

LARGE Volume Models and Superstring Cosmophysics

Joseph Conlon, Oxford University

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Plan of these two lectures:

1. The LARGE volume scenario
 - ▶ Construction
 - ▶ Moduli spectrum
 - ▶ Structure of susy breaking
2. Applications to cosmology

Why *String* Cosmophysics?

String theory is a theory of quantum gravity whose natural scale is $M_P \sim 2.4 \times 10^{18} \text{GeV}$.

Interesting cosmological scales

- ▶ Inflation: $V_{inf} \lesssim (10^{16} \text{GeV})^4$
- ▶ ...
- ▶ QCD phase transition: $V \sim (200 \text{MeV})^4$
- ▶ BBN: $V_{BBN} \sim (1 \text{MeV})^4$
- ▶ Late-time acceleration: $V_\Lambda \sim (10^{-3} \text{eV})^4$

All are $\ll M_P$.

Why care about string theory?

Why String Cosmophysics?

String theory is a ten dimensional theory and we need to compactify.

The geometric parameters of the extra dimensions turn into 4-dimensional scalar fields called *moduli*.

These moduli are ubiquitous in string compactifications.

- ▶ Moduli are good inflaton candidates. Unstabilised, they destroy candidate inflationary models.
- ▶ Moduli last. They are gravitationally coupled and dominate the energy density.
- ▶ Moduli dynamics break supersymmetry.

Why String Cosmophysics: Inflation

Inflation requires a flat potential with $\eta \sim \frac{M_P^2 V''}{V} \ll 1$ along the inflaton direction and no runaways.

Flatness of inflation requires control of Planck-suppressed operators:

$$V = V_0 \left(1 + \frac{\phi^2}{M_P^2} + \dots \right)$$

would generate $\mathcal{O}(1)$ correction to η .

Decompactification moduli (dilaton and volume) couple to everything.

Unstabilised, these decompactify inflaton potential.

Why String Cosmophysics: Moduli Lifetimes

Moduli are gravitationally coupled and are long-lived:

$$\Gamma \sim \frac{1}{4\pi} \frac{m_\phi^3}{M_P^2} \sim \left(\frac{m_\phi}{30\text{TeV}} \right)^{-3} 1\text{sec}$$

$$T_{reheat} \sim (1\text{MeV}) \left(\frac{m_\phi}{30\text{TeV}} \right)^{3/2}$$

Moduli oscillate after inflation and redshift as matter.

They decay late with low reheating temperatures, and can forbid leptogenesis, thermal WIMP dark matter, electroweak baryogenesis....

Why String Cosmophysics?

If string theory is true:

- ▶ You may be able to study the initial singularity without worrying about moduli
- ▶ You cannot study the post-inflationary universe without worrying about moduli

In particular I want to emphasise that conventional moduli dynamics often render impossible many ideas such as thermal leptogenesis, electroweak baryogenesis, thermal WIMP dark matter, Big Bang Nucleosynthesis.....

If our universe involves string theory, we need to study moduli.

To study moduli we need to study moduli stabilisation.

Moduli Stabilisation

Moduli stabilisation is about creating a potential for moduli with a stable minimum.

Nature is hierarchical, and interesting moduli stabilisation scenarios generate hierarchies.

I am going to focus on the LARGE volume scenario in IIB flux compactifications.

By breaking supersymmetry and stabilising the volume at exponentially large values, this gives good control and attractive phenomenology.

LARGE Volume Models

Consider IIB flux compactifications.

The leading order 4-dimensional supergravity theory is

$$K = -2 \ln(\mathcal{V}) - \ln \left(i \int \Omega \wedge \bar{\Omega} \right) - \ln(S + \bar{S}),$$
$$W = \int G_3 \wedge \Omega.$$

This fixes dilaton and complex structure but is no-scale with respect to the Kähler moduli.

No-scale models have

- ▶ Vanishing cosmological constant
- ▶ Broken supersymmetry
- ▶ Unfixed flat directions

- ▶ The effective supergravity theory is

$$K = -2\ln(\mathcal{V}) - \ln\left(i \int \Omega \wedge \bar{\Omega}\right) - \ln(S + \bar{S})$$

$$W = \int (F_3 + iSH_3) \wedge \Omega \equiv \int G_3 \wedge \Omega.$$

- ▶ This stabilises the dilaton and complex structure moduli.

$$D_S W = D_U W = 0.$$

$$W = \int G_3 \wedge \Omega = W_0.$$

The theory has an important **no-scale** property.

$$\hat{K} = -2 \ln(\mathcal{V}(T + \bar{T})) - \ln\left(i \int \Omega \wedge \bar{\Omega}(U)\right) - \ln(S + \bar{S}),$$

$$W = \int G_3 \wedge \Omega(S, U).$$

$$V = e^{\hat{K}} \left(\sum_{U,S} \hat{K}^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} + \sum_T \hat{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right)$$

$$= e^{\hat{K}} \left(\sum_{U,S} \hat{K}^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} \right) = 0.$$

$$\begin{aligned}\hat{K} &= -2 \ln (\mathcal{V}(T_i + \bar{T}_i)), \\ W &= W_0. \\ V &= e^{\hat{K}} \left(\sum_T \hat{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right) \\ &= 0\end{aligned}$$

No-scale model :

- ▶ vanishing vacuum energy
- ▶ broken susy
- ▶ T unstabilised

No-scale is broken perturbatively and non-pertubatively.

$$\hat{K} = -2 \ln(\mathcal{V}) - \ln \left(i \int \Omega \wedge \bar{\Omega} \right) - \ln(S + \bar{S}),$$
$$W = \int G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i}.$$

Non-perturbative effects (D3-instantons / gaugino condensation) allow the T -moduli to be stabilised by solving $D_T W = 0$.

For consistency, this requires

$$W_0 = \left\langle \int G_3 \wedge \Omega \right\rangle \ll 1.$$

$$\begin{aligned}\hat{K} &= -2 \ln(\mathcal{V}), \\ W &= W_0 + \sum_i A_i e^{-a_i T_i}.\end{aligned}$$

Solving $D_T W = \partial_T W + (\partial_T K)W = 0$ gives

$$\text{Re}(T) \sim \frac{1}{a} \ln(W_0)$$

For $\text{Re}(T)$ to be large, W_0 must be *enormously* small.

Going beyond no-scale the appropriate 4-dimensional supergravity theory is

$$K = -2 \ln \left(\mathcal{V} + \frac{\xi}{g_s^{3/2}} \right) - \ln \left(i \int \Omega \wedge \bar{\Omega} \right) - \ln (S + \bar{S}),$$
$$W = \int G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i}.$$

Key ingredients are:

- (1) the inclusion of stringy α' corrections to the Kähler potential
- (2) nonperturbative instanton corrections in the superpotential.

LARGE Volume Models

The simplest model (the Calabi-Yau $\mathbb{P}^4_{[1,1,1,6,9]}$) has two moduli and a 'Swiss-cheese' structure:

$$\mathcal{V} = \left(\tau_b^{3/2} - \tau_s^{3/2} \right).$$

Computing the moduli scalar potential, we get for $\mathcal{V} \gg 1$,

$$V = \frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s \tau_s}}{\mathcal{V}} - \frac{a_s |A_s W| \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{\xi |W|^2}{g_s^{3/2} \mathcal{V}^3}.$$

The minimum of this potential can be found analytically.

Moduli Stabilisation: LARGE Volume

$$V = \underbrace{\frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s \tau_s}}{\mathcal{V}} - \frac{a_s |A_s W_0| \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2}}_{\text{Integrate out heavy mode } \tau_s} + \frac{\xi |W_0|^2}{g_s^{3/2} \mathcal{V}^3}.$$

$$V = -\frac{|W_0|^2 (\ln \mathcal{V})^{3/2}}{\mathcal{V}^3} + \frac{\xi |W_0|^2}{g_s^{3/2} \mathcal{V}^3}.$$

A minimum exists at

$$\mathcal{V} \sim |W_0| e^{a_s \tau_s}, \quad \tau_s \sim \frac{\xi^{2/3}}{g_s}.$$

This minimum is **non-supersymmetric AdS** and at **exponentially large volume**.

LARGE Volume Models

The locus of the minimum satisfies

$$\mathcal{V} \sim |W|e^{c/g_s}, \quad \tau_s \sim \ln \mathcal{V}.$$

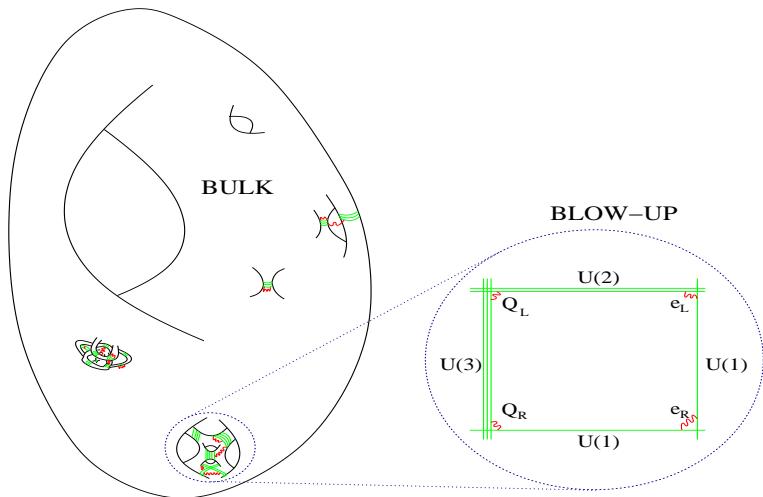
The minimum is at **exponentially large volume** and **non-supersymmetric**.

The large volume lowers the string scale and supersymmetry scale through

$$m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}, \quad m_{3/2} \sim \frac{M_P}{\mathcal{V}}.$$

An appropriate choice of volume will generate TeV scale soft terms and allow a supersymmetric solution of the hierarchy problem.

LARGE Volume Models



Question: LVS uses an α' correction to the effective action.

If some α' corrections are important, won't all will be?

Truncation is self-consistent because minimum exists at exponentially large volumes.

The inverse volume is the expansion parameter and so it is consistent to only include the leading α' corections.

Moduli Stabilisation: LARGE Volume

Higher α' corrections are suppressed by more powers of volume.

Example:

$$\begin{aligned} \int d^{10}x \sqrt{g} G_3^2 \mathcal{R}^3 & : \int d^{10}x \sqrt{g} \mathcal{R}^4 \\ \int d^4x \sqrt{g_4} \left(\int d^6x \sqrt{g_6} G_3^2 \mathcal{R}^3 \right) & : \int d^4x \sqrt{g_4} \left(\int d^6x \sqrt{g_6} \mathcal{R}^4 \right) \\ \int d^4x \sqrt{g_4} (\mathcal{V} \times \mathcal{V}^{-1} \times \mathcal{V}^{-1}) & : \int d^4x \sqrt{g_4} (\mathcal{V} \times \mathcal{V}^{-4/3}) \\ \int d^4x \sqrt{g_4} (\mathcal{V}^{-1}) & : \int d^4x \sqrt{g_4} (\mathcal{V}^{-1/3}) \end{aligned}$$

Moduli Stabilisation: LARGE Volume

Loop corrections are also suppressed by more powers of volume: there exists an 'extended no scale structure'

$$\begin{aligned}W &= W_0, \\K_{full} &= K_{tree} + K_{loop} + K_{\alpha'} \\&= -3 \ln(T + \bar{T}) + \underbrace{\frac{c_1}{(T + \bar{T})(S + \bar{S})}}_{loop} + \underbrace{\frac{c_2(S + \bar{S})^{3/2}}{(T + \bar{T})^{3/2}}}_{\alpha'}.\end{aligned}$$
$$\begin{aligned}V_{full} &= V_{tree} + V_{loop} + V_{\alpha'} \\&= \underbrace{0}_{tree} + \underbrace{\frac{c_2(S + \bar{S})^{3/2}}{(T + \bar{T})^{3/2}}}_{\alpha'} + \underbrace{\frac{c_1}{(S + \bar{S})(T + \bar{T})^2}}_{loop}\end{aligned}$$

Moduli Stabilisation: LARGE Volume

The mass scales present are (for $\mathcal{V} \sim 3 \times 10^7 l_s^6$)

Planck scale: $M_P = 2.4 \times 10^{18} \text{ GeV.}$

String scale: $M_S \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{15} \text{ GeV.}$

KK scale $M_{KK} \sim \frac{M_P}{\mathcal{V}^{2/3}} \sim 10^{14} \text{ GeV.}$

Gravitino mass $m_{3/2} \sim \frac{M_P}{\mathcal{V}} \sim 10^{11} \text{ GeV.}$

Small modulus $m_{\tau_s} \sim m_{3/2} \ln \left(\frac{M_P}{m_{3/2}} \right) \sim 10^{12} \text{ GeV.}$

Complex structure moduli $m_U \sim m_{3/2} \sim 10^{11} \text{ GeV.}$

Volume modulus $m_{T_b} \sim \frac{M_P}{\mathcal{V}^{3/2}} \sim 4 \times 10^6 \text{ GeV.}$

M_{soft}

To be determined....

The Matter Sector

Where should the matter live?

An important feature of the LARGE volume models is the distinction between small and large cycles.

There is a large cycle τ_b corresponding to the overall volume modulus with $\tau_b \sim \mathcal{V}^{2/3}$ and small cycles $\tau_{s,i}$ with volumes close to the string scale.

Gauge theories are realised by D-branes wrapping cycles.

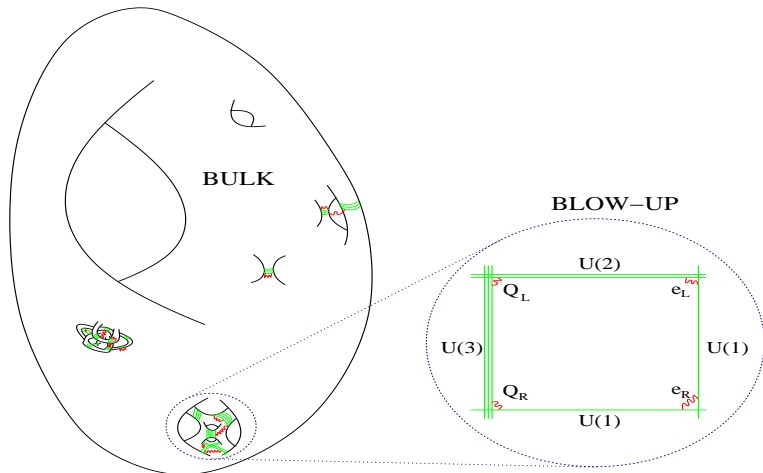
For a brane wrapping a cycle τ_i

$$\alpha_a^{-1} = \tau_i$$

As $\alpha_{SM} \sim 25$, this implies the Standard Model cannot be realised on the large cycle: it must be realised locally via branes at singularities or wrapping small cycles.

Moduli Stabilisation: LARGE Volume

SM on local cycle:



For branes at singularities, blow-up moduli are FI term of U(1) gauge theories.

$$V_D \sim \left(\sum_i q |\phi_i|^2 + t_{blowup} \right)^2$$

For vanishing matter field vevs the blow-up fields are stabilised at the singularity.

Moduli Stabilisation: LARGE Volume

The details of the matter sector depend on the brane construction.

There are various ways to do this: intersecting branes, branes at singularities, F-theory...

The exact details of matter spectrum and constructions will not concern us particularly here: we will assume that the MSSM or an appropriate extension can be realised.

In any string construction an important general question is the scale of the soft terms.

The standard moduli cosmology and moduli problems follow from:

$$m_{\text{moduli}} \sim m_{3/2} \sim m_{\text{soft}} \sim 1\text{TeV}$$

and

$$V_{\text{typical}} \sim m_{3/2}^2 M_P^2$$

In LVS the second relation is false and the first subtle.

LVS has a very different structure of supersymmetry breaking than KKLT.

In KKLT, AdS minimum is supersymmetric and susy breaking occurs entirely through antibrane/uplifting.

In LVS, susy breaking occurs in AdS minimum and is inherited from pure flux compactifications.

LVS inherits *no-scale structure* of susy breaking.

Recall the general formalism for computing soft terms:

$$W = \hat{W}(\Phi) + \mu(\Phi)H_1H_2 + \frac{1}{6}Y_{\alpha\beta\gamma}(\Phi)C^\alpha C^\beta C^\gamma + \dots,$$

$$K = \hat{K}(\Phi, \bar{\Phi}) + \tilde{K}_{\alpha\bar{\beta}}(\Phi, \bar{\Phi})C^\alpha C^{\bar{\beta}} + [ZH_1H_2 + h.c.] + \dots,$$

$$f_a = f_a(\Phi).$$

Φ : Moduli fields

C^α : Matter fields: quarks, leptons....

Couplings to visible matter determine the soft terms.

LARGE Volume: Soft Terms

Soft scalar masses $m_{\tilde{ij}}^2$ and trilinears $A_{\alpha\beta\gamma}$ are given by

$$\begin{aligned}\tilde{m}_{\alpha\bar{\beta}}^2 &= (m_{3/2}^2 + V_0)\tilde{K}_{\alpha\bar{\beta}} \\ &\quad - \bar{F}^{\bar{m}}F^n \left(\partial_{\bar{m}}\partial_n\tilde{K}_{\alpha\bar{\beta}} - (\partial_{\bar{m}}\tilde{K}_{\alpha\bar{\gamma}})\tilde{K}^{\bar{\gamma}\delta}(\partial_n\tilde{K}_{\delta\bar{\beta}}) \right) \\ A'_{\alpha\beta\gamma} &= e^{\hat{K}/2}F^m \left[\hat{K}_m Y_{\alpha\beta\gamma} + \partial_m Y_{\alpha\beta\gamma} \right. \\ &\quad \left. - \left((\partial_m\tilde{K}_{\alpha\bar{\rho}})\tilde{K}^{\bar{\rho}\delta}Y_{\delta\beta\gamma} + (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma) \right) \right]. \\ M_a &= \frac{F^m\partial_m f_a}{\text{Re}(f_a)}\end{aligned}$$

Need **moduli F-terms** and **matter metrics \tilde{K}** and **gauge kinetic function f_a**

LARGE Volume: Soft Terms

The moduli F-terms have a hierarchical structure:

$$\text{Bulk volume modulus } |F^{\tau_b}| \sim m_{3/2} M_P \sim \left(\frac{M_P^2}{\mathcal{V}} \right)$$

$$\text{Small Kähler modulus } |F^{\tau_s}| \sim m_{3/2} M_{string} \sim \left(\frac{M_P^2}{\mathcal{V}^{3/2}} \right)$$

$$\text{Dilaton modulus } |F^S| \sim m_{3/2}^2 \sim \left(\frac{M_P^2}{\mathcal{V}^2} \right)$$

$$\text{Complex structure modulus } |F^U| \sim m_{3/2}^2 \sim \left(\frac{M_P^2}{\mathcal{V}^2} \right)$$

$$\text{Blow-up moduli } |F^i| \sim m_{3/2}^2 \sim \left(\frac{M_P^2}{\mathcal{V}^2} \right)$$

The dominant F-term belongs to the overall volume modulus

For a local model at a singularity,

$$f_a = S + \lambda_i T_i$$

where T_i are the singularity blow-up moduli.

For a local model on an 'instanton cycle' T_s

$$f_a = T_s + \lambda_a S$$

where T_s is the small cycle modulus.

This second case is hard to realise due to instanton/SM chirality tension.

We assume MSSM realised by branes at singularities.

The \mathcal{V} dependence of matter metrics $\tilde{K}_{\alpha\bar{\beta}}$ can be computed if we know where the Standard Model is realised.

The physical Yukawa couplings are

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{\tilde{K}_{\alpha\bar{\alpha}} \tilde{K}_{\beta\bar{\beta}} \tilde{K}_{\gamma\bar{\gamma}}}}$$

Geometry implies these couplings are local.

We know

- ▶ $Y_{\alpha\beta\gamma}$ does not depend on \mathcal{V} .
- ▶ The overall term $e^{\hat{K}/2} \sim \frac{1}{\mathcal{V}}$.

LARGE Volume: Soft Terms

The local interactions care only about the local geometry and decouple from the bulk volume.

Physical locality then implies the physical Yukawa couplings $\hat{Y}_{\alpha\beta\gamma}$ do not depend on the bulk volume.

For local fields C^α the relation

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{\tilde{K}_{\alpha\bar{\alpha}} \tilde{K}_{\beta\bar{\beta}} \tilde{K}_{\gamma\bar{\gamma}}}}$$

implies

$$\tilde{K}_{\alpha\bar{\alpha}} \sim \frac{1}{\mathcal{V}^{2/3}}.$$

LARGE Volume: Soft Terms

The fact that F^b is the dominant F-term and $\tilde{K}_{\alpha\bar{\alpha}} \sim \frac{1}{V^{2/3}}$ means that the leading susy breaking structure is essentially that of no-scale models:

$$W = W_0$$
$$K = -3 \ln(T_b + \bar{T}_b) + \frac{C\bar{C}}{(T_b + \bar{T}_b)}$$

The gauge kinetic function is independent of the volume modulus T_b .

No-scale has many remarkable properties and cancellations.

No-scale has many remarkable properties and cancellations.

1. Gaugino mass vanishes as f_a does not depend on volume modulus.
2. Anomaly-mediated gaugino mass also vanishes.

$$\begin{aligned} m_{1/2} &= -\frac{g^2}{16\pi^2} \left[(3T_G - T_R) m_{3/2} - (T_G - T_R) K_i F^i \right. \\ &\quad \left. - \frac{2T_R}{d_R} F^i \partial_i (\ln \det Z) + 2T_G F^I \partial_I \ln \left(\frac{1}{g_0^2} \right) \right] \\ &= 0. \end{aligned}$$

No-scale has many remarkable properties and cancellations.

1. Leading order scalar mass vanishes.

$$m_{\tilde{Q}}^2 = m_{3/2}^2 - F^i F^{\bar{j}} \partial_i \partial_{\bar{j}} \ln \tilde{K} = 0.$$

2. In that the calculation exists, anomaly mediated scalar masses also vanish.

The upshot is that soft terms are suppressed significantly below the gravitino mass scale.

This violates one 'genericity' assumption $m_{\text{soft}} \sim m_{3/2}$.

At what order do soft terms arise?

As

$$f_a = S + \lambda_a T_i,$$

gaugino masses are set by $F^S, F^{T_i} \sim \mathcal{V}^{-2}$.

These generate gaugino masses of

$$M_{gaugino} \sim \frac{M_P}{\mathcal{V}^2}$$

One potential exception to these cases is if field redefinitions occur at the singularity, which may lead to $m_{soft} \sim m_{3/2}$. The conditions for, and effects of, these are poorly understood at the moment.

LARGE Volume: Soft Terms

Scalar masses are more subtle.

Cancellations in scalar mass formulae persist to the extent that physical Yukawas are independent of the bulk volume.

Calculation pushes scalar masses to

$$m_{\text{scalar}}^2 \lesssim \frac{M_P^2}{\mathcal{V}^3}$$

A full analysis of whether a further suppression of soft terms exists requires α' corrections to the matter kinetic terms that have not been computed.

We shall **assume** the most appealing case with $m_{\tilde{Q}} \sim M_a \sim \frac{M_P}{\mathcal{V}}$.

Moduli Stabilisation: LARGE Volume

The mass scales present are then (for $\mathcal{V} \sim 3 \times 10^7 l_s^6$)

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Volume modulus $m_{\tau_b} \sim \frac{M_P}{\mathcal{V}^{3/2}} \sim 4 \times 10^6 \text{ GeV.}$

Soft terms $M_{\text{soft}} \sim \frac{M_P}{\mathcal{V}^2} \sim 10^3 \text{ GeV.}$

Tomorrow we will examine cosmological consequences of this moduli spectrum.

Summary of LARGE Volume Models

- ▶ The stabilised volume is naturally exponentially large.
- ▶ The Calabi-Yau has a ‘Swiss cheese’ structure.
- ▶ This lowers the gravitino mass through

$$m_{3/2} = \frac{M_P W_0}{\mathcal{V}}.$$

- ▶ The minimum breaks supersymmetry at a hierarchically low scale.
- ▶ Soft terms are highly suppressed compared to the gravitino mass.
- ▶ The moduli have a distinctive and well-defined spectrum with interesting cosmological applications...