building
4. Hyperweak Gauge Groups

# Moduli Stabilisation: Fluxes

In IIB, the moduli definitions are

- Kähler moduli:

$$T_i = \tau_i + ic_i, \text{ where } \tau_i = \int_{\Sigma_4} \sqrt{g} \text{ and } c_i = \int_{\Sigma_4} C_4.$$

$T_i$  are complexified 4-cycle volumes.

- Complex structure moduli:

$U_i$  are the geometric complex structure moduli of the Calabi-Yau.

- Dilaton modulus:

$S = \frac{1}{g_s} + ic_0$  combines the string coupling and the RR 0-form.

# Moduli Stabilisation: Fluxes

We work in (orientifolds of) type IIB string theory with D3 and D7 branes.

The IIB field content includes 3-form field strengths  $F_3 = dC_2$ ,  $H_3 = dB_2$  from RR and NS-NS sector.

$$G_3 = F_3 + SH_3.$$

The extra dimensions contain

1. D3/D7 D-branes
2. O3/O7 orientifold branes
3. 3-form fluxes  $G_3 = F_3 + SH_3$ .

# Moduli Stabilisation: Fluxes

Under these conditions it can be shown (GKP 2001) that the 10-D metric is

$$ds_{10}^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{MN} dy^M dy^N$$

The metric is *warped Calabi-Yau*.

The warp factor scales as

$$e^{2A(y)} \sim 1 + \frac{1}{\mathcal{V}^{2/3}}$$

and vanishes in the infinite volume limit.

The dilaton and complex structure moduli are fixed by the fluxes. The Kähler moduli are not fixed.

# Moduli Stabilisation: Fluxes

- We want to work in 4-dimensional supergravity.
- The fluxes carry energy generating a potential for the moduli associated with these cycles.
- This energy is expressed through a superpotential

$$W = \int (F_3 + iSH_3) \wedge \Omega \equiv \int G_3 \wedge \Omega.$$

- This generates a potential for the dilaton and complex structure moduli.

# Moduli Stabilisation: Fluxes

- The effective supergravity theory is

$$K = -2 \ln(\mathcal{V}) - \ln \left( i \int \Omega \wedge \bar{\Omega} \right) - \ln(S + \bar{S})$$

$$W = \int (F_3 + iSH_3) \wedge \Omega \equiv \int G_3 \wedge \Omega.$$

- This stabilises the dilaton and complex structure moduli.

$$D_S W = D_U W = 0.$$

$$W = \int G_3 \wedge \Omega = W_0.$$

# Moduli Stabilisation: Fluxes

The theory has an important **no-scale** property.

$$\hat{K} = -2 \ln (\mathcal{V}(T + \bar{T})) - \ln \left( i \int \Omega \wedge \bar{\Omega}(U) \right) - \ln (S + \bar{S}),$$

$$W = \int G_3 \wedge \Omega(S, U).$$

$$V = e^{\hat{K}} \left( \sum_{U,S} \hat{K}^{\alpha\bar{\beta}} D_{\alpha} W D_{\bar{\beta}} \bar{W} + \sum_T \hat{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right)$$
$$= e^{\hat{K}} \left( \sum_{U,S} \hat{K}^{\alpha\bar{\beta}} D_{\alpha} W D_{\bar{\beta}} \bar{W} \right) = 0.$$

# Moduli Stabilisation: Fluxes

$$\hat{K} = -2 \ln (\mathcal{V}(T_i + \bar{T}_i)) ,$$

$$W = W_0 .$$

$$V = e^{\hat{K}} \left( \sum_T \hat{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right)$$
$$= 0$$

No-scale model :

- vanishing vacuum energy
- broken susy
- $T$  unstabilised

No-scale is broken perturbatively and non-pertubatively.



# Moduli Stabilisation: KKLT

$$\hat{K} = -2 \ln(\mathcal{V}) - \ln \left( i \int \Omega \wedge \bar{\Omega} \right) - \ln(S + \bar{S}),$$

$$W = \int G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i}.$$

Non-perturbative effects (D3-instantons / gaugino condensation) allow the  $T$ -moduli to be stabilised by solving  $D_T W = 0$ .

For consistency, this requires

$$W_0 = \left\langle \int G_3 \wedge \Omega \right\rangle \ll 1.$$

# Moduli Stabilisation: KKLT

$$\hat{K} = -2 \ln(\mathcal{V}),$$
$$W = W_0 + \sum_i A_i e^{-a_i T_i}.$$

Solving  $D_T W = \partial_T W + (\partial_T K)W = 0$  gives

$$\text{Re}(T) \sim \frac{1}{a} \ln(W_0)$$

For  $\text{Re}(T)$  to be large,  $W_0$  must be *enormously* small.

# Moduli Stabilisation: KKLT

KKLT stabilisation has three phenomenological problems:

1. No susy hierarchy: fluxes prefer  $W_0 \sim 1$  and  $m_{3/2} \gg 1\text{TeV}$ .
2. Susy breaking not well controlled - depends entirely on uplifting.
3.  $\alpha'$  expansion not well controlled - volume is small and there are large flux backreaction effects.

# Moduli Stabilisation: LARGE Volume

$$\hat{K} = -2 \ln \left( \mathcal{V} + \frac{\xi}{2g_s^{3/2}} \right), \quad \left( \xi = \frac{\chi(\mathcal{M})\zeta(3)}{2(2\pi)^3} \right)$$

$$W = W_0 + \sum_i A_i e^{-a_i T_i}.$$

- Include perturbative as well as non-perturbative corrections to the scalar potential.
- Add the leading  $\alpha'$  corrections to the Kähler potential.
- These descend from the  $\mathcal{R}^4$  term in 10 dimensions.
- This leads to dramatic changes in the large-volume vacuum structure.

# Moduli Stabilisation: LARGE Volume

The simplest model  $\mathbb{P}^4_{[1,1,1,6,9]}$  has two Kähler moduli.

$$\mathcal{V} = \left( \frac{T_b + \bar{T}_b}{2} \right)^{3/2} - \left( \frac{T_s + \bar{T}_s}{2} \right)^{3/2} \equiv \left( \tau_b^{3/2} - \tau_s^{3/2} \right).$$

If we compute the scalar potential, we get for  $\mathcal{V} \gg 1$ ,

$$V = \frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s \tau_s}}{\mathcal{V}} - \frac{a_s |A_s W_0| \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{\xi |W_0|^2}{g_s^{3/2} \mathcal{V}^3}.$$

# Moduli Stabilisation: LARGE Volume

$$V = \underbrace{\frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s \tau_s}}{\mathcal{V}} - \frac{a_s |A_s W_0| \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2}}_{\text{Integrate out } \tau_s} + \frac{\xi |W_0|^2}{g_s^{3/2} \mathcal{V}^3}.$$

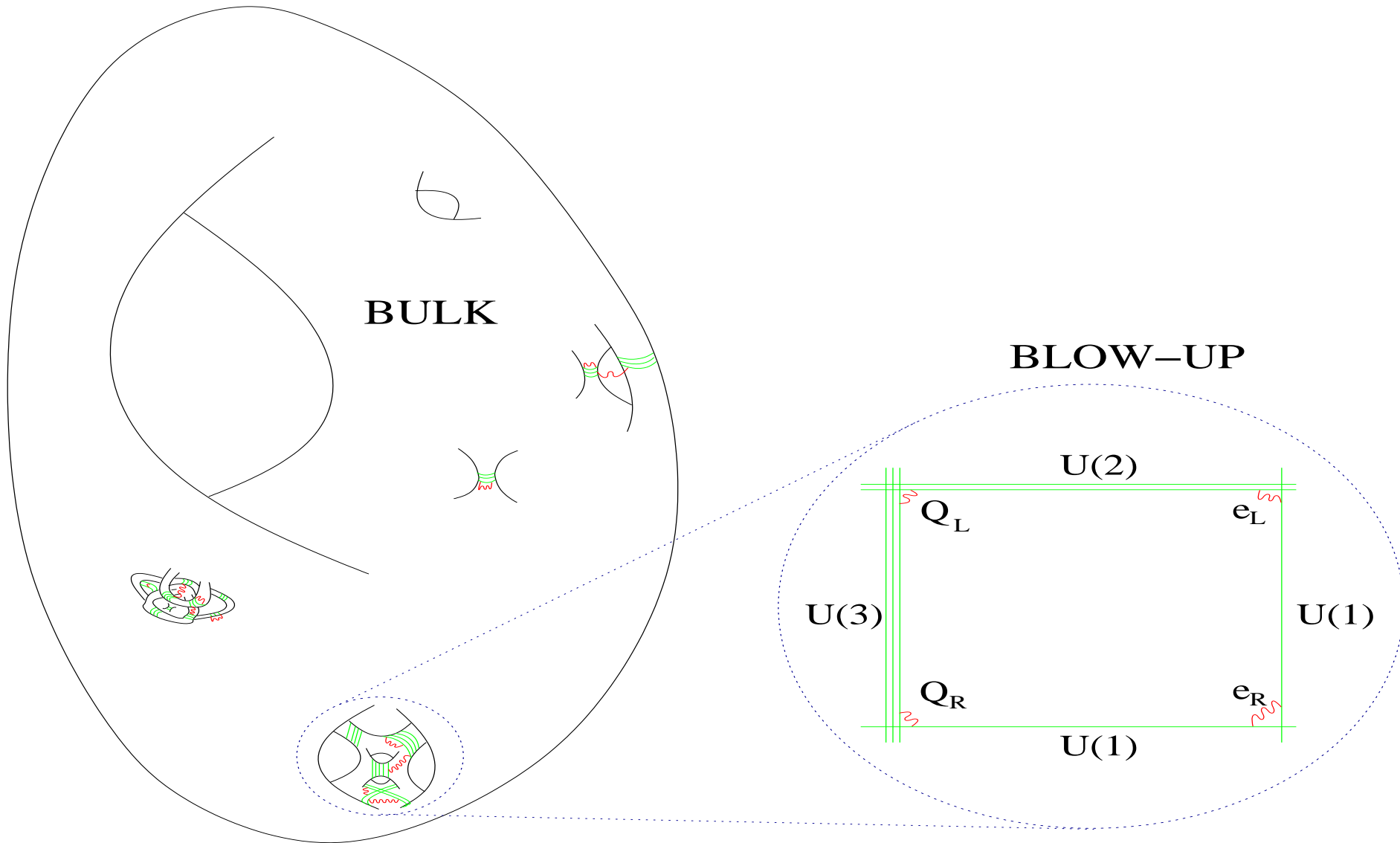
$$V = -\frac{|W_0|^2 (\ln \mathcal{V})^{3/2}}{\mathcal{V}^3} + \frac{\xi |W_0|^2}{g_s^{3/2} \mathcal{V}^3}.$$

A minimum exists at

$$\mathcal{V} \sim |W_0| e^{a_s \tau_s}, \quad \tau_s \sim \frac{\xi^{2/3}}{g_s}.$$

This minimum is **non-supersymmetric AdS** and at **exponentially large volume**.

# Moduli Stabilisation: LARGE Volume



# Moduli Stabilisation: LARGE Volume

Higher  $\alpha'$  corrections are suppressed by more powers of volume.

Example:

$$\begin{aligned} \int d^{10}x \sqrt{g} \mathcal{G}_3^2 \mathcal{R}^3 & : \int d^{10}x \sqrt{g} \mathcal{R}^4 \\ \int d^4x \sqrt{g_4} \left( \int d^6x \sqrt{g_6} \mathcal{G}_3^2 \mathcal{R}^3 \right) & : \int d^4x \sqrt{g_4} \left( \int d^6x \sqrt{g_6} \mathcal{R}^4 \right) \\ \int d^4x \sqrt{g_4} (\mathcal{V} \times \mathcal{V}^{-1} \times \mathcal{V}^{-1}) & : \int d^4x \sqrt{g_4} (\mathcal{V} \times \mathcal{V}^{-4/3}) \\ \int d^4x \sqrt{g_4} (\mathcal{V}^{-1}) & : \int d^4x \sqrt{g_4} (\mathcal{V}^{-1/3}) \end{aligned}$$



# Moduli Stabilisation: LARGE Volume

- The minimum of the potential is **non-supersymmetric AdS** and at **exponentially large volume**.
- The large volume lowers the string scale and gravitino mass through

$$m_s = \frac{M_P}{\sqrt{\mathcal{V}}}, \quad m_{3/2} = \frac{M_P W_0}{\mathcal{V}}.$$

- To solve the gauge hierarchy problem, need  $\mathcal{V} \sim 10^{15}$ .
- D7-branes wrapped on small cycle carry the Standard Model: need  $T_s \sim 20(2\pi\sqrt{\alpha'})^4$ .
- The vacuum is pseudo no-scale and breaks susy...

# Moduli Stabilisation: LARGE Volume

Supersymmetry is broken in the vacuum.

The supersymmetry breaking scale is set by the gravitino mass

$$M_{3/2} = \frac{W_0 M_P}{\mathcal{V}}$$

The stabilised volume is

$$\mathcal{V} \sim W_0 e^{\frac{c}{g_s}}, \quad (c \text{ constant})$$

and is essentially arbitrary.

We like TeV supersymmetry and so set  $\mathcal{V} \sim 10^{15}$  and explore the consequences!

# Moduli Stabilisation: LARGE Volume

The mass scales present are:

Planck scale:	$M_P = 2.4 \times 10^{18} \text{ GeV.}$
String scale:	$M_S \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11} \text{ GeV.}$
KK scale	$M_{KK} \sim \frac{M_P}{\mathcal{V}^{2/3}} \sim 10^9 \text{ GeV.}$
Gravitino mass	$m_{3/2} \sim \frac{M_P}{\mathcal{V}} \sim 1 \text{ TeV.}$
Moduli	$m_U, m_{T_S}, m_S \sim m_{3/2} \sim 1 \text{ TeV.}$
Volume modulus	$m_{\tau_b} \sim \frac{M_P}{\mathcal{V}^{3/2}} \sim 1 \text{ MeV.}$

# Axions

Axions are a well-motivated solution to the strong CP problem of the Standard Model.

The Lagrangian for QCD contains a term

$$\mathcal{L}_{QCD} = \frac{1}{4\pi g^2} \int d^4x F_{\mu\nu}^a F^{a,\mu\nu} + \frac{\theta}{8\pi^2} \int F^a \wedge F^a.$$

The  $\theta$  term

$$\mathcal{L}_\theta = \frac{\theta}{8\pi^2} \int F^a \wedge F^a.$$

gives rise to the strong CP problem.

This term vanishes in perturbation theory and is only non-vanishing for non-perturbative instanton computations.

# Axions

$\theta$  is an angle and in principle takes any value  $\theta \in (-\pi, \pi)$ .

It can be shown that non-vanishing  $\theta$  generates an electric dipole moment for the neutron.

The measured absence of a neutron electric dipole moment implies that

$$|\theta_{QCD}| \lesssim 10^{-10}.$$

No symmetry of the Standard Model requires  $|\theta|$  to be so small.

The problem of why  $|\theta_{QCD}| \ll \pi$  is the *strong CP* problem of the Standard Model.

# Axions

The best solution to the strong CP problem is due to Peccei and Quinn.

The angle  $\theta$  is promoted to a dynamical field with Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\mu \theta \partial^\mu \theta + \int \frac{\theta}{8\pi^2 f_a} F^a \wedge F^a.$$

$f_a$  is called the *axion decay constant* and measures the strength of the non-renormalisable axion-matter coupling.

# Axions

Non-perturbative QCD effects generate a potential for  $\theta$ ,

$$V_\theta \sim \Lambda_{QCD}^4 \left( 1 - \cos \left( \frac{\theta}{f_a} \right) \right) = \frac{\Lambda_{QCD}^4}{2f_a^2} (\theta^2 + \dots)$$

The axion field  $\theta$  gets a mass

$$m_\theta = \frac{\Lambda_{QCD}^2}{f_a} \sim 0.01 \text{eV} \left( \frac{10^9 \text{GeV}}{f_a} \right)$$

Axion phenomenology depends entirely on the value of  $f_a$ .

# Axions

Constraints from supernova cooling and direct searches imply  $f_a \gtrsim 10^9 \text{ GeV}$  (axions cannot couple too strongly to matter).

Avoiding the overproduction of axion dark matter during a hot big bang prefers  $f_a \lesssim 10^{12} \text{ GeV}$  (axions cannot couple too weakly to matter).

There exists an axion ‘allowed window’,

$$10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}.$$

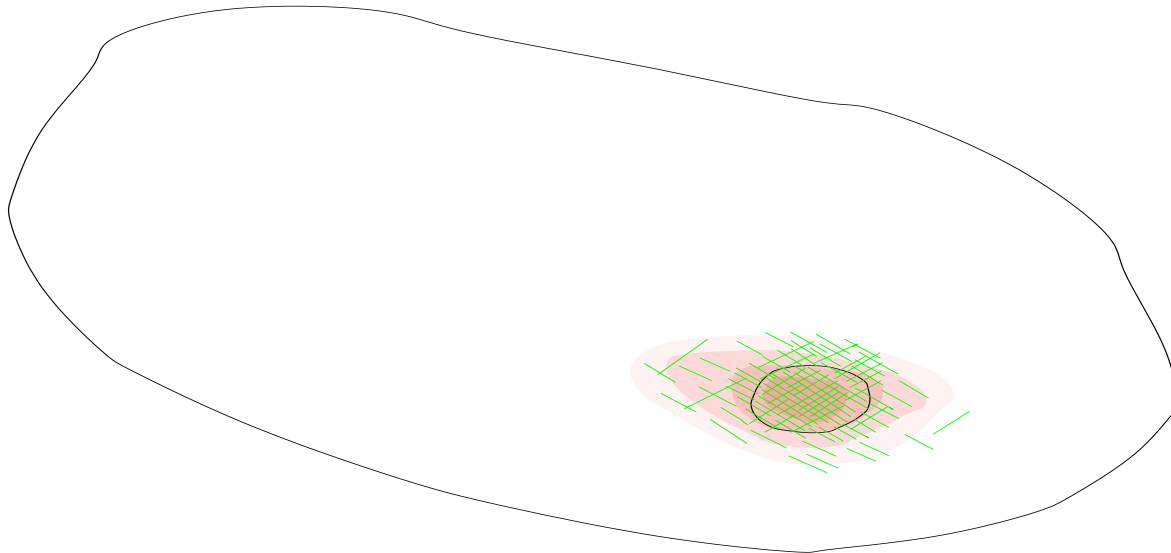


# Axions

- In string theory, the axionic coupling comes from the Chern-Simons interaction

$$\int_{\mathbb{M}_4 \times \Sigma_4} F \wedge F \wedge C_4 \rightarrow \int_{\mathbb{M}_4} \frac{c}{f_a} F \wedge F.$$

- $f_a$  measures the axion-matter coupling.



# Axions

- The axion coupling is a local coupling and does not see the overall volume.
- The coupling is set by the string scale:

$$f_a \sim m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}.$$

- This generates the axion scale,

$$f_a \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11} \text{ GeV}.$$

# Axions

TeV-scale supersymmetry required

$$M_{susy} \sim m_{3/2} = \frac{M_P}{\mathcal{V}} = 1\text{TeV}.$$

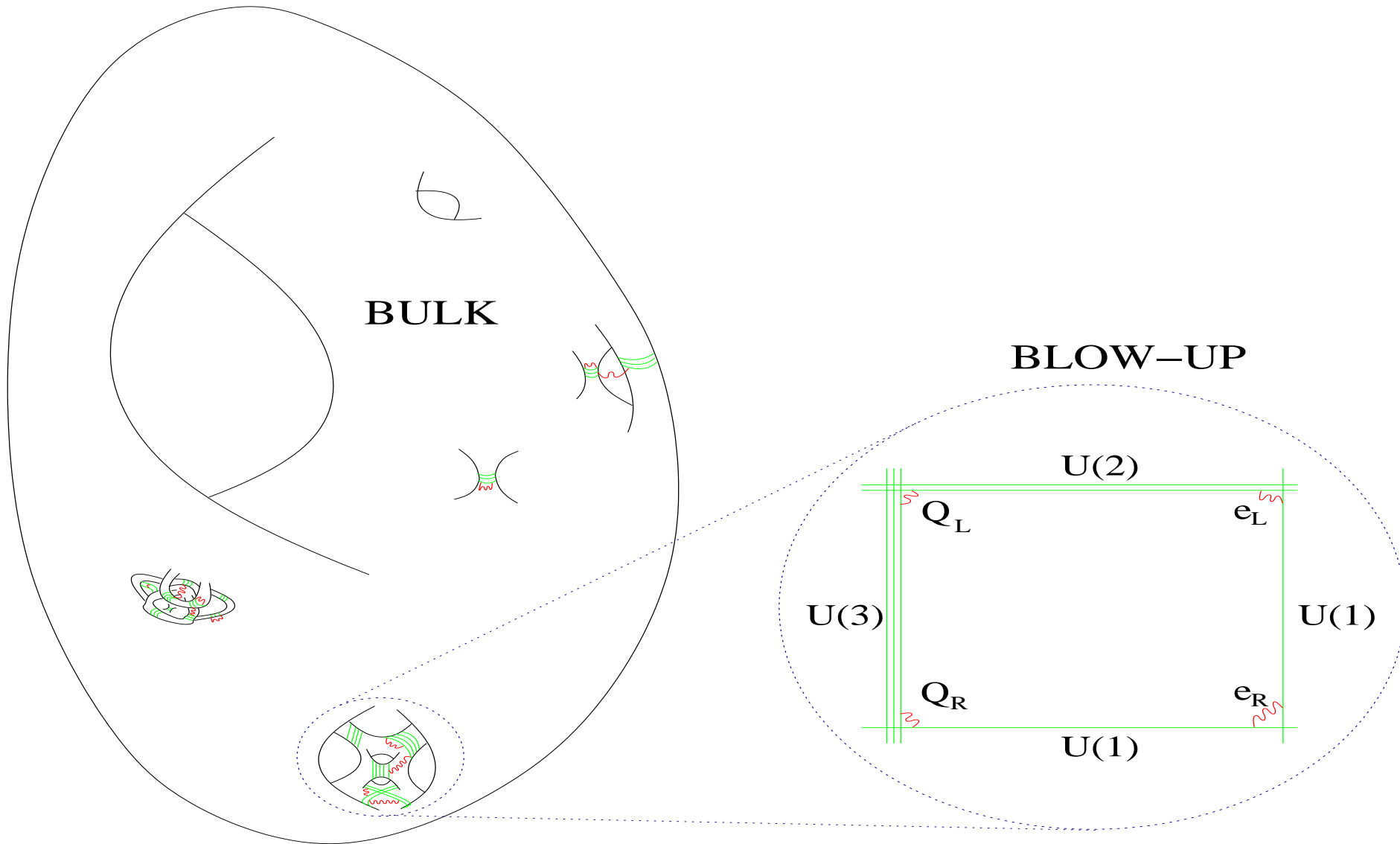
This gives  $\mathcal{V} \sim 10^{15} l_s^6$ .

The axion scale is therefore

$$f_a \sim \sqrt{M_{susy} M_P} \sim 10^{11} \text{GeV}$$

The existence of an axion in the allowed window correlates to the existence of supersymmetry at the TeV scale.

# Model Building



# Model Building

- Standard Model must be local and live on the 'hole in the cheese'
- Local geometry consists of magnetised D7 branes/ F-theory wrapped on a collapsing 4-cycle
- Geometrically collapsing 4-cycle is a del Pezzo surface.
- Model-building must come from D-branes on a (resolved) del Pezzo singularity.

# Model Building

There are two limits:

## 1. The geometric limit:

The del Pezzo is blown up to finite size.

The stability conditions are geometric ( $J \wedge F = 0$ ).

Model Building: [Beasley-Heckman-Vafa \(08\)](#), [Tatar-Watari \(08\)](#),  
[Blumenhagen-Braun-Grimm-Weigand \(08\)](#)

## 2. The singular limit:

The del Pezzo is collapsed to give a del Pezzo singularity.

Both 'branes' and 'antibranes' are supersymmetric.

Model-building: [Aldazabal-Ibanez-Quevedo-Uranga \(00\)](#), [Berenstein-Jejjala-Leigh \(01\)](#), [Verlinde-Wijnholt \(05\)](#), [JC-Maharana-Quevedo \(08\)](#)

# Model Building

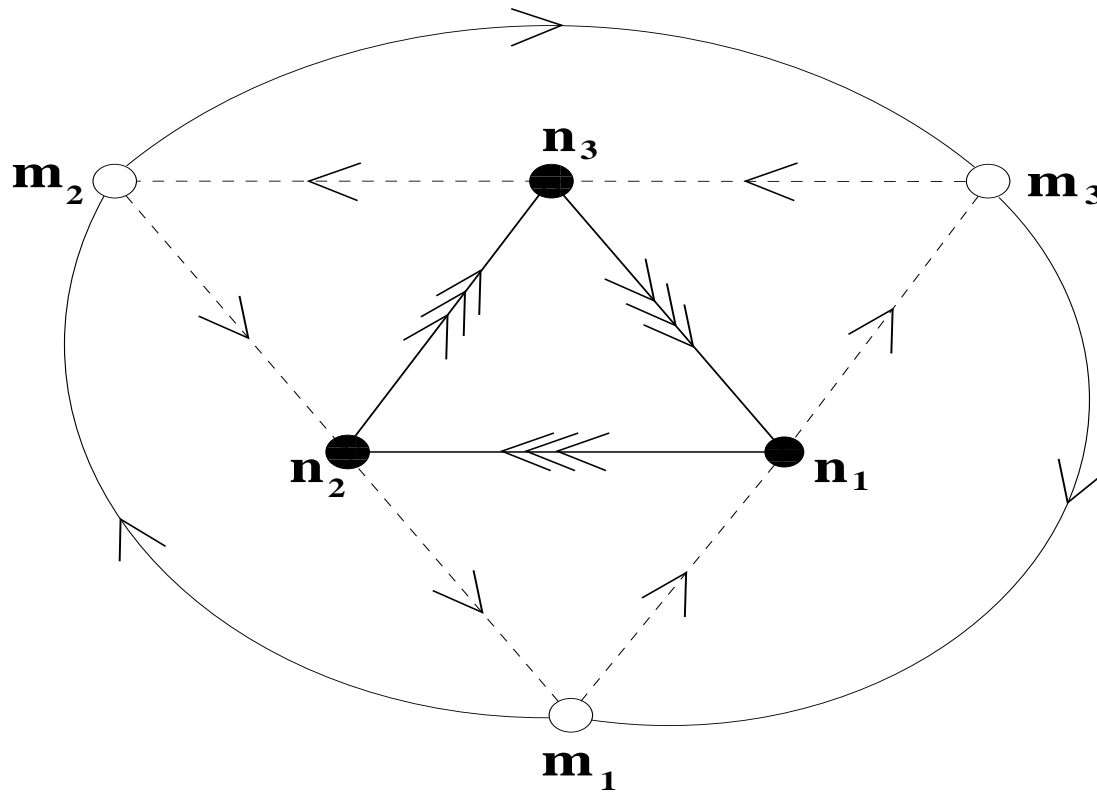
We work at the singular limit. Why?

- Moduli stabilisation of 'Standard Model' cycle requires D-terms.
- D-terms drive del Pezzo cycle to vanishing limit at edge of Kähler cone.
- Geometric branes cross lines of marginal stability and become unstable.
- We must therefore use the susy branes at singularities!

# Model Building

We use the quivers for del Pezzo  $n$  ( $dP_n$ ) singularities.

The  $dP_0 \equiv \mathbb{C}^3/\mathbb{Z}_3$  quiver is

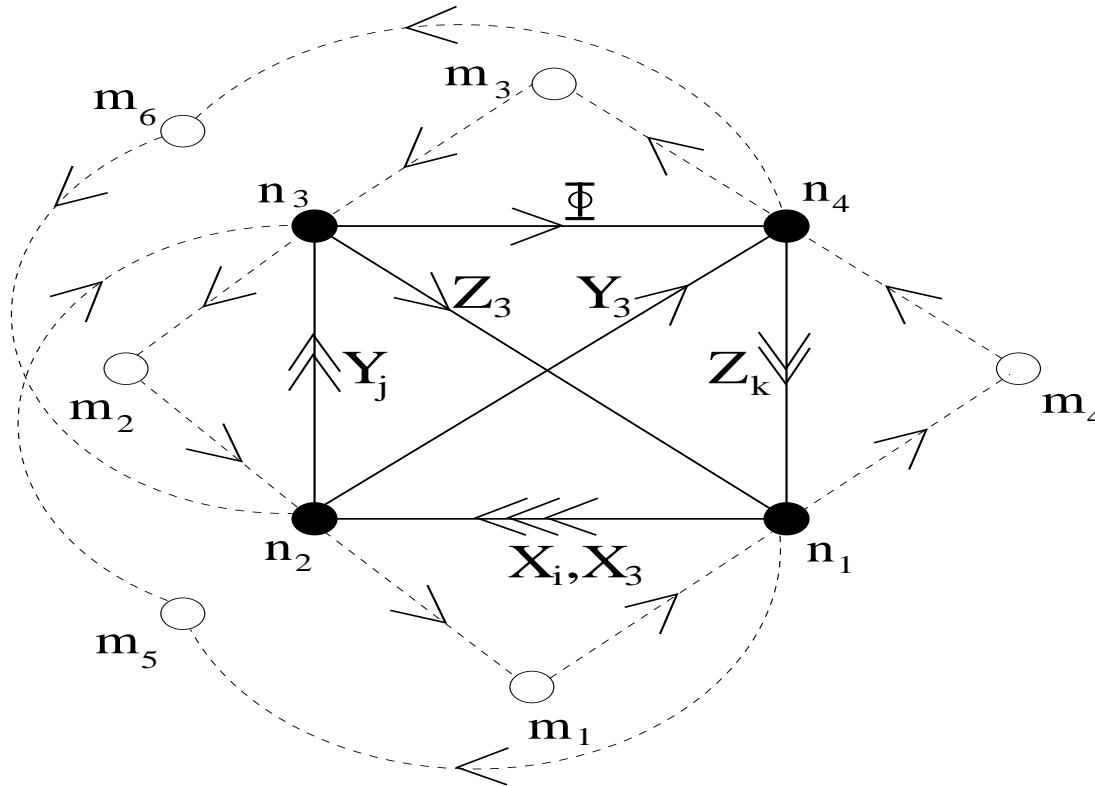


This has an  $SU(3)$  family symmetry for 33 interactions.



# Model Building

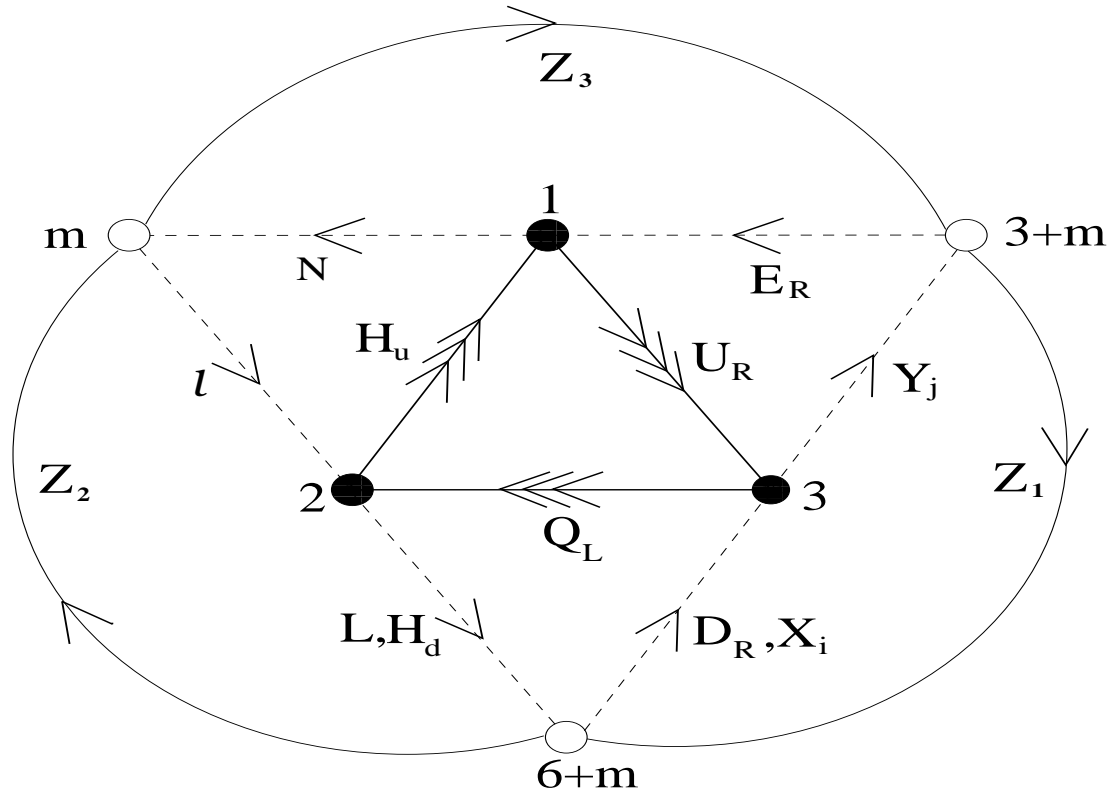
The  $dP_1 \equiv \mathbb{C}^3/\mathbb{Z}_3$  quiver is



This has an  $SU(2) \times U(1)$  family symmetry for 33 interactions.

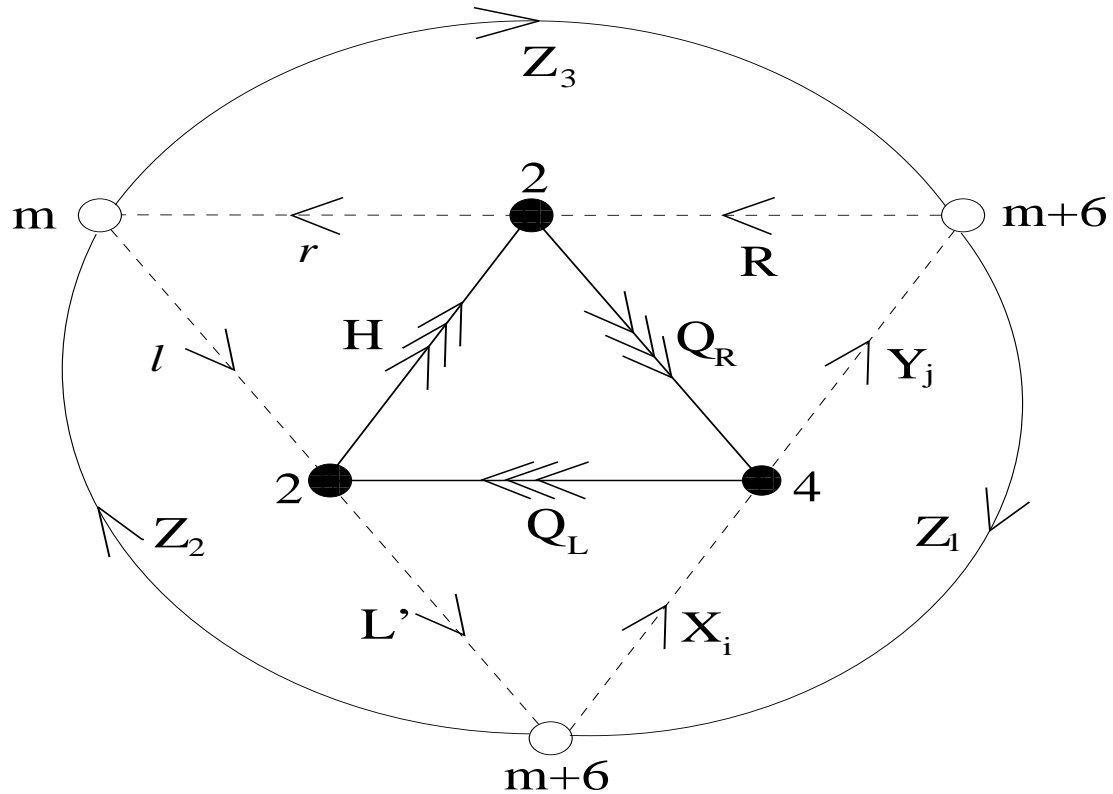
# Model Building

A Standard-Model like spectrum for  $dP_0$ :



# Model Building

A Pati-Salam-Model like spectrum for  $dP_0$ :



# Model Building

Both these models have (33) Yukawa textures

$$Y_{ijk} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & M \\ 0 & -M & 0 \end{pmatrix}.$$

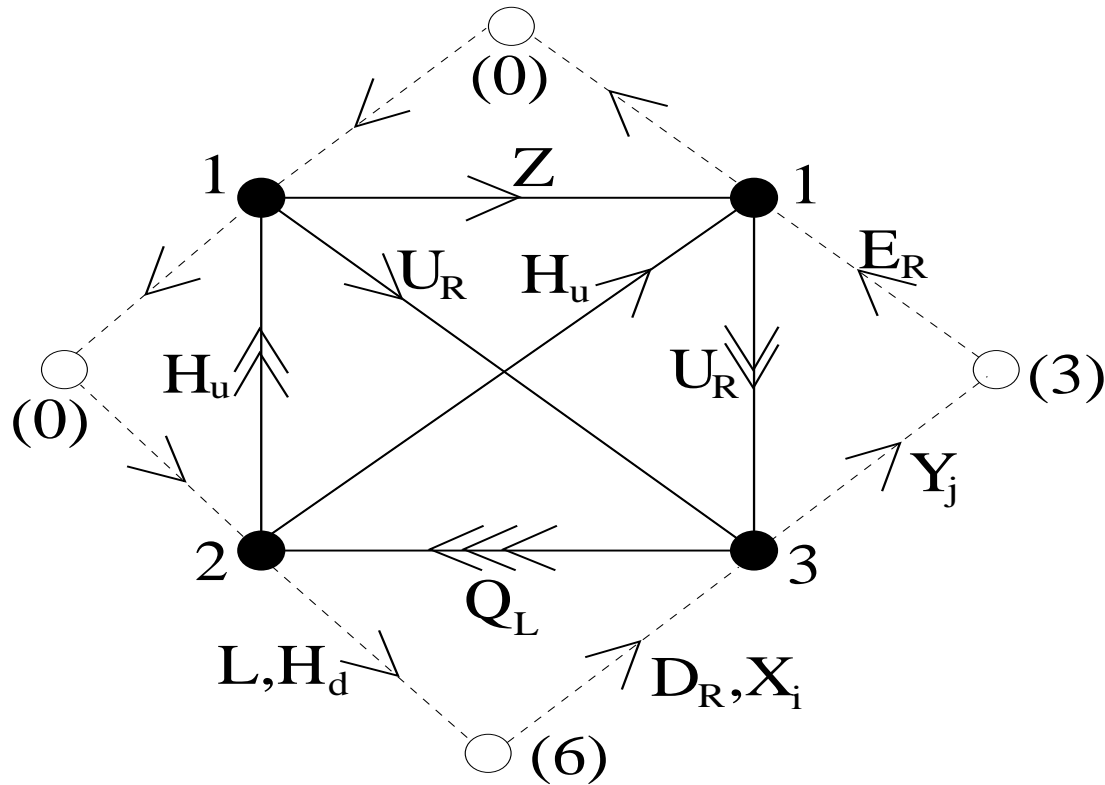
The  $(M, M, 0)$  Yukawa structure follows from the  $SU(3)$  family symmetry.

This can be improved by going to models based on  $dP_1$  singularities.

Here the family symmetry is reduced to  $SU(2) \times U(1)$ .

# Model Building

A Standard-Model like spectrum for  $dP_1$ :



# Model Building

For models based on  $dP_1$ , the (33) superpotential is

$$W = \epsilon_{ij} X_i Y_j Z_3 - \epsilon_{ij} X_i Y_3 Z_k + \frac{\Phi}{\Lambda} X_3 \epsilon_{ij} Y_i Z_j.$$

The  $SU(3)$  family symmetry is broken down to  $SU(2) \times U(1)$ .

The Yukawa mass spectrum is now  $(M, m, 0)$ , where  $m \sim \langle \frac{\Phi}{\Lambda} \rangle$ .

In contrast to  $dP_0$  this now allows a heavy third generation.

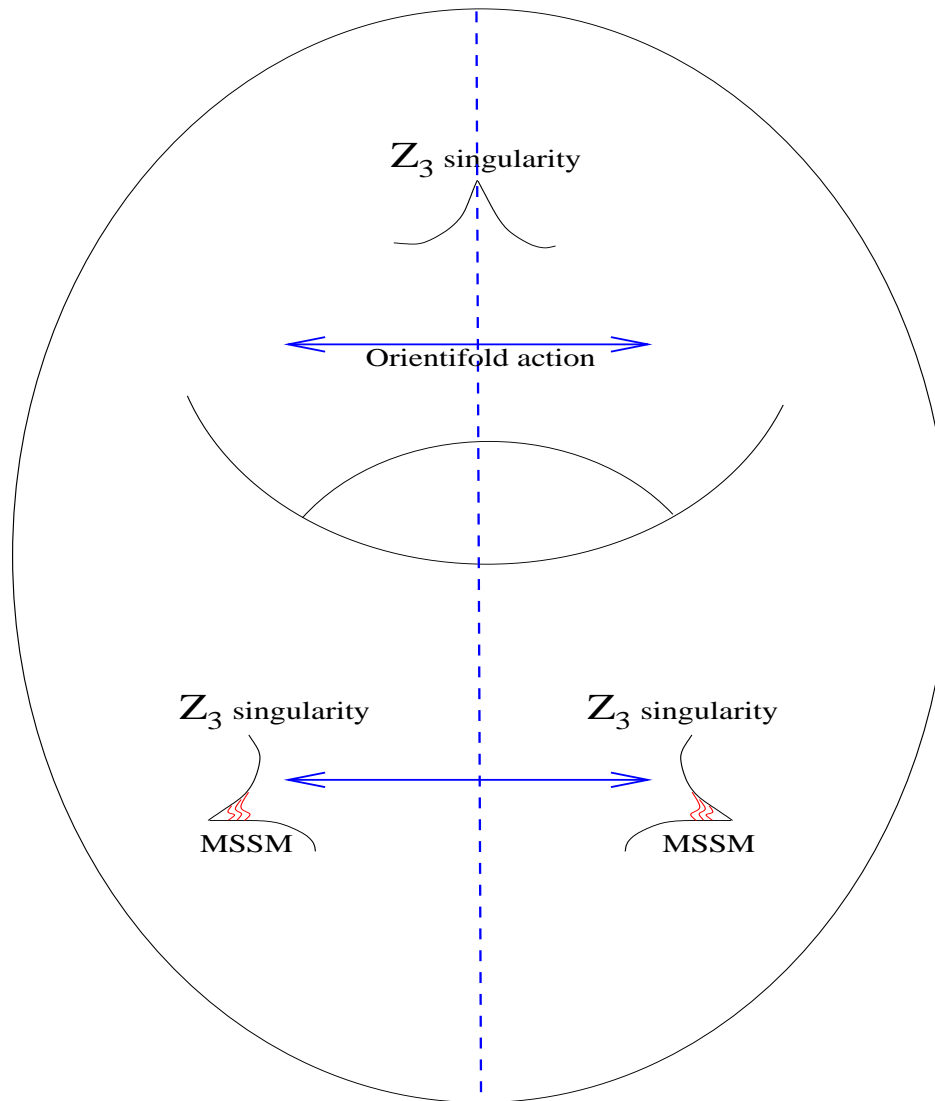
# Model Building

None of the above models are perfect. However

- It is easy to get quasi-realistic spectra and couplings.
- Three generations of chiral SM matter arises naturally (cf heterotic string!).
- Flavour symmetries for Yukawas arise naturally.
- Relatively few exotics are present.

Punchline: attractive SM-like models with relative ease.

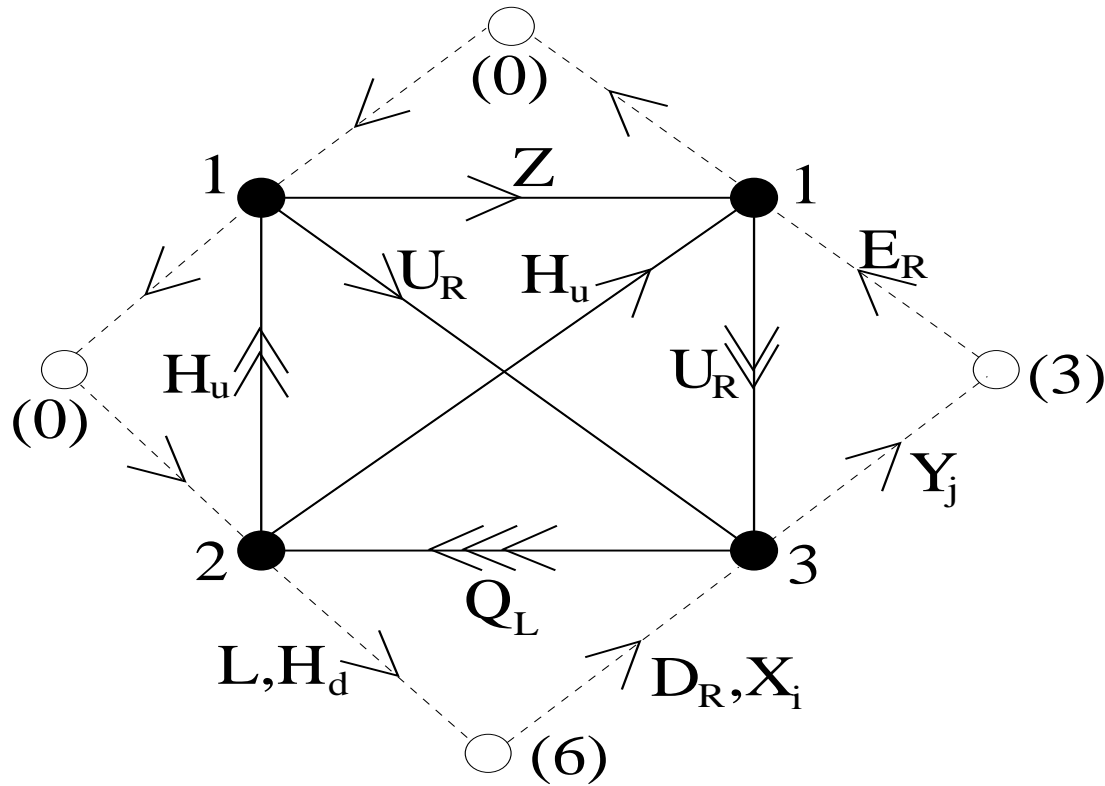
# Model Building





# Hyper-Weak Gauge Groups

A Standard-Model like spectrum for  $dP_1$ :



# Hyper-Weak Gauge Groups

- In LARGE volume models, the Standard Model is a local construction and branes only wrap small cycles.
- There are also bulk cycles associated to the overall volume. These have cycle size

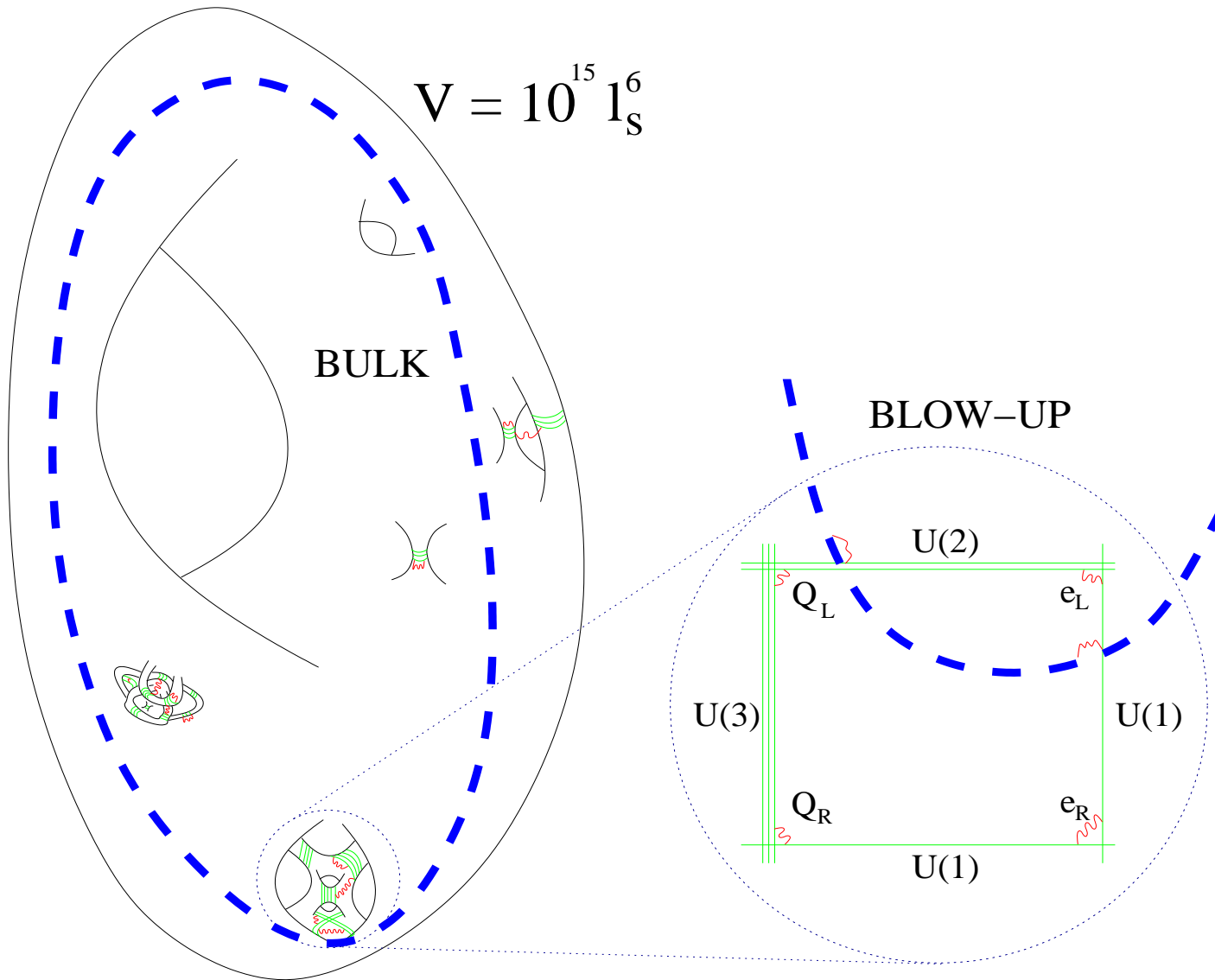
$$\tau_b \sim \mathcal{V}^{2/3} \sim 10^{10}.$$

- There is no reason not to have D7 branes wrapping these cycles!
- The gauge coupling for such branes is

$$\frac{g^2}{4\pi} = \frac{1}{\tau_b}.$$

with  $g \sim 10^{-4}$ .

# Hyper-Weak Gauge Groups



# Hyper-Weak Gauge Groups

In LARGE volume models it is a generic expectation that there will exist additional gauge groups with very weak coupling

$$\alpha^{-1} \sim 10^9.$$

Two phenomenological questions to ask:

1. How heavy is the hyper-weak  $Z'$  gauge boson?
2. How does Standard Model matter couple to the hyper-weak force?

# Hyper-Weak Gauge Groups

Hyper-weak  $Z'$  may get a mass by a Higgs mechanism in either visible or hidden sectors.

- If hyperweak gauge group is broken by weak-scale vevs,

$$M_{Z'} \sim gv \sim 10^{-4} \times 10 \rightarrow 100\text{GeV} \sim 1 \rightarrow 10\text{MeV}.$$

- If hyperweak gauge group is broken by chiral condensate  $\langle \bar{q}q \rangle \sim \Lambda_{QCD}^3$ ,

$$M_{Z'} \sim gv \sim 10^{-4} \times 100\text{MeV} \sim 10\text{keV}.$$

- If hyperweak gauge group is broken by hidden sector physics  $\langle \phi_{hid} \rangle \sim v$ ,  $M_{Z'} \sim gv \sim 10^{-4} \times v$ .

# Hyper-Weak Gauge Groups

- There may also be kinetic mixing between the new gauge boson and the photon (cf  $Z/\gamma$  mixing).
- If the new gauge boson is light, the mixing allows the new boson to couple to electromagnetic currents:

$$\mathcal{L}_{int} = \frac{(\bar{\psi}\gamma^\mu\psi)A_\mu}{\sqrt{1-\lambda^2}} - \frac{(\bar{\psi}\gamma^\mu\psi)\lambda Z'_\mu}{\sqrt{1-\lambda^2}}$$

where  $\lambda$  is the mixing parameter.

- This can give milli-charged fermions under the new gauge boson  $Z'$ .

# Hyper-Weak Gauge Groups

Bounds on an MeV-scale gauge boson: axial and vector couplings  $g_V(\bar{\psi}\gamma^\mu\psi)Z'_\mu$  or  $g_A(\bar{\psi}\gamma^5\gamma^\mu\psi)Z'_\mu$ .

Coupling	Bound	Experimental measurement
$g_e^V$	$10^{-4}m_U$	$g_e - 2$
$g_e^A$	$5 \cdot 10^{-5}m_U$	$g_e - 2$
$g_\mu^V$	$10^{-3}$	$g_\mu - 2$
$g_\mu^A$	$5 \cdot 10^{-6}m_U$	$g_\mu - 2$
$ g_e g_\nu $	$10^{-11}m_U^2$	$\nu - e$ scattering
$g_{c(b)}^A$	$10^{-6}m_U$	$B(\psi(\Upsilon) \rightarrow \gamma + \text{invisible})$
$ g_e^A g_q^V $	$10^{-14}m_U^2$	atomic parity violation

(Based on Fayet hep-ph/0702176)

# Conclusions

- Moduli stabilisation is an important problem in string theory.
- LARGE volume models are an attractive method of moduli stabilisation.
- These models have interesting consequences for
  1. Low-energy supersymmetry
  2. QCD axions in the allowed window
  3. Model-building using branes at singularities
  4. Possible new hyper-weak gauge groups (e.g. gauged  $(B - L)$ ,  $\alpha_{B-L} \lesssim 10^{-10}$ ).