Dark Radiation and a $0.1 - 1$ keV Cosmic Axion Background

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COSMO 2013, Cambridge, 5th September 2013

Thanks to my collaborators

1208.3562 Michele Cicoli, JC, Fernando Quevedo
'Dark Radiation in LARGE Volume Models'

1304.1804 JC, David Marsh
'The Cosmophenomenology of Axionic Dark Radiation'

1305.3603 JC, David Marsh
'Searching for a 0.1-1 keV Cosmic Axion Background'

Stephen Angus, JC, David Marsh, Andrew Powell, Lukas Witkowski
'In preparation'

also see 1305.3562 Angus, JC, Haisch, Powell
'Loop Corrections to ΔN_{eff} in large volume models' (Angus talk today)

Talk Structure

1. Prelude: What is Dark Radiation?

2. Theoretical motivation for dark radiation: reheating and modulus decays

3. Experimental motivation for dark radiation: CMB and BBN

4. A 0.1 - 1 keV Cosmic Axion Background

5. Observing a Cosmic Axion Background and the Cluster Soft Excess

I
PRELUDE

Prelude

This is the age of precision cosmology.

- 80% of the non-relativistic matter content of the universe does not come from the Standard Model.

  This implies the existence of new non-relativistic species not present in the Standard Model.

- ?? % of the relativistic radiation content of the universe does not come from the Standard Model ($0 <$?? $\lesssim 10$)

  If dark matter, why not dark radiation?

  What new non-Standard Model relativistic species exist?

Prelude

The observable sensitive to non-Standard Model radiation is N_{eff}.

N_{eff} measures the 'effective number of neutrino species' at BBN/CMB: in effect, any hidden radiation decoupled from photon plasma (not necessarily connected to neutrinos).
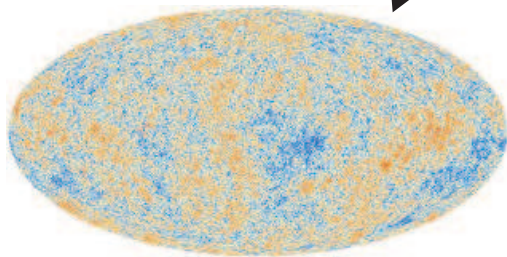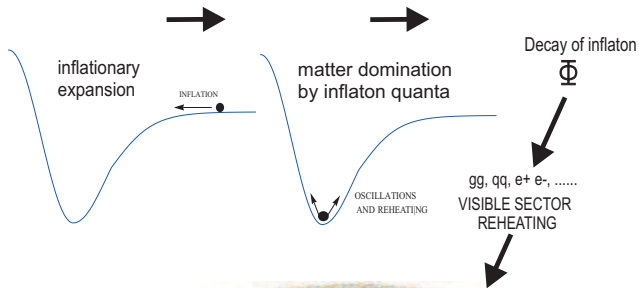
At CMB times,

$$\rho_{total} = \rho_\gamma \left(1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{eff} \right).$$

For a canonical Hot Big Bang, $N_{eff} = 3.046$: $\Delta N_{eff} = N_{eff} - 3.046$ represents dark radiation - additional radiation decoupled from SM thermal bath.

II
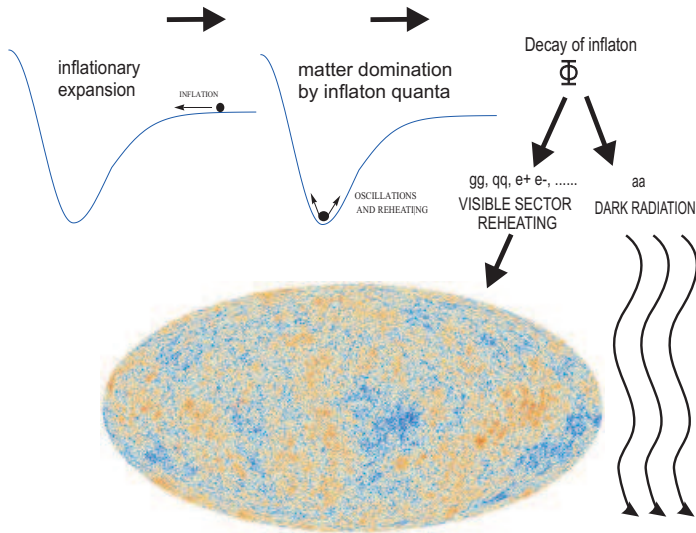# THEORY MOTIVATION

The Standard Cosmology

The Standard Cosmology $+ \Delta N_{eff}$

inflationary expansion

INFLATION

matter domination by inflaton quanta

OSCILLATIONS AND REHEATING

Decay of inflaton
$\bar{\Phi}$

gg, qq, e+ e-, ...
VISIBLE SECTOR REHEATING

aa
DARK RADIATION

Candidates for Dark Radiation

Dark radiation occurs whenever reheating involves decays to a massless hidden sector as well as the Standard Model.

Last decaying scalar

$$\Phi$$

gg, qq, e+ e-, ...

VISIBLE
SECTOR REHEATING

aa
DARK RADIATION
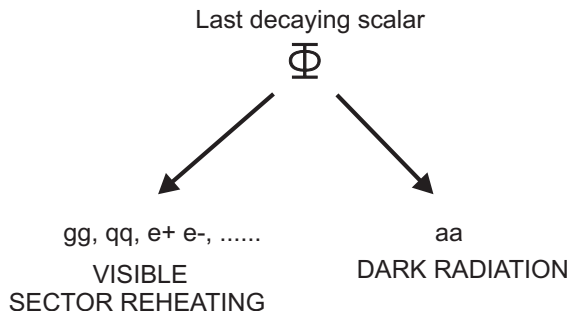
Such massless hidden sectors exist in many BSM constructions - QCD axion, axion-like particles, hidden photons, WISPs, chiral fermions....(talk by Ringwald)

Origin of Dark Radiation

How should we think about dark radiation?

In the inflationary universe, reheating proceeds from decays of a scalar field.

Often analysed in a simplified framework with a single field responsible both inflation and reheating. However:
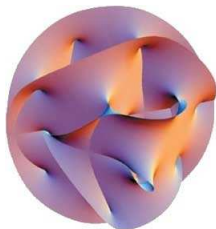
- Non-relativistic matter redshifts as $\rho_\Phi \sim a^{-3}$
- Radiation redshifts as $\rho_\gamma \sim a^{-4}$

$$3 < 4$$

Reheating is dominated by the LAST scalar to decay NOT the first.

Origin of Dark Radiation

In string theory, there are extra dimensions, and the size and shape of these extra dimensions are parametrised by moduli - the 'normal modes'. (McAllister, Burgess talk)

Moduli are massive scalars that interact only via 'gravitational' couplings suppressed by M_P^{-1}.

Moduli are generically displaced from their final minimum during inflation, and subsequently oscillate as non-relativistic matter ($\rho \sim a^{-3}$) before decaying.

Theoretical Motivation

Gravitational interactions mean that moduli are long-lived:

$$\Gamma \sim \frac{1}{8\pi}\frac{m_\Phi^3}{M_P^2}$$

$$\tau = \Gamma^{-1} \sim 8\pi\frac{M_P^2}{m_\Phi^3} = \left(\frac{100\text{TeV}}{m_\Phi}\right)^3 0.1\text{s}$$

$$T_{decay} \sim \left(\frac{m_\Phi}{100\text{TeV}}\right)^{3/2} 3 \text{ MeV}$$

As moduli are long-lived they come to dominate the energy density of the universe and reheating is driven by moduli decay.

A generic expectation of string compactifications is that the universe passes through a stage where it is modulus-dominated, and reheating comes from the decays of these moduli.

Theoretical Motivation

We expect reheating to be driven by the late-time decays of massive Planck-coupled particles.

Last decaying scalar

Φ

gg, qq, e+ e-, ...
VISIBLE
SECTOR REHEATING

aa
DARK RADIATION
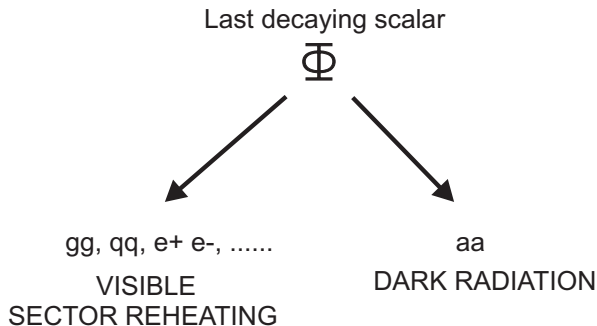
Dark radiation arises from hidden sector decays of moduli

Ideal subject for string phenomenology!

Theoretical Motivation

As gravitationally coupled particles, moduli generally couple to everything with M_P^{-1} couplings and there is no reason to expect vanishing couplings to hidden sectors.

Visible sector : $\dfrac{\Phi}{4M_P} F_{\mu\nu}^{color} F^{color,\mu\nu}, \dfrac{\partial_\mu \partial^\mu \Phi}{M_P} H_u H_d, \ldots$

Hidden sector : $\dfrac{\Phi}{2M_P} \partial_\mu a \partial^\mu a, \dfrac{\Phi}{4M_P} F_{\mu\nu}^{hidden} F^{hidden,\mu\nu} \ldots$

This is supported by explicit studies of string effective field theories
JC Cicoli Quevedo, Higaki (Kamada) [Nakayama] Takahashi, JC Marsh..

Axionic decay modes naturally arise with BR($\Phi \to aa$) $\sim 0.01 \to 1$.

Magnitude of Dark Radiation

Decays to any massless weakly coupled hidden sectors (axions, ALPs, hidden photons, RR $U(1)$s etc), gives dark radiation.

Visible/hidden branching ratio sets magnitude of dark radiation.

$$\Phi \to hidden \text{ with branching ratio } f_{hidden}$$
$$\Phi \to gg, \gamma\gamma, qq, \ldots \text{ with branching ratio } 1 - f_{hidden}$$

$$
\begin{aligned}
\Delta N_{eff} &= \frac{43}{7} \frac{f_{hidden}}{1 - f_{hidden}} \left(\frac{g(T_{\nu \, dec})}{g(T_{reheat})} \right)^{1/3} \\
&\simeq 3.43 \frac{f_{hidden}}{1 - f_{hidden}} \qquad (T_{reheat} = 1\text{GeV})
\end{aligned}
$$

For reheating with $T \sim 1$GeV, $\Delta N_{eff} \sim 0.5$ requires $f_{hidden} \sim 0.15$.

III
EXPERIMENTAL
MOTIVATION

Experimental Motivation: Physics

Both the CMB and primordial BBN abundances are sensitive to additional radiation in the early universe (which changes the expansion rate).

In the CMB, ΔN_{eff} modifies the damping tail of the CMB and is probed by the ratio between the damping scale and the sound horizon.

At BBN times, extra radiation modifies the expansion rate at a given temperature.

This affects the primordial Helium and Deuterium abundances: $(D/H)_p$ (where N_{eff} is degenerate with $\Omega_b h^2$) and Y_p.

Recent observations have tended to hint at the $1 \div 3\sigma$ level for $\Delta N_{eff} > 0$.

Experimental Motivation: CMB

Various (non-independent) measurements (talks by Dunkley, Efstathiou), 1 σ error bars:

- CMB + BAO
  - 3.55 ± 0.60 (WMAP9 + eCMB + BAO, 1212.5226)
  - 3.50 ± 0.47 (SPT + CMB + BAO, 1212.6267)
  - 2.87 ± 0.60 (WMAP7 + ACT + BAO, 1301.0824)
  - 3.30 ± 0.27 (Planck + eCMB + BAO, 1303.5076)

- CMB + BAO + H_0
  - 3.84 ± 0.40 (WMAP9 + eCMB + BAO + H0, 1212.5226)
  - 3.71 ± 0.35 (SPT + CMB + BAO + H0, 1212.6267)
  - 3.52 ± 0.39 (WMAP7 + ACT + BAO+ H0, 1301.0824)
  - 3.52 ± 0.24 (Planck + eCMB + BAO + H0, 1303.5076)

Experimental Motivation: CMB and H_0

Planck fit to ΛCDM predicts value of H_0 in local universe as

$$H_0 = 67.3 \pm 1.2 \text{km s}^{-1}\text{Mpc}^{-1}$$

This is a ΛCDM-dependent prediction not a measurement - must be tested directly by observations.

This prediction is $\sim 2.5\sigma$ discrepant from direct measurements of H_0:

$$
\begin{aligned}
H_0 &= 73.8 \pm 2.4 \text{ (Riess et al 2011)} \\
H_0 &= 74.3 \pm 1.5 \pm 2.1 \text{ (Freedman et al, 2012)} \\
H_0 &= 76.0 \pm 1.9 \text{ (?recent claim - 1306.6276)}
\end{aligned}
$$

Tension can be relieved by increasing ΔN_{eff} as this is degenerate with H_0.

Experimental Motivation: BBN

An independent probe of N_{eff} is via BBN primordial abundances - new determinations of Y_p and $(D/H)_P$ appeared last month.

$$Y_P = 0.254 \pm 0.003 \quad \text{(1308.2100, Izotov et al)}$$
$$(D/H)_P = (2.53 \pm 0.04) \times 10^{-5} \quad \text{(1308.3240, Cooke et al)}$$

Updated bounds: $(D/H)_P +$ CMB

$$N_{eff} = 3.28 \pm 0.28 \quad \text{(updates } 3.02 \pm 0.27 \text{ from Planck XVI)}$$

BBN alone $(D/H)_P + Y_P$:

$$N_{eff} = 3.57 \pm 0.18 \quad \text{(1308.3240, Cooke et al)}$$
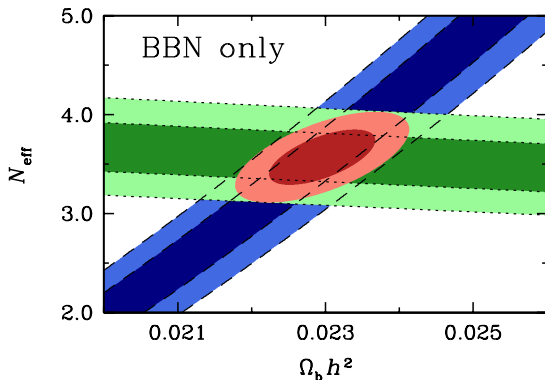
Based on Y_P and $(D/H)_P$: $N_{eff} = 3.57 \pm 0.18$ (1308.3240)

(figure from Cooke, Pettini, Jorgensen, Murphy, Steidel 1308.3240)

Experimental Motivation: Summary

- Essentially all determinations of N_{eff} have come in high, although not with decisive significance.
- Two independent channels $CMB + H_0$ and BBN hint for $\Delta N_{eff} \neq 0$ at $2 \div 3\sigma$ level.
- Central value roughly $\Delta N_{eff} \sim 0.5$
- Not 'evidence' let alone 'discovery', but consistent hints for a very well motivated extension of ΛCDM.
- Potential improvements from many directions: CMB polarisation, direct H_0 measurements, primordial abundances, low energy nuclear scattering....

IV
A COSMIC AXION
BACKGROUND

Properties of Dark Radiation

String theory says we expect reheating to be driven by the late-time decays of massive Planck-coupled particles.

Dark radiation arises from hidden sector decays of moduli

Ideal subject for string phenomenology!

Properties of Dark Radiation
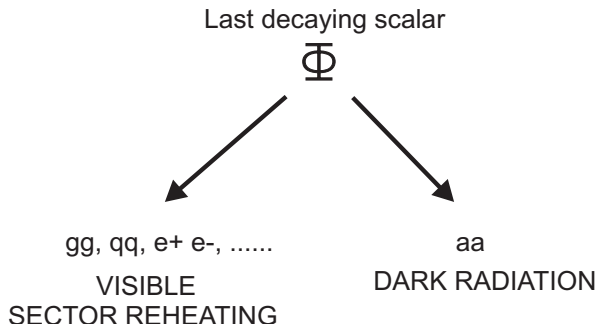
Independent of susy breaking scale in string models reheating is driven by decays of the lightest moduli, and dark radiation arises from hidden sector decays of these moduli.

Typical moduli couplings $\frac{\Phi}{4M_P}F_{\mu\nu}F^{\mu\nu}$ or $\frac{\Phi}{M_P}\partial_\mu a\partial^\mu a$ give

$$H_{decay} \sim \Gamma \sim \frac{1}{8\pi}\frac{m_\Phi^3}{M_P^2}$$

$$T_{reheat} \sim \left(3H_{decay}^2 M_P^2\right)^{1/4} \sim \frac{m_\phi^{3/2}}{M_P^{1/2}} \sim 0.6\,\text{GeV}\left(\frac{m_\phi}{10^6\,\text{GeV}}\right)^{3/2}$$

$$E_{axion} = \left(\frac{m_\Phi}{2}\right) = \quad 5\times 10^5\,\text{GeV}\left(\frac{m_\Phi}{10^6\,\text{GeV}}\right)$$

A Cosmic Axion Background
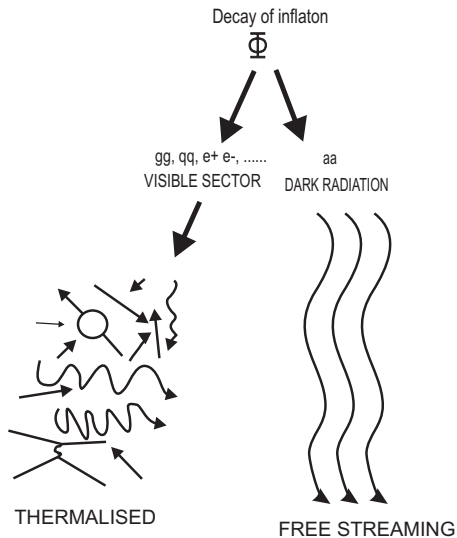
A Cosmic Axion Background

$$\Phi \to gg, \dots : \qquad \text{Decays thermalise} \qquad T_\gamma \sim T_{reheat} \sim \frac{m_\Phi^{3/2}}{M_P^{\frac{1}{2}}}$$

$$\Phi \to aa : \quad \text{Axions never thermalise} \quad E_a = \frac{m_\Phi}{2}$$

Thermal bath cools into the CMB while axions never thermalise and freestream to the present day:

Ratio of axion energy to photon temperature is

$$\frac{E_a}{T_\gamma} \sim \left(\frac{M_P}{m_\Phi}\right)^{\frac{1}{2}} \sim 10^6 \left(\frac{10^6 \text{GeV}}{m_\Phi}\right)^{\frac{1}{2}}$$

Retained through cosmic history!

Properties of Dark Radiation

Ratio of axion energy to photon temperature is

$$\frac{E_a}{T_\gamma} \sim \left(\frac{M_P}{m_\Phi}\right)^{\frac{1}{2}} \sim 10^6 \left(\frac{10^6 \text{GeV}}{m_\Phi}\right)^{\frac{1}{2}}$$

No absolute prediction, but a lightest modulus mass $m \sim 10^6 \text{GeV}$ arises in many string models - often correlated with SUSY approaches to the weak hierarchy problem.

- ▶ KKLT hep-th/0503216 Choi et al
- ▶ Sequestered LVS 0906.3297 Blumenhagen et al
- ▶ 'G2 MSSM' 0804.0863 Acharya et al

NB Moduli problem requires $m_\Phi \gtrsim 10^5 \text{TeV}$.

A Cosmic Axion Background

The 'Hot Big Bang' looks like

Particle Energies
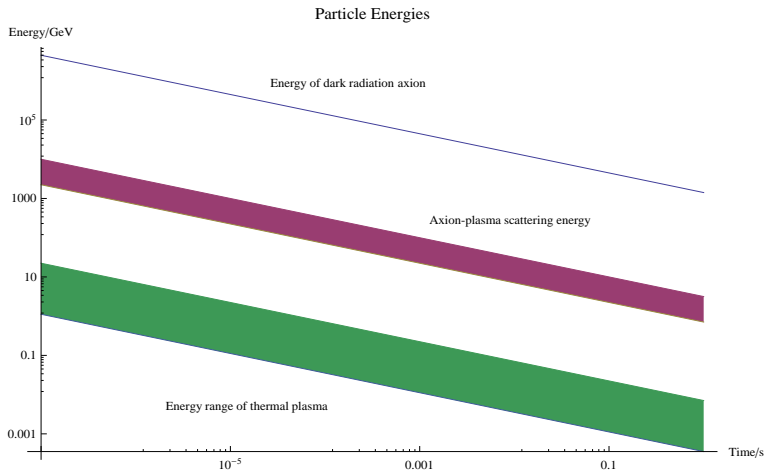
A Cosmic Axion Background

Axions originate at $z \sim 10^{12}(t \sim 10^{-6}$ s) and freestream to today.

PREDICTION: Cosmic Axion Background

Energy: $E \sim 0.1 \div 1 \text{keV}$ Flux: $\sim \left(\frac{\Delta N_{eff}}{0.57} \right) 10^{6} \text{cm}^{-2} \text{s}^{-1}$.
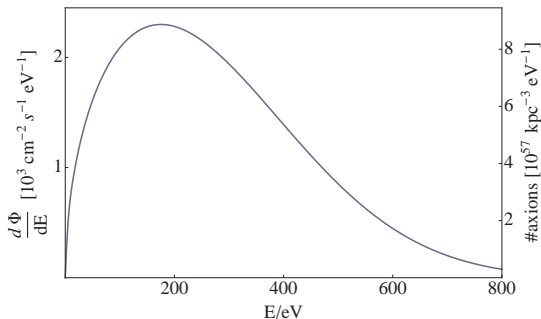
A Cosmic Axion Background

The current energy of such axionic dark radiation is

$$E_a \sim 200 \text{eV} \left(\frac{10^6 \text{ GeV}}{m_\Phi} \right)^{\frac{1}{2}}$$

The expectation that there is a dark analogue of the CMB at $E \gg T_{CMB}$ comes from very simple and general properties of moduli.

It is not tied to precise models of moduli stabilisation or choice of string theory etc.

It just requires the existence of massive particles only interacting gravitationally.

For $10^5 \text{GeV} \lesssim m_\Phi \lesssim 10^8 \text{GeV}$ CAB lies today in EUV/soft X-ray wavebands.

V
OBSERVING A COSMIC
AXION BACKGROUND

How to see a CAB?

Axion-photon conversions come from axion coupling to electromagnetism (cf Ringwald talk):

$$\mathcal{L}_{a-\gamma} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4M}aF_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2.$$

For general axion-like particles $M \equiv g_{a\gamma\gamma}^{-1}$ and m_a are unspecified.

We take $m_a = 0$ (in practice $\lesssim 10^{-12}$eV) and keep M free.

Direct bounds (axion production in supernovae) are $M \gtrsim 10^{11}$GeV.

Seeing Axions

Axion-to-photon conversion probability for axion energy E_a in transverse magnetic field $B_\perp$ of domain size L is:

$$P(a \to \gamma) = \sin^2(2\theta) \sin^2 \left(\frac{\Delta}{\cos 2\theta} \right)$$

where

$$\theta \approx 2.8 \cdot 10^{-5} \times \left(\frac{10^{-3} \text{cm}^{-3}}{n_e} \right) \left(\frac{B_\perp}{1 \ \mu\text{G}} \right) \left(\frac{E_a}{200 \ \text{eV}} \right) \left(\frac{10^{14} \ \text{GeV}}{M} \right),$$

$$\Delta = 0.27 \times \left(\frac{n_e}{10^{-3} \text{cm}^{-3}} \right) \left(\frac{200 \ \text{eV}}{E_a} \right) \left(\frac{L}{1 \ \text{kpc}} \right).$$

Seeing Axions

Axions convert to photons in coherent magnetic field domain: want large magnetic fields supported over large volumes.

Galaxy Clusters:

- The largest virialised structures in the universe
- Typical size 1 Mpc, typical mass $10^{14} \div 10^{15} M_{sun}$.
- Large magnetic fields $B \sim 1 \div 10 \mu G$ coherent over $L \sim 1 \div 10$ kpc.
- Hot intracluster gas, $T_{gas} \sim 2 \div 10$ keV.
- By mass 1 per cent galaxies, 10 per cent gas, 90 per cent dark matter.
- Sit at the 'large magnetic fields over large volumes' frontier of particle physics.

Focus on Coma - large, well observed, nearby cluster at $|b| \gg 0$.
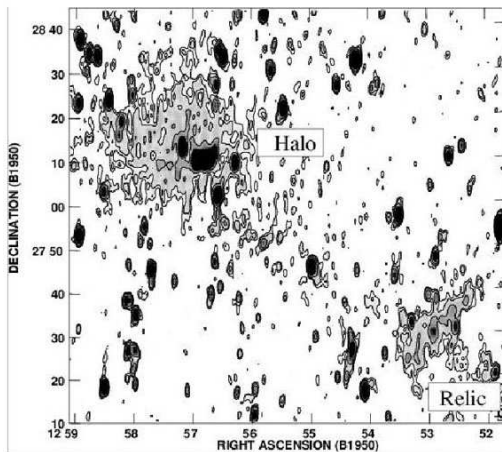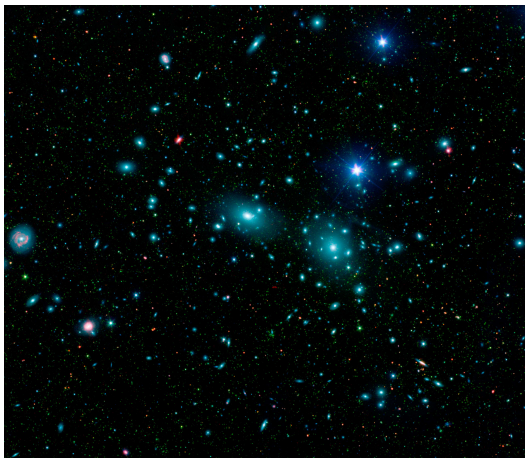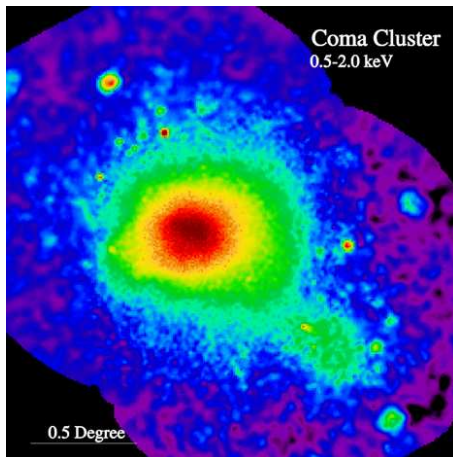
A Cosmic Axion Background

Energy density of Cosmic Axion Background is

$$\rho_{CAB} = \left(\frac{\Delta N_{eff}}{0.57} \right) 1.6 \times 10^{60} \text{erg Mpc}^{-3}$$

Typical cluster X-ray luminosity (typical scale 1 Mpc) is

$$\mathcal{L} \sim 10^{42 \div 45} \text{erg s}^{-1}$$

For field B over a domain L, in small mixing approximation,

$$
\begin{aligned}
P(a \to \gamma) &= \frac{B^2 L^2}{4M^2} \text{ per domain} \\
&= 2.0 \cdot 10^{-18} \text{s}^{-1} \times \left(\frac{B_\perp}{3\ \mu\text{G}} \frac{10^{13}\ \text{GeV}}{M} \right)^2 \left(\frac{L}{1\ \text{kpc}} \right).
\end{aligned}
$$

A Cosmic Axion Background

$a \to \gamma$ conversion generates a soft X-ray luminosity

$$\mathcal{L}_{Mpc^3} = 3.6 \cdot 10^{41} \text{ erg Mpc}^{-3}\text{s}^{-1} \times$$
$$\times \left(\frac{\Delta N_{eff}}{0.57}\right)\left(\frac{B}{\sqrt{2}\ \mu G}\frac{10^{13}\text{ GeV}}{M}\right)^2\left(\frac{L}{1\text{ kpc}}\right),$$

Extremely luminous - for $\Delta N_{eff} \sim 0.5$ and $M \sim 10^{11}$ GeV, $a \to \gamma$ luminosity outshines entire cluster!

Counterpart - for $M \sim 10^{11}$ GeV observable signal can remain even with $\Delta N_{eff} \sim 10^{-4}$.

Axions that are everywhere are much easier to detect than axions that must be first produced in stars or supernovae.

The Cluster Soft Excess

In fact there exists a long-standing (since 1996) EUV/soft x-ray excess from galaxy clusters (Lieu 1996, review Durret 2008). E.g Coma has

$$\mathcal{L}_{excess} \sim 10^{43} \text{erg s}^{-1}$$

What is observed is excess emission above low-energy tail from thermal Bremmstrahlung emission from hot intracluster gas.

Observed by many missions - principally EUVE and ROSAT, also XMM-Newton, Suzaku, Chandra.

Statistical significance ($> 100\sigma$) not an issue.

Possible astrophysical explanations (thermal warm gas/Inverse Compton-CMB scattering of relativistic electrons) all have problems.
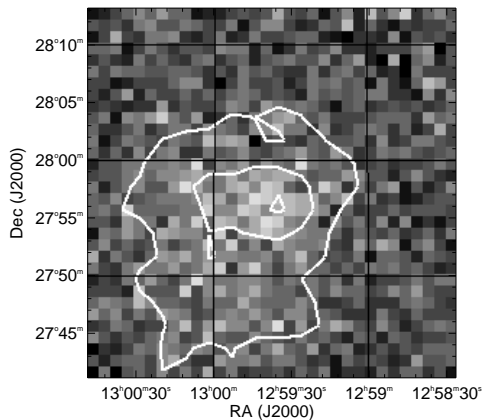
from astro-ph/0403081 Bowyer et al, soft excess in Coma as observed by EUVE

Soft excess extends well beyond hot gas and cluster virial radius:
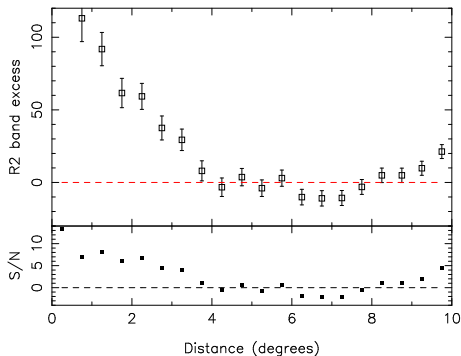
from 0903.3067 Bonamente et al, ROSAT R2 band (0.14–0.28keV) observation of
Coma

Cluster Soft Excess: Astrophysical Explanations

Two main proposals for astrophysical explanations:

1. A warm thermal gas with $T \sim 0.2$keV.

   Interpret soft excess as thermal bremmstrahlung emission from this warm gas.

2. A large non-thermal relativistic electron population with $E \sim 200 - 300$ MeV.

   Interpret soft excess as inverse Compton scattering of electrons on CMB.

Both have problems (in back-up slides).

The Cluster Soft Excess and a CAB

This excess may be generated by $a \to \gamma$ conversion in cluster magnetic field.

Necessary energy and luminosity easy to obtain ($M \lesssim 10^{13}$GeV) and also consistent with large spatial extent of excess.

Many predictions:

- Soft excess magnitude and morphology fully determined by cluster magnetic field and electron density
- Spatial extent of excess conterminous with magnetic field
- No thermal emission lines (e.g. O_{VII}) associated to excess
- Energy of excess is constant across clusters, varying with redshift as $E_a \sim (1 + z)$.

In progress: detailed model of axion propagation through turbulent Coma magnetic field.

Axion Propagation through Coma

Magnetic field model is best fit to Faraday rotation (Bonafede et al 1002.0594):

- Magnetic field has Kolmogorov spectrum, $|B(k)| \sim k^{-11/3}$, generated between $k_{max} = \frac{2\pi}{2\text{kpc}}$ and $k_{min} = \frac{2\pi}{34\text{kpc}}$.
- Spatial magnetic field has Gaussian statistics.
- Central magnetic field $\langle B \rangle_{r<291kpc} = 4.7\mu$G
- Equipartition radial scaling of B, $B(r) \sim n_e(r)^{1/2}$
- Electron density taken from β-model with $\beta = 0.75$,

$$n_e(r) = 3.44 \times 10^{-3} \left(1 + \left(\frac{r}{291\text{kpc}}\right)^2\right)^{-\frac{3\beta}{2}} \text{cm}^{-3}$$

- Numerical 2000^3 magnetic field with 0.5kpc resolution.

Numerical propagation of axions with $E = 25\text{eV} \div 25000\text{eV}$ and determination of $P(a \to \gamma)$.

Discussion

For $M \sim 10^{14}$ GeV conversion probabilities range from $10^{-5} \div 10^{-8}$.

Conversion probabilities are highly sensitive to axion energy and magnetic field profile.

For low energies ($E_a \lesssim 100$ eV), central deficit in $P(a \to \gamma)$.

For high energies, ($E_a \gtrsim 1000$ eV), central excess in $P(a \to \gamma)$.

Radial behaviour of axion conversion probabilities depends sensitively on axion energy and on magnetic field model.

$E \sim \mathcal{O}(200$ eV) is a critical energy where turnover occurs between central deficit and central dominance for $a \to \gamma$ conversion.

Still work in progress....

Conclusions

- Dark radiation is an extension of standard cosmology with exceptional theoretical motivation and good experimental motivation.

- Its existence requires only a decay mode of the 'inflaton' to a massless hidden sector.

- In string models dark radiation is naturally generated through the modulus decay $\Phi \to aa$.

- For typical moduli masses this predicts a Cosmic Axion Background freestreaming from $z \sim 10^{12}$ to now have $E_a \sim 0.1 \div 1$ keV.

- CAB can be observed through $a \to \gamma$ conversion in magnetic fields and may already be visible through long-standing astrophysics EUV excess from galaxy clusters.

In this era of fin de siecle depression over the unreasonable success of the Standard Models of particle physics and cosmology.....

Conclusions

In this era of fin de siecle depression over the unreasonable success of the Standard Models of particle physics and cosmology.....

has a Cosmic Axion Background been observed and missed for the last 17 years?

Thank you for listening

Astrophysics: $T \sim 0.2$keV warm gas

The original proposal. However:

1. Such a gas is pressure unstable against the hot ICM gas.

   It rapidly cools away on a timescale much shorter than cluster timescales.

2. A thermal $T \sim 0.2$keV gas would also have thermal emission lines - particularly O_{VII} at 560 eV.

   No such lines have been observed - some early claimed detections have gone away.

A more promising propsal: a large population of non-thermal electrons scattering off the CMB. However:

1. If this population continues to $E \sim 2\text{GeV}$, its synchrotron radio emission is above level of Coma radio halo.

   This necessitates a sharp spectral cutoff between $\sim 200\text{MeV}$ and $\sim 2\text{GeV}$.

2. This population necessarily produces gamma rays through non-thermal bremmstrahlung.

   It was predicted that these gamma rays would be easily observable by Fermi (Atoyan + Volk 2000)

   But - Fermi does not see any clusters:

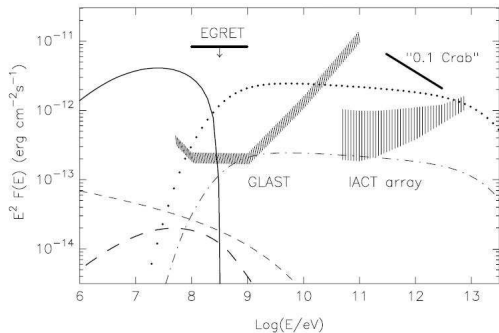 $$\mathcal{F}^{Coma}_{>100 \, MeV} < 1.1 \times 10^{-9} \text{ph cm}^{-2} \text{ s}^{-1}$$

FIG. 6.—Expected γ-ray fluxes expected from the Coma Cluster. The

from Atoyan + Volk, 2000

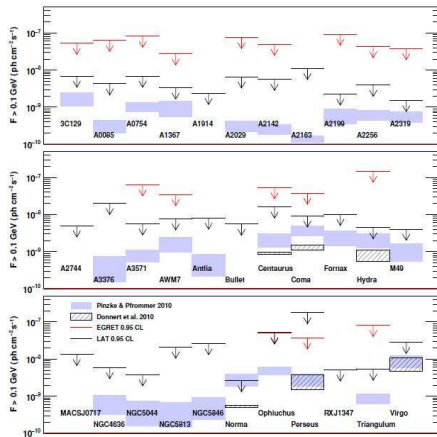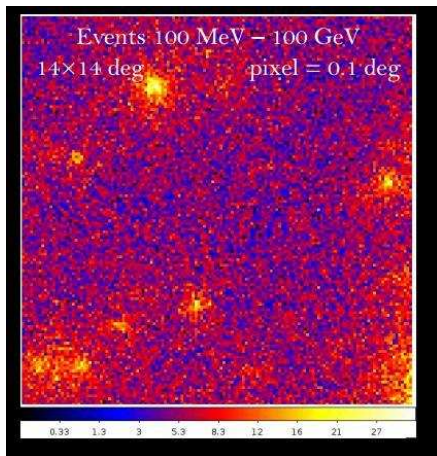# Astrophysics: non-thermal $E \sim 150$ MeV electrons

Figure 1. Photon flux upper limits derived from *Fermi*-LAT observations of galaxy clusters (assuming unresolved gamma-ray emission) are compared with EGRET

Fermi 18 month limits, Ackermann 2010

Events 100 MeV − 100 GeV
14×14 deg   pixel = 0.1 deg

0.33   1.3   3   5.3   8.3   12   16   21   27

(Ando + Zandanel, to appear 2013)