String Phenomenology and Moduli Stabilisation

Joseph P. Conlon (Cavendish Laboratory & DAMTP, Cambridge)

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Talk Structure

- Maladies of Particle Physics
- String Phenomenology
- Moduli Stabilisation: Fluxes and KKLT
- Moduli Stabilisation: LARGE Volume Models

Maladies of Particle Physics

Nature likes hierarchies:

- The Planck scale, $M_P = 2.4 \times 10^{18}$GeV.
- The inflationary scale, $M \sim 10^{13} \to 10^{16}$GeV.
- The axion scale, 10^9GeV $\lesssim f_a \lesssim 10^{12}$GeV
- The weak scale : $M_W \sim 100$GeV
- The fermion masses, $m_e \sim 0.5$MeV $\to m_t \sim 170$GeV.
- The neutrino mass scale, 0.05eV $\lesssim m_\nu \lesssim 0.3$eV.
- The cosmological constant, $\Lambda \sim (10^{-3}\text{eV})^4$

These demand an explanation!

Maladies of Particle Physics

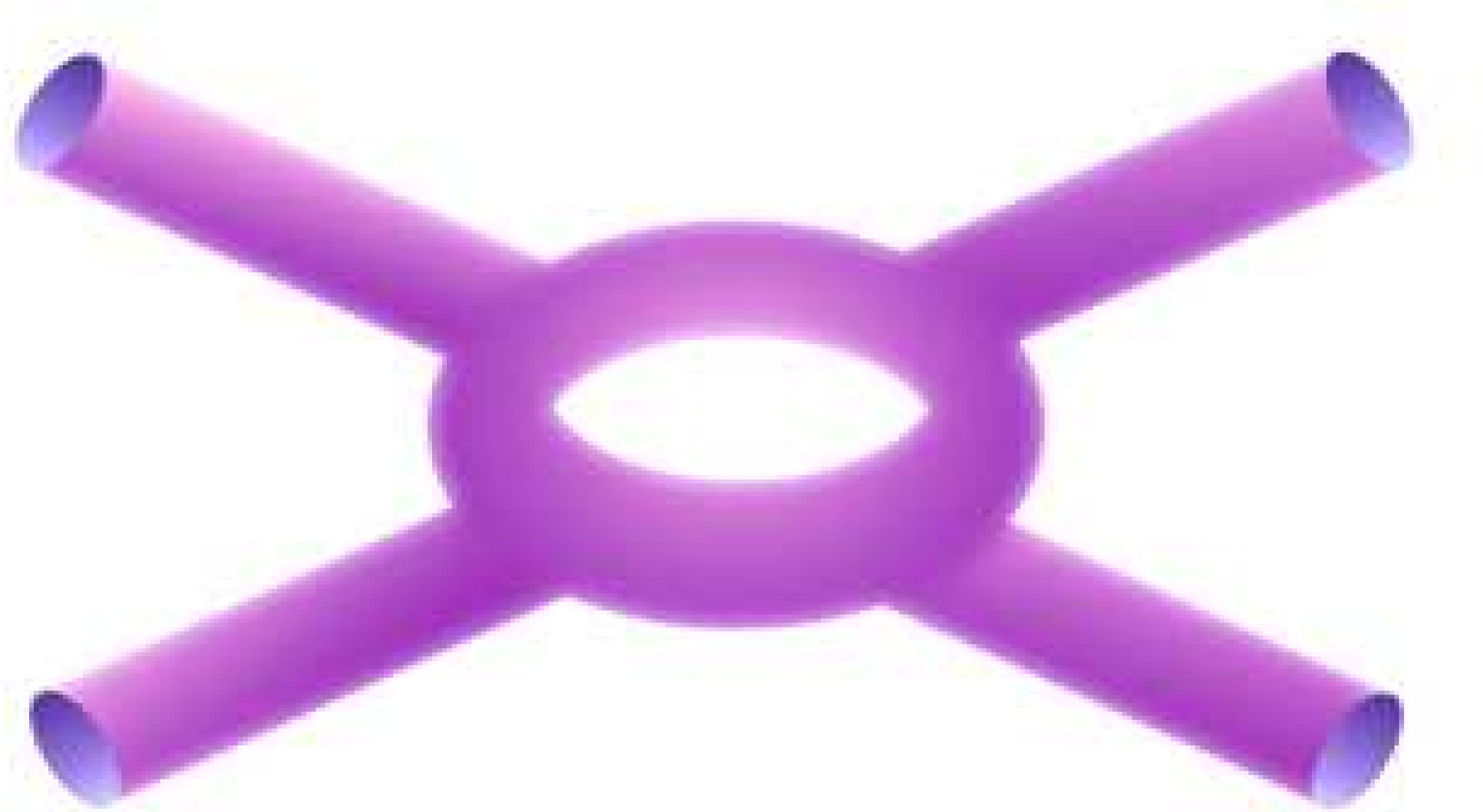
We can also ask

1. Why is the Standard Model gauge group $SU(3) \times SU(2) \times U(1)$?
2. Why are there three generations?
3. What sets α_{EW}, α_{strong}, α_Y?
4. Why four dimensions?
5. Where does the flavour structure come from?

None of these questions can be answered within the Standard Model.

To address them, a more fundamental approach is needed.

String Phenomenology



String Phenomenology

String theory represents a candidate fundamental theory.

String theory is famous as a theory of quantum gravity.

It has also given important insights into algebraic geometry, black holes and quantum field theory.

Can string theory do the same for particle physics?

If nature is stringy, string theory should give insights into all the fundamental problems mentioned previously.

String phenomenology aims to use string theory to address these fundamental problems of particle physics.

String Phenomenology

The overall aim of these two talks is to describe the LARGE volume models:

- an intermediate string scale $m_s \sim 10^{11}$GeV
- stabilised exponentially large extra dimensions ($\mathcal{V} \sim 10^{15} l_s^6$).

and the insight they provide into the axionic, weak, neutrino and flavour hierarchies.

Different hierarchies come as different powers of the (LARGE) volume.

Talk structure

My first talk will describe moduli stabilisation:

What is it? Why is it necessary? How to do it - and where do the LARGE volume models come from?

For reasons of time I will focus on IIB string compactifications.

The second talk (tomorrow) will describe phenomenological applications of LARGE volume models to different areas of particle physics.

Please ask questions!

String Phenomenology

- String theory lives in ten dimensions.
- The world is four-dimensional, so ten-dimensional string theory needs to be compactified.
- To preserve $\mathcal{N} = 1$ supersymmetry, we compactify on a six-dimensional Calabi-Yau manifold - Ricci-flat and Kähler.
- All scales, matter, particle spectra and couplings come from the geometry of the extra dimensions.

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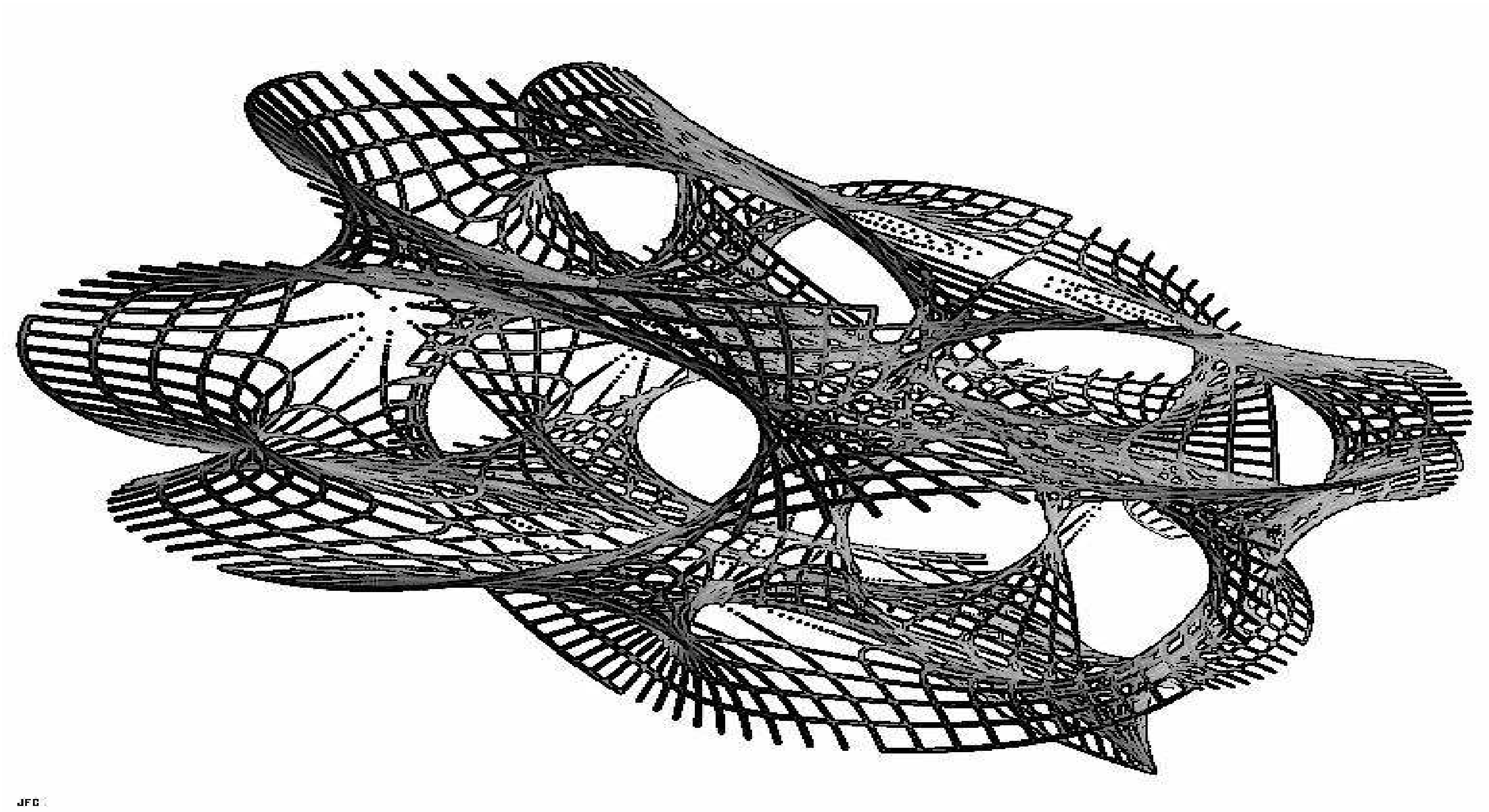
String Phenomenology



String Phenomenology

- The spectrum of light particles is determined by higher-dimensional topology.
- There are very many Calabi-Yaus - at least 30,000 distinct pairs of Hodge numbers and 240,000,000 different toric Calabi-Yaus.
- However many compactification properties are model-independent.
- In particular, string compactifications generically produce many uncharged scalar particles.
- These moduli parametrise the size and shape of the extra dimensions (Kähler and complex moduli).

String Phenomenology

Recall:

- A Calabi-Yau manifold is a Kähler manifold with vanishing first Chern class.
- The Ricci-flat Kähler metric is uniquely determined by the complex structure and the Kähler class.
- The geometric moduli are $h^{1,1}$ Kähler moduli (T_i) and $h^{2,1}$ complex structure moduli (U_j).
- The moduli values entirely specify the Calabi-Yau metric.
- In string theory the Kähler moduli are complexified: $T_i = \tau_i + ic_i$.
- There is also the dilaton modulus S(the string coupling).

String Phenomenology

In IIB, the moduli definitions are

- Kähler moduli:

  $T_i = \tau_i + i c_i$, where $\tau_i = \int_{\Sigma_4} \sqrt{g}$ and $c_i = \int_{\Sigma_4} C_4$.

  T_i are complexified 4-cycle volumes.

- Complex structure moduli:

  U_i are the geometric complex structure moduli of the Calabi-Yau.

- Dilaton modulus:

  $S = \frac{1}{g_s} + i c_0$ combines the string coupling and the RR 0-form.

String Phenomenology

- The moduli determine the geometry of the Calabi-Yau
- All scales, interactions and couplings come from the geometry of the extra dimensions.

  example:

$$M_{string} = \frac{g_s M_P}{\sqrt{\mathcal{V}}}$$

- To say anything about particle physics scales, we need to stabilise the moduli.
- Moduli stabilisation is a prerequisite to string phenomenology.

String Phenomenology

- Moduli are naively massless scalar fields which may take large classical vevs.
- They are uncharged and interact gravitationally.
- Such massless scalars generate long-range, unphysical fifth forces.

  To avoid fifth forces moduli must be given masses.
- It is essential to generate potentials for moduli and stabilise them.
- The LARGE volume models are an appealing scenario of moduli stabilisation.

Moduli Stabilisation: Fluxes

- Stabilising moduli requires a source of vacuum energy that depends on the moduli.
- Fluxes are quantised,

$$\int H_3 = 2\pi(2\pi\sqrt{\alpha'})^2 n, \quad n \in \mathbb{Z}.$$

- Fluxes carry an energy density

$$E_{flux} = \int \sqrt{g} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma}$$

The energy density depends on the geometry of the quantisation cycles.

Moduli Stabilisation: Fluxes

We work in (orientifolds of) type IIB string theory with D3 and D7 branes.

The IIB field content includes 3-form field strengths $F_3 = dC_2$, $H_3 = dB_2$ from RR and NS-NS sector.

$$G_3 = F_3 + SH_3.$$

The extra dimensions contain

1. D3/D7 D-branes
2. O3/O7 orientifold branes
3. 3-form fluxes $G_3 = F_3 + SH_3$.

Moduli Stabilisation: Fluxes

Under these conditions it can be shown (GKP 2001) that the 10-D metric is

$$ds_{10}^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{MN} dy^M dy^N$$

The metric is *warped Calabi-Yau*.

The warp factor scales as

$$e^{2A(y)} \sim 1 + \frac{1}{\mathcal{V}^{2/3}}$$

and vanishes in the infinite volume limit.

The dilaton and complex structure moduli are fixed by the fluxes. The Kähler moduli are not fixed.

Moduli Stabilisation: Fluxes

- We want to work in 4-dimensional supergravity.
- The fluxes carry energy generating a potential for the moduli associated with these cycles.
- This energy is expressed through a superpotential

$$W = \int (F_3 + iSH_3) \wedge \Omega \equiv \int G_3 \wedge \Omega.$$

- This generates a potential for the dilaton and complex structure moduli.

Moduli Stabilisation: Fluxes

- The effective supergravity theory is

$$
\begin{aligned}
K &= -2\ln(\mathcal{V}) - \ln\left(i\int \Omega\wedge\bar{\Omega}\right) - \ln(S+\bar{S}) \\
W &= \int (F_3 + iSH_3)\wedge\Omega \equiv \int G_3\wedge\Omega.
\end{aligned}
$$

- This stabilises the dilaton and complex structure moduli.

$$D_S W = D_U W = 0.$$

$$W = \int G_3 \wedge \Omega = W_0.$$

Moduli Stabilisation: Fluxes

The theory has an important no-scale property.

$$
\begin{aligned}
\hat{K} &= -2\ln\left(\mathcal{V}(T+\bar{T})\right) - \ln\left(i\int \Omega \wedge \bar{\Omega}(U)\right) - \ln\left(S+\bar{S}\right), \\
W &= \int G_3 \wedge \Omega\,(S,U). \\
V &= e^{\hat{K}}\left(\sum_{U,S} \hat{K}^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}}\bar{W} + \sum_{T} \hat{K}^{i\bar{j}} D_i W D_{\bar{j}}\bar{W} - 3|W|^2\right) \\
&= e^{\hat{K}}\left(\sum_{U,S} \hat{K}^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}}\bar{W}\right) = 0.
\end{aligned}
$$

Moduli Stabilisation: Fluxes

$$
\begin{aligned}
\hat{K} &= -2\ln\left(\mathcal{V}(T_i + \bar{T}_i)\right), \\
W &= W_0\,. \\
V &= e^{\hat{K}}\left(\sum_T \hat{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2\right) \\
&= 0
\end{aligned}
$$

No-scale model :

- vanishing vacuum energy
- broken susy
- T unstabilised

No-scale is broken perturbatively and non-pertubatively.

Moduli Stabilisation: KKLT

$$\begin{aligned}\hat{K} &= -2\ln\left(\mathcal{V}\right) - \ln\left(i\int \Omega\wedge\bar{\Omega}\right) - \ln\left(S+\bar{S}\right), \\ W &= \int G_3\wedge\Omega + \sum_i A_i e^{-a_i T_i}.\end{aligned}$$

Non-perturbative effects (D3-instantons / gaugino condensation) allow the T-moduli to be stabilised by solving $D_T W = 0$.

For consistency, this requires

$$W_0 = \left\langle \int G_3 \wedge \Omega \right\rangle \ll 1.$$

Moduli Stabilisation: KKLT

$$\hat{K} = -2\ln(\mathcal{V}),$$
$$W = W_0 + \sum_i A_i e^{-a_i T_i}.$$

Solving $D_T W = \partial_T W + (\partial_T K)W = 0$ gives

$$\text{Re}(T) \sim \frac{1}{a}\ln(W_0)$$

For Re(T) to be large, W_0 must be *enormously* small.

Moduli Stabilisation: KKLT

KKLT stabilisation has three phenomenological problems:

1. No susy hierarchy: fluxes prefer $W_0 \sim 1$ and $m_{3/2} \gg 1$TeV.
2. Susy breaking not well controlled - depends entirely on uplifting.
3. α' expansion not well controlled - volume is small and there are large flux backreaction effects.

Moduli Stabilisation: LARGE Volume

$$\hat{K} = -2\ln\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right), \qquad \left(\xi = \frac{\chi(\mathcal{M})\zeta(3)}{2(2\pi)^3}\right)$$

$$W = W_0 + \sum_i A_i e^{-a_i T_i}.$$

- Include perturbative as well as non-perturbative corrections to the scalar potential.
- Add the leading α' corrections to the Kähler potential.
- These descend from the $\mathcal{R}^4$ term in 10 dimensions.
- This leads to dramatic changes in the large-volume vacuum structure.

Moduli Stabilisation: LARGE Volume

The simplest model $\mathbb{P}^4_{[1,1,1,6,9]}$ has two Kähler moduli.

$$\mathcal{V} = \left(\frac{T_b + \bar{T}_b}{2}\right)^{3/2} - \left(\frac{T_s + \bar{T}_s}{2}\right)^{3/2} \equiv \left(\tau_b^{3/2} - \tau_s^{3/2}\right).$$

If we compute the scalar potential, we get for $\mathcal{V} \gg 1$,

$$V = \frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s |A_s W_0| \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\xi |W_0|^2}{g_s^{3/2}\mathcal{V}^3}.$$

Moduli Stabilisation: LARGE Volume

$$V = \underbrace{\frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s |A_s W_0| \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2}}_{\text{Integrate out } \tau_s} + \frac{\xi |W_0|^2}{g_s^{3/2} \mathcal{V}^3}.$$

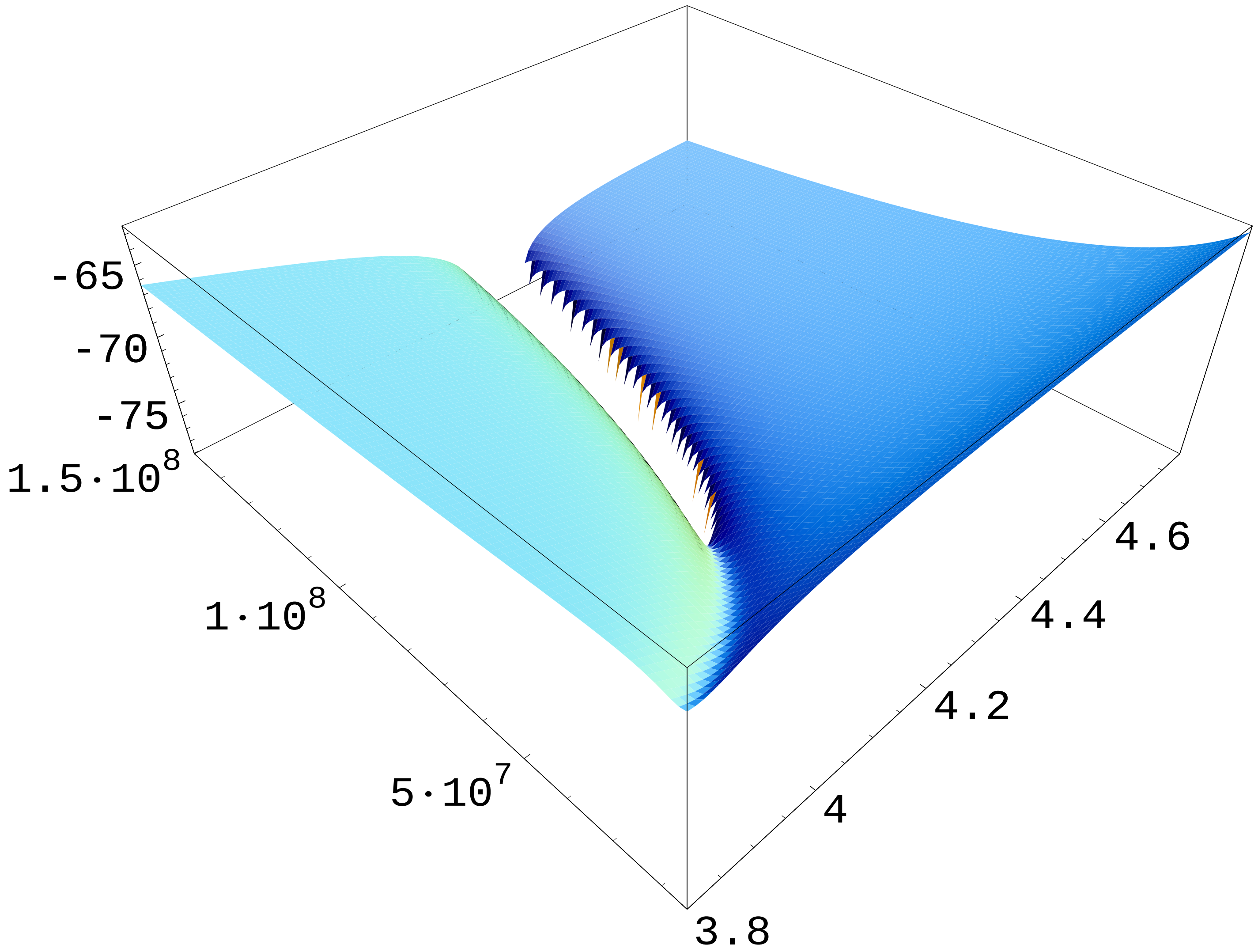
$$V = -\frac{|W_0|^2 \left(\ln \mathcal{V}\right)^{3/2}}{\mathcal{V}^3} + \frac{\xi |W_0|^2}{g_s^{3/2} \mathcal{V}^3}.$$

A minimum exists at

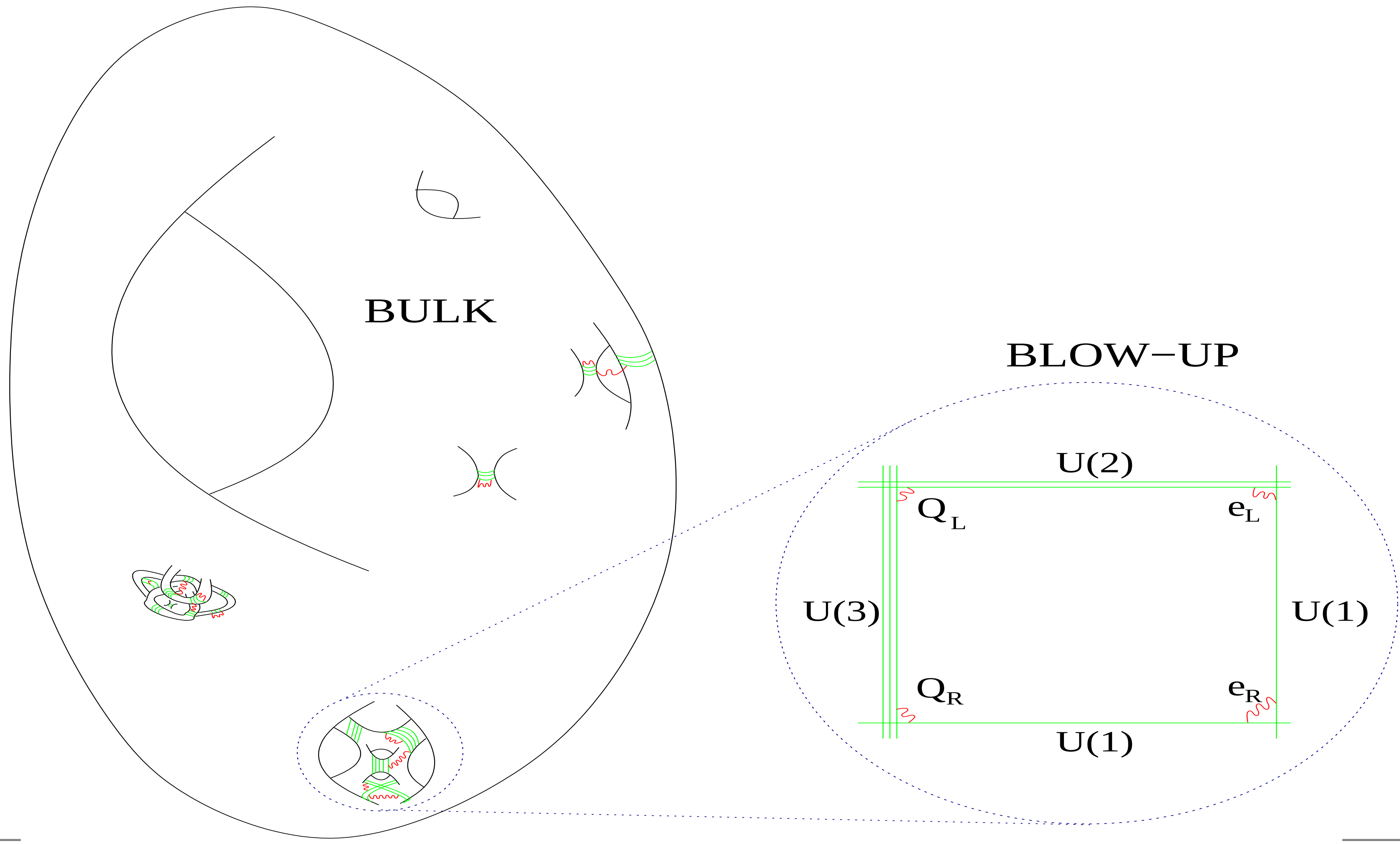
$$\mathcal{V} \sim |W_0| e^{a_s \tau_s}, \qquad \tau_s \sim \frac{\xi^{2/3}}{g_s}.$$

This minimum is non-supersymmetric AdS and at exponentially large volume.

Moduli Stabilisation: LARGE Volume



Moduli Stabilisation: LARGE Volume



Moduli Stabilisation: LARGE Volume

- The stabilised volume is exponentially large.
- The large volume lowers the string scale and gravitino mass through

$$m_s = \frac{M_P}{\sqrt{\mathcal{V}}}, \qquad m_{3/2} = \frac{M_P W_0}{\mathcal{V}}.$$

- To solve the gauge hierarchy problem, need $\mathcal{V} \sim 10^{15}$.
- D7-branes wrapped on small cycle carry the Standard Model: need $T_s \sim 20(2\pi\sqrt{\alpha'})^4$.
- The vacuum is pseudo no-scale and breaks susy...

Moduli Stabilisation: LARGE Volume

The locus of the minimum satisfies

$$\mathcal{V} \sim |W_0| e^{c/g_s}, \qquad \tau_s \sim \ln \mathcal{V}.$$

The minimum is non-supersymmetric AdS and at exponentially large volume.

The large volume lowers the string scale and gravitino mass through

$$m_s = \frac{M_P}{\sqrt{\mathcal{V}}}, \qquad m_{3/2} = \frac{M_P}{\mathcal{V}}.$$

Moduli Stabilisation: LARGE Volume

Higher α' corrections are suppressed by more powers of volume.

Example:

$$\begin{aligned}
\int d^{10}x\sqrt{g}\mathcal{G}_3^2\mathcal{R}^3 &: \int d^{10}x\sqrt{g}\mathcal{R}^4 \\
\int d^4x\sqrt{g_4}\left(\int d^6x\sqrt{g_6}\mathcal{G}_3^2\mathcal{R}^3\right) &: \int d^4x\sqrt{g_4}\left(\int d^6x\sqrt{g_6}\mathcal{R}^4\right) \\
\int d^4x\sqrt{g_4}\left(\mathcal{V}\times\mathcal{V}^{-1}\times\mathcal{V}^{-1}\right) &: \int d^4x\sqrt{g_4}\left(\mathcal{V}\times\mathcal{V}^{-4/3}\right) \\
\int d^4x\sqrt{g_4}\left(\mathcal{V}^{-1}\right) &: \int d^4x\sqrt{g_4}\left(\mathcal{V}^{-1/3}\right)
\end{aligned}$$

Moduli Stabilisation: LARGE Volume

Loop corrections are suppressed by more powers of volume: there exists an ‘extended no scale structure’

$$
\begin{aligned}
W &= W_0, \\
K_{full} &= K_{tree} + K_{loop} + K_{\alpha'} \\
&= -3\ln(T+\bar{T}) + \underbrace{\frac{c_1}{(T+\bar{T})(S+\bar{S})}}_{loop} + \underbrace{\frac{c_2(S+\bar{S})^{3/2}}{(T+\bar{T})^{3/2}}}_{\alpha'} . \\
V_{full} &= V_{tree} + V_{loop} + V_{\alpha'} \\
&= \underbrace{0}_{tree} + \underbrace{\frac{c_2(S+\bar{S})^{3/2}}{(T+\bar{T})^{3/2}}}_{\alpha'} + \underbrace{\frac{c_1}{(S+\bar{S})(T+\bar{T})^2}}_{loop}
\end{aligned}
$$

Moduli Stabilisation: LARGE Volume

This is consistent with the Coleman-Weinberg expectation:

$$\delta V_{loop} = \Lambda^4 \mathrm{STr}(M^0) + \Lambda^2 \mathrm{STr}(M^2) + \mathrm{STr}(M^4)\ln(\Lambda^2/M^2)$$

At very large volume $\mathcal{V} \gg 1$,

$$\mathrm{STr}(M^2) \sim m_{3/2}^2 \sim \frac{M_P^2}{\mathcal{V}^2}, \qquad \Lambda^2 \sim M_{KK}^2 \sim \frac{M_P^2}{\mathcal{V}^{4/3}}.$$

consistently giving

$$\delta V_{loop} \sim \frac{M_P^4}{\mathcal{V}^{10/3}}.$$

Moduli Stabilisation: LARGE Volume

The mass scales present are:

Planck scale:	$M_P = 2.4 \times 10^{18}$GeV.
String scale:	$M_S \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11}$GeV.
KK scale	$M_{KK} \sim \frac{M_P}{\mathcal{V}^{2/3}} \sim 10^9$GeV.
Gravitino mass	$m_{3/2} \sim \frac{M_P}{\mathcal{V}} \sim 30$TeV.
Small modulus	$m_{\tau_s} \sim m_{3/2} \ln\left(\frac{M_P}{m_{3/2}}\right) \sim 1000$TeV.
Complex structure moduli	$m_U \sim m_{3/2} \sim 30$TeV.
Soft terms	$m_{susy} \sim \frac{m_{3/2}}{\ln(M_P/m_{3/2})} \sim 1$TeV.
Volume modulus	$m_{\tau_b} \sim \frac{M_P}{\mathcal{V}^{3/2}} \sim 1$MeV.

Summary of LARGE Volume Models

- The stabilised volume is naturally exponentially large.
- The Calabi-Yau has a ‘Swiss cheese’ structure.
- This lowers the gravitino mass through

$$m_{3/2} = \frac{M_P W_0}{\mathcal{V}}.$$

- The minimum breaks supersymmetry at a hierarchically low scale.
- The LARGE volume is a powerful tool to generate various hierarchies.
- This will be the focus of the second lecture.