Hierarchy Problems and Supersymmetry Breaking in String Theory

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Talk Structure

- Hierarchies in Nature
- String Phenomenology and Moduli Stabilisation
- Large Volume Models
- Supersymmetry Breaking
- Axions
- Neutrino Masses
- Conclusions

Hierarchies in Nature

Nature likes hierarchies:

- The Planck scale, $M_P = 2.4 \times 10^{18}$GeV.
- The GUT/inflation scale, $M \sim 10^{16}$GeV.
- The axion scale, 10^9GeV $\lesssim f_a \lesssim 10^{12}$GeV
- The weak scale : $M_W \sim 100$GeV
- The QCD scale $\Lambda_{QCD} \sim 200$MeV
- The neutrino mass scale, 0.05eV $\lesssim m_\nu \lesssim 0.3$eV.
- The cosmological constant, $\Lambda \sim (10^{-3}\text{eV})^4$

These demand an explanation!

Hierarchies in Nature

This talk will argue that

- an intermediate string scale $m_s \sim 10^{11}$GeV
- stabilised exponentially large extra dimensions ($\mathcal{V} \sim 10^{15} l_s^6$).

explains the the axionic, weak and neutrino hierarchies.

Different hierarchies will come as different powers of the (large) volume.

Moduli Stabilisation

- String theory lives in ten dimensions.
- Compactify on a Calabi-Yau manifold to give a four-dimensional theory.
- The geometry determines the four-dimensional particle spectrum.
- The spectrum always includes uncharged scalar particles - moduli - describing the size and shape of the extra dimensions.

Moduli Stabilisation

- Moduli are naively massless scalars which couple gravitationally.
- These generate fifth forces and so must be given masses.
- Generating potentials for moduli is the field of moduli stabilisation.
- This talk is on the large-volume models which represent a particular moduli stabilisation scenario.

Moduli Stabilisation: Fluxes

- Fluxes carry an energy density depending on the cycle geometry.
- This energy generates a potential for the cycle moduli.
- In IIB compactifications, 3-form fluxes generate a superpotential

$$W = \int (F_3 + iSH_3) \wedge \Omega \equiv \int G_3 \wedge \Omega.$$

- This stabilises the dilaton and complex structure moduli.

$$D_S W = D_U W = 0.$$

$$W = \int G_3 \wedge \Omega = W_0.$$

Moduli Stabilisation: Fluxes

$$
\begin{aligned}
\hat{K} &= -2\ln\left(\mathcal{V}(T_i+\bar{T}_i)\right), \\
W &= W_0\,. \\
V &= e^{\hat{K}}\left(\sum_T \hat{K}^{i\bar{j}} D_i W D_{\bar{j}}\bar{W} - 3|W|^2\right) \\
&= 0
\end{aligned}
$$

No-scale model :

- vanishing vacuum energy
- broken susy
- T unstabilised

Moduli Stabilisation: KKLT

$$\hat{K} = -2\ln(\mathcal{V}),$$
$$W = W_0 + \sum_i A_i e^{-a_i T_i}.$$

- Non-perturbative ADS superpotential.
- The T-moduli are stabilised by solving $D_T W = 0$.
- This gives a susy AdS vacuum which is uplifted by anti-branes/magnetic fluxes/IASD 3-form fluxes/ISS/something else.
- Susy breaking is sourced by the uplift.

Moduli Stabilisation: KKLT

KKLT stabilisation has three phenomenological problems:

1. No susy hierarchy: fluxes prefer $W_0 \sim 1$ and $m_{3/2} \gg 1$TeV.
2. Susy breaking not well controlled - depends entirely on uplifting.
3. α' expansion not well controlled - volume is small and there are large flux backreaction effects.

Moduli Stabilisation: Large-Volume

$$\hat{K} = -2\ln\left(\mathcal{V} + \frac{\xi}{g_s^{3/2}}\right),$$

$$W = W_0 + \sum_i A_i e^{-a_i T_i}.$$

- Add the leading α' corrections to the Kähler potential.
- This leads to dramatic changes in the large-volume vacuum structure.

Moduli Stabilisation: Large-Volume

The simplest model $\mathbb{P}^4_{[1,1,1,6,9]}$ has two Kähler moduli.

$$\mathcal{V} = \left(\frac{T_b + \bar{T}_b}{2}\right)^{3/2} - \left(\frac{T_s + \bar{T}_s}{2}\right)^{3/2} \equiv \left(\tau_b^{3/2} - \tau_s^{3/2}\right).$$

If we compute the scalar potential, we get for $\mathcal{V} \gg 1$,

$$V = \frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s |A_s W_0| \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\xi |W_0|^2}{g_s^{3/2}\mathcal{V}^3}.$$

Moduli Stabilisation: Large-Volume

$$V = \underbrace{\frac{\sqrt{\tau_s} a_s^2 |A_s|^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s |A_s W_0| \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2}}_{\text{Integrate out } \tau_s} + \frac{\xi |W_0|^2}{g_s^{3/2} \mathcal{V}^3}.$$

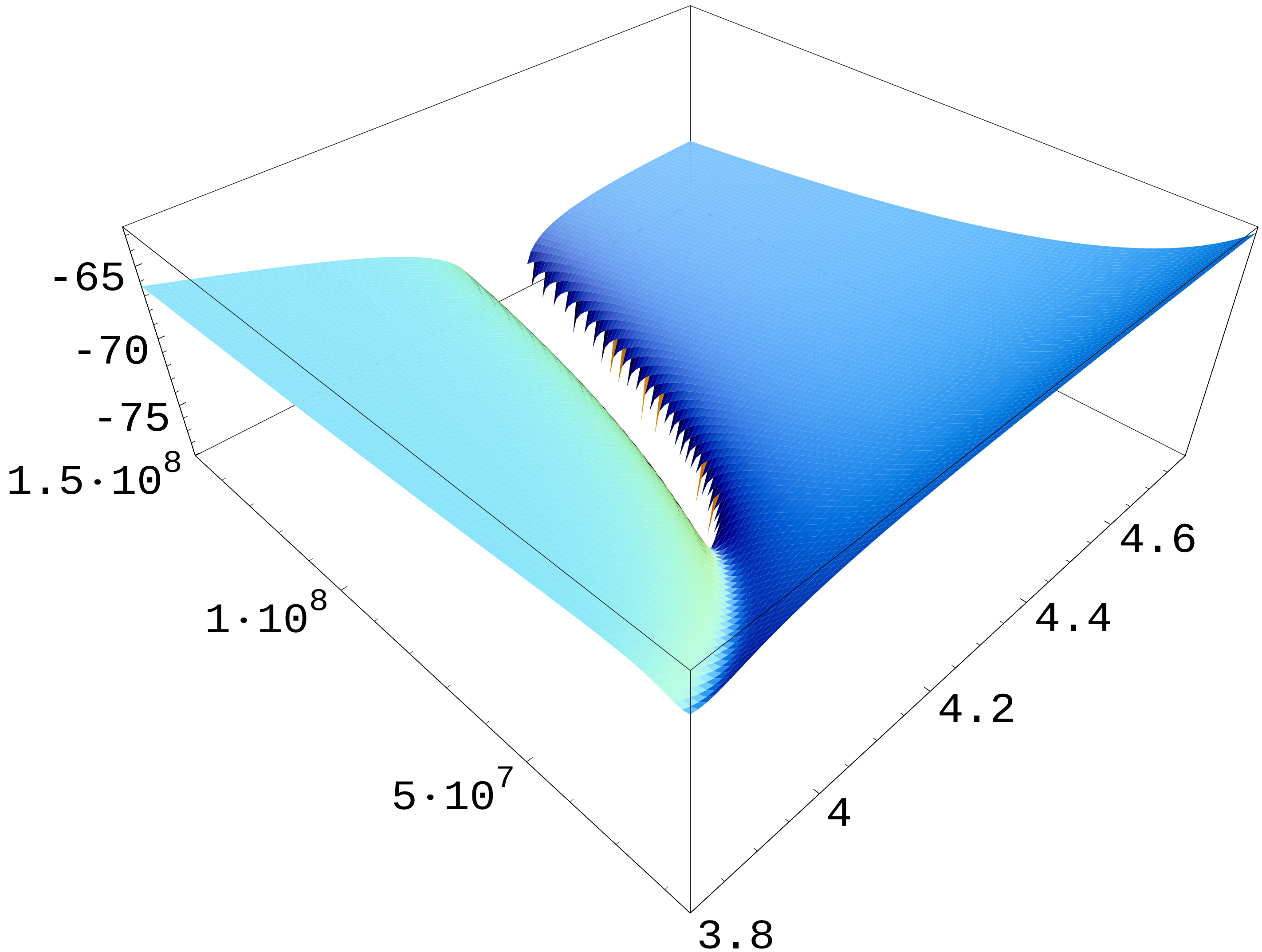
$$V = -\frac{|W_0|^2 (\ln \mathcal{V})^{3/2}}{\mathcal{V}^3} + \frac{\xi |W_0|^2}{g_s^{3/2} \mathcal{V}^3}.$$

A minimum exists at

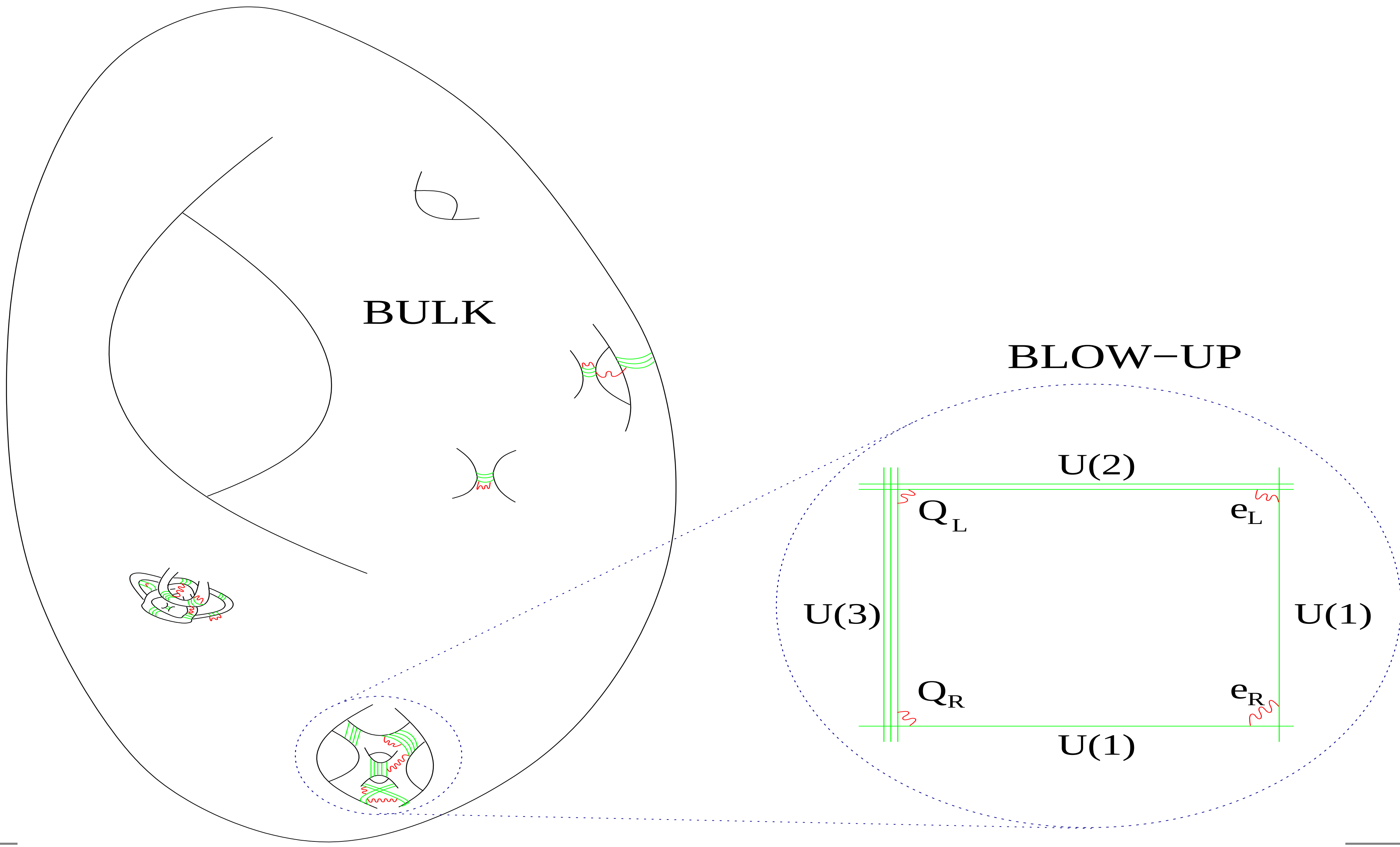
$$\mathcal{V} \sim |W_0| e^{a_s \tau_s}, \qquad \tau_s \sim \frac{\xi^{2/3}}{g_s}.$$

This minimum is non-supersymmetric AdS and at exponentially large volume.

Moduli Stabilisation: Large-Volume



Moduli Stabilisation: Large-Volume



Moduli Stabilisation: Large-Volume

- The stabilised volume is exponentially large.
- The large volume lowers the string scale and gravitino mass through

$$m_s = \frac{M_P}{\sqrt{\mathcal{V}}}, \qquad m_{3/2} = \frac{M_P}{\mathcal{V}}.$$

- To solve the gauge hierarchy problem, need $\mathcal{V} \sim 10^{15}$.
- The vacuum is pseudo no-scale and breaks susy...

Moduli Stabilisation: Large-Volume

The mass scales present are:

Planck scale:	$M_P = 2.4 \times 10^{18}$GeV.
String scale:	$M_S \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11}$GeV.
KK scale	$M_{KK} \sim \frac{M_P}{\mathcal{V}^{2/3}} \sim 10^{9}$GeV.
Gravitino mass	$m_{3/2} \sim \frac{M_P}{\mathcal{V}} \sim 30$TeV.
Small modulus	$m_{\tau_s} \sim m_{3/2} \ln\left(\frac{M_P}{m_{3/2}}\right) \sim 1000$TeV.
Complex structure moduli	$m_U \sim m_{3/2} \sim 30$TeV.
Soft terms	$m_{susy} \sim \frac{m_{3/2}}{\ln(M_P/m_{3/2})} \sim 1$TeV.
Volume modulus	$m_{\tau_b} \sim \frac{M_P}{\mathcal{V}^{3/2}} \sim 1$MeV.

SUSY Breaking and Soft Terms

Supersymmetry will (hopefully) be discovered at the LHC.

It is parametrised by

- Soft scalar masses, $m_i^2 \phi_i^2$
- Gaugino masses, $M_a \lambda^a \lambda^a$,
- Trilinear scalar A-terms, $A_{\alpha\beta\gamma}\phi^\alpha \phi^\beta \phi^\gamma$
- B-terms, BH_1H_2.

SUSY Breaking and Soft Terms

- To compute soft terms, we expand K and W in powers of matter fields C^α,

$$W = \hat{W}(\Phi) + \mu(\Phi) H_1 H_2 + \frac{1}{6} Y_{\alpha\beta\gamma}(\Phi) C^\alpha C^\beta C^\gamma + \ldots,$$

$$K = \hat{K}(\Phi, \bar{\Phi}) + \tilde{K}_{\alpha\bar{\beta}}(\Phi, \bar{\Phi}) C^\alpha C^{\bar{\beta}} + [Z H_1 H_2 + h.c.] + \ldots,$$

$$f_a = f_a(\Phi).$$

SUSY Breaking and Soft Terms

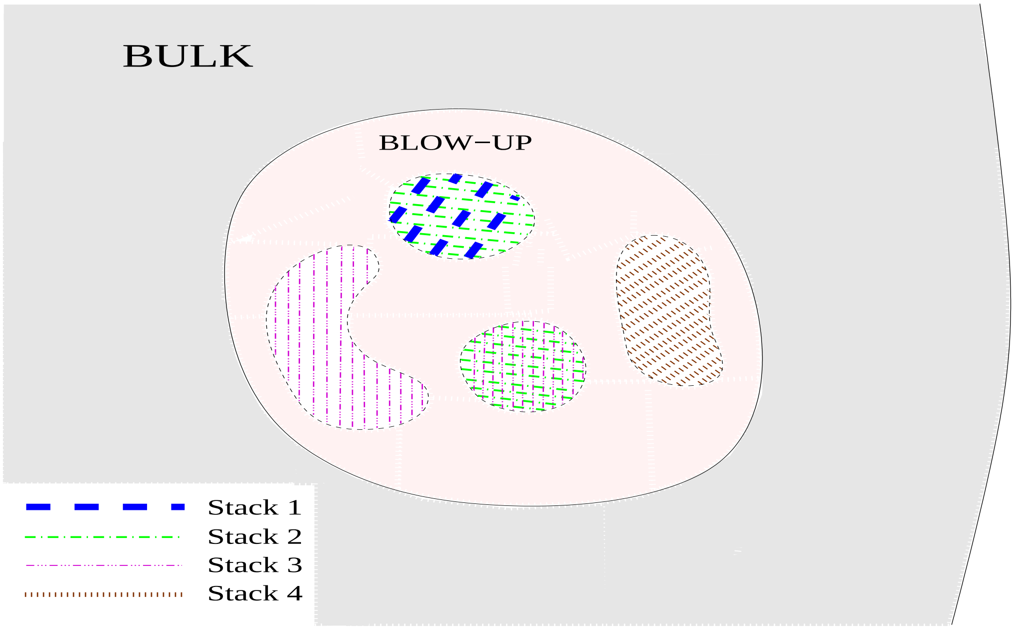
- Soft scalar masses $m^2_{i\bar{j}}$ and trilinears $A_{\alpha\beta\gamma}$ are given by

$$\begin{aligned}
\tilde{m}^2_{\alpha\bar{\beta}} &= (m^2_{3/2} + V_0)\tilde{K}_{\alpha\bar{\beta}} \\
&\quad -\bar{F}^{\bar{m}}F^n\left(\partial_{\bar{m}}\partial_n\tilde{K}_{\alpha\bar{\beta}} - (\partial_{\bar{m}}\tilde{K}_{\alpha\bar{\gamma}})\tilde{K}^{\bar{\gamma}\delta}(\partial_n\tilde{K}_{\delta\bar{\beta}})\right) \\
A'_{\alpha\beta\gamma} &= e^{\hat{K}/2}F^m\Big[\hat{K}_m Y_{\alpha\beta\gamma} + \partial_m Y_{\alpha\beta\gamma} \\
&\quad -\left((\partial_m\tilde{K}_{\alpha\bar{\rho}})\tilde{K}^{\bar{\rho}\delta}Y_{\delta\beta\gamma} + (\alpha\leftrightarrow\beta) + (\alpha\leftrightarrow\gamma)\right)\Big].
\end{aligned}$$

- To compute soft terms, we need to know $\tilde{K}_{\alpha\bar{\beta}}(\Phi)$.

The brane geometry

- We assume the Standard Model comes from a stack of (magnetised) branes all wrapping a blowup cycle.
- Chiral fermions stretch between differently magnetised branes.



SUSY Breaking and Soft Terms

- In the dilute flux approximation we find

$$\tilde{K}_{\alpha\bar{\beta}} = \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}} k_{\alpha\bar{\beta}}(U, \bar{U})$$

Soft terms are

$$\begin{aligned} M_i &= \frac{F^s}{2\tau_s} \equiv M, \\ m_{\alpha\bar{\beta}} &= \frac{M}{\sqrt{3}} \tilde{K}_{\alpha\bar{\beta}}, \\ A_{\alpha\beta\gamma} &= -M\,\hat{Y}_{\alpha\beta\gamma}, \\ B &= -\frac{4M}{3}. \end{aligned}$$

Mirror Mediation

- These soft terms are flavour-universal.

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- An invalid argument:

  In gravity mediation flavour and susy breaking are both Planck-scale physics.

  Therefore susy breaking is sensitive to flavour

  Therefore squark masses are non-universal.

Mirror Mediation

- These soft terms are flavour-universal.
- An invalid argument:

  In gravity mediation flavour and susy breaking are both Planck-scale physics.

  Therefore susy breaking is sensitive to flavour

  Therefore squark masses are non-universal.
- In string theory, we have Kähler (T) and complex structure (U) moduli. These are decoupled at leading order.

$$\mathcal{K} = -2\ln\left(\mathcal{V}(T)\right) - \ln\left(i\int \Omega \wedge \bar{\Omega}(U)\right) - \ln(S+\bar{S}).$$

- The kinetic terms for T and U fields do not mix.

Mirror Mediation

- Due to the shift symmetry $T \to T + i\epsilon$, the T moduli make no perturbative appearance in the superpotential.
- It is the U moduli that source flavour...

$$W = \ldots + \frac{1}{6} Y_{\alpha\beta\gamma}(U) C^{\alpha} C^{\beta} C^{\gamma} + \ldots .$$

- ...and the T moduli that break supersymmetry,

$$D_T W \neq 0, F^T \neq 0, \qquad D_U W = 0, F^U = 0.$$

- At leading order, susy breaking (Kähler moduli) and flavour (complex structure moduli) decouple.

Soft Terms: Spectra

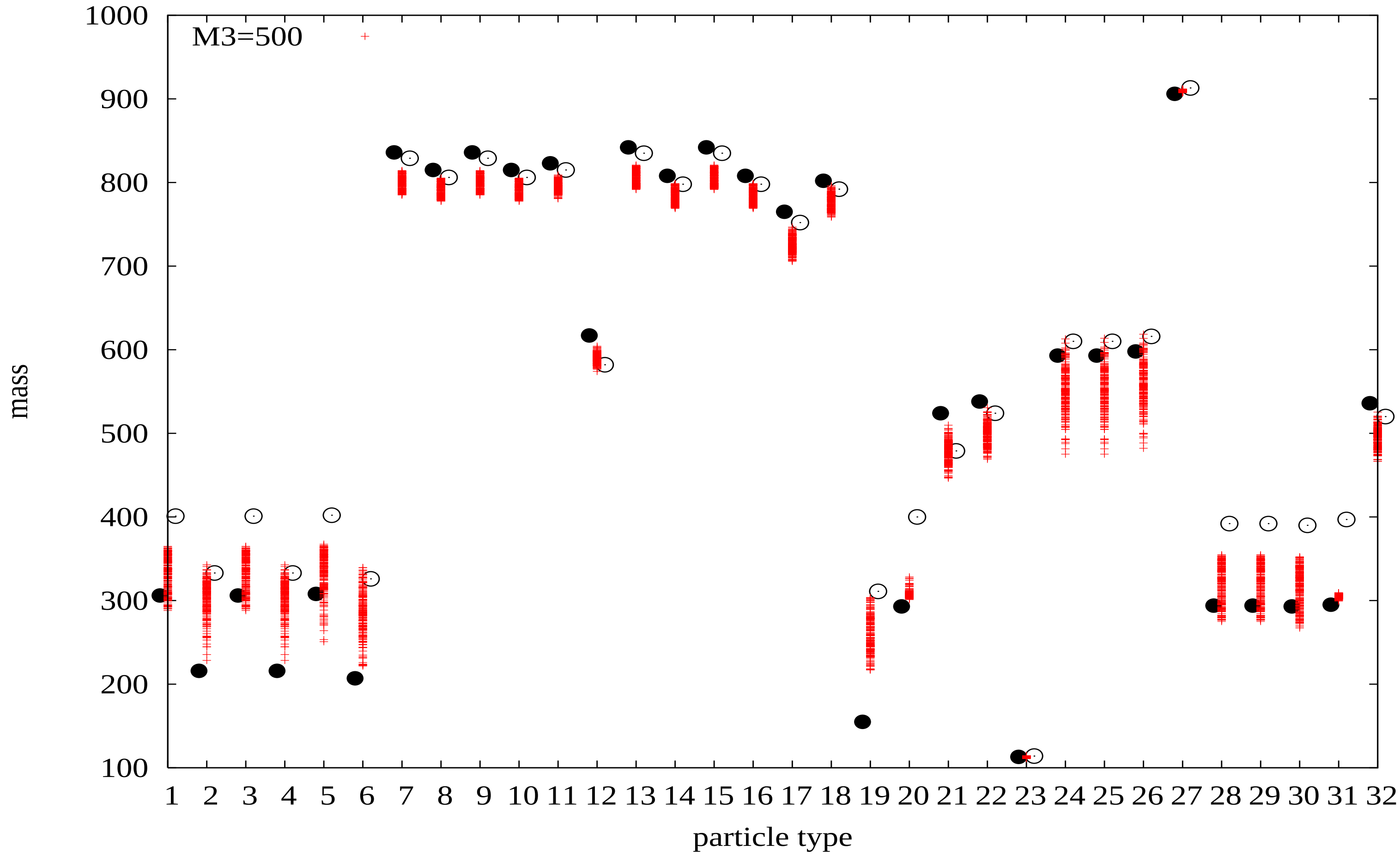
- Magnetic fluxes are needed for chirality.
- These alter the gauge kinetic functions

$$f_a = \frac{T}{4\pi} \rightarrow f_a = \frac{T}{4\pi} + h_a(F)S.$$

- Fluxes perturb the soft terms.
- We generate many such spectra, with high-scale soft terms allowed to fluctuate by $\pm 20\%$.

Soft Terms: Spectra

- We run the soft terms to low energy using SoftSUSY:



Soft Terms: Spectra

- The spectrum is more compressed compared to mSUGRA: the squarks are lighter and sleptons heavier.
- This arises because the RG running starts at the intermediate rather than GUT scale.
- The gaugino mass ratios are

$$M_1 : M_2 : M_3 = 1.5 \rightarrow 2 : 2 : 6.$$

Axions

- Axions are a well-motivated solution to the strong CP problem.
- The QCD Lagrangian is

$$\mathcal{L}_{QCD} = \frac{1}{g^2} \int d^4x F^a_{\mu\nu} F^{a,\mu\nu} + \theta \int F^a \wedge F^a.$$

- The strong CP problem:

  Naively $\theta \in (-\pi, \pi)$ - experimentally $|\theta| \lesssim 10^{-10}$.

- The axionic (Peccei-Quinn) solution is to promote θ to a dynamical field, $\theta(x)$.

Axions

- The canonical Lagrangian for θ is

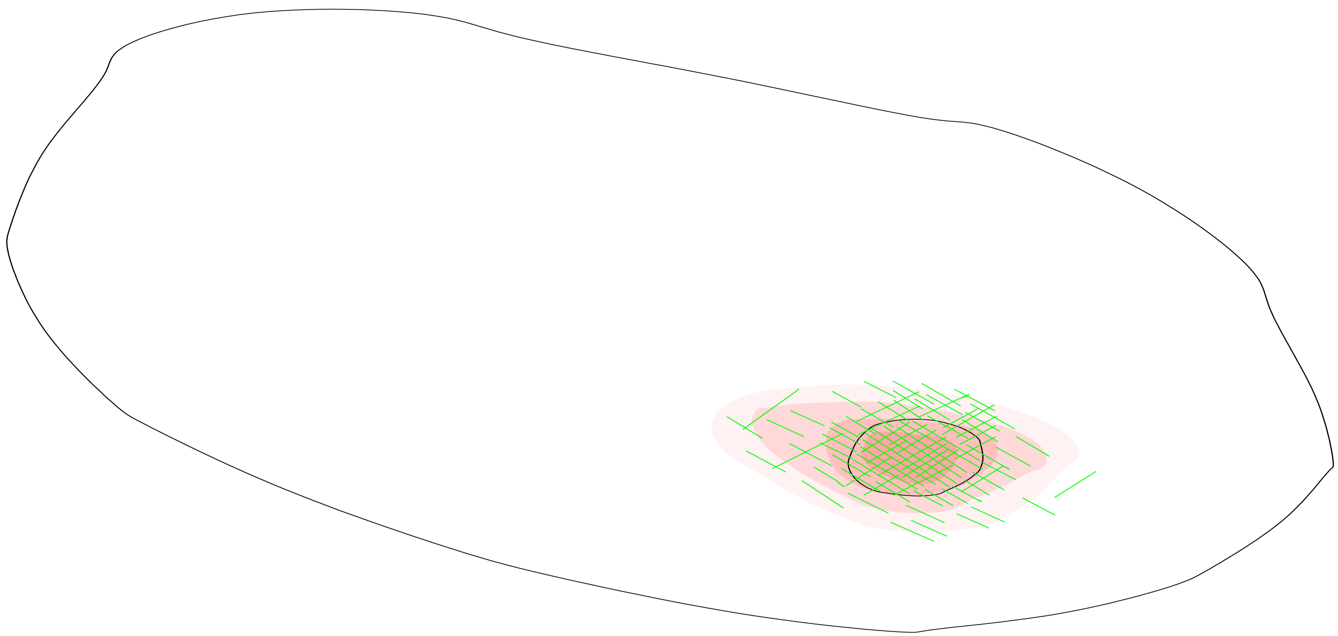
$$\mathcal{L} = \frac{1}{2}\partial_\mu \theta \partial^\mu \theta + \int \frac{\theta}{f_a} F^a \wedge F^a.$$

  f_a is the axionic decay constant.
- Constraints on supernova cooling and direct searches imply $f_a \gtrsim 10^9$GeV.
- Avoiding the overproduction of axion dark matter prefers $f_a \lesssim 10^{12}$GeV.
- There exists an axion 'allowed window',

$$10^9\text{GeV} \lesssim f_a \lesssim 10^{12}\text{GeV}.$$

Axions

- For D7 branes, the axionic coupling comes from the RR form in the brane Chern-Simons action.
- The axion decay constant f_a measures the coupling of the axion to matter.



Axions

- The coupling of the axion to matter is a local coupling and does not see the overall volume.
- This coupling can only see the string scale:

$$f_a \sim m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}.$$

(This is confirmed by a full analysis)

- This generates the axion scale,

$$f_a \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11}\text{GeV}.$$

Neutrino Masses

- Neutrino masses exist:

$$0.05\text{eV} \lesssim m_\nu^H \lesssim 0.3\text{eV}.$$

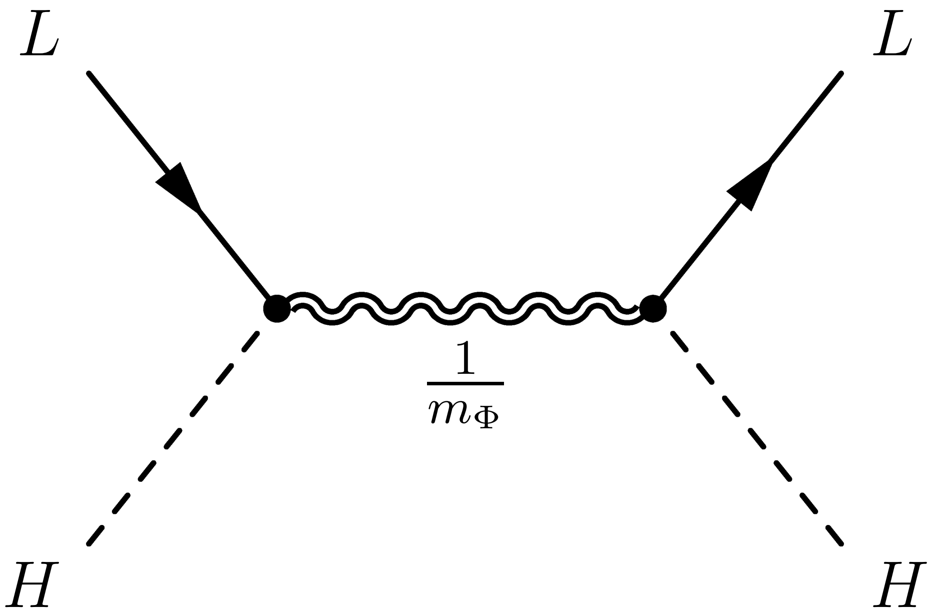
- In the seesaw mechanism, this corresponds to a Majorana mass scale for right-handed neutrinos

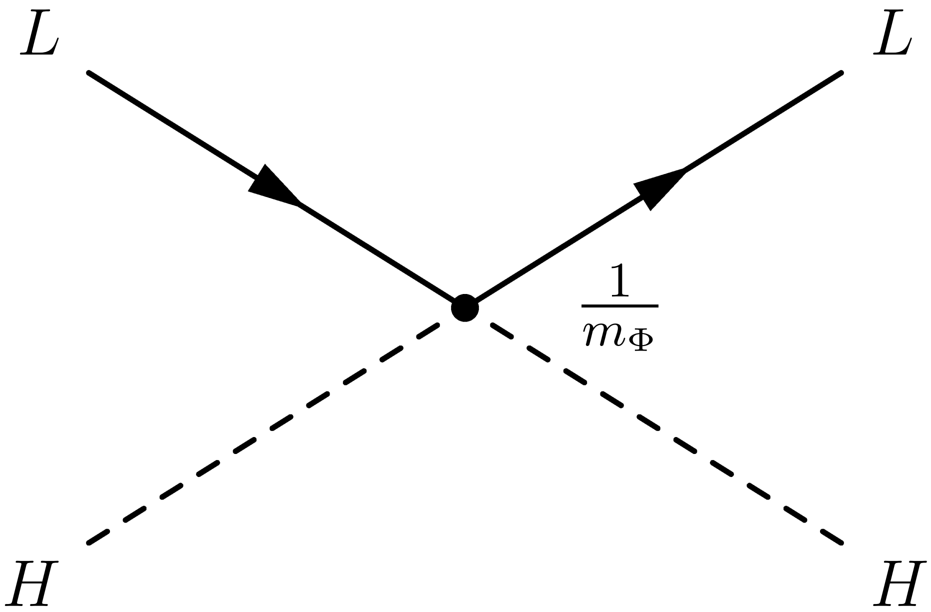
$$M_{\nu_R} \sim 3 \times 10^{14}\text{GeV}.$$

- Equivalently, this is the suppression scale Λ of the dimension five MSSM operator

$$\begin{aligned} \mathcal{O}_{m_\nu} &= \frac{1}{\Lambda} H_2 H_2 L L \\ \Rightarrow m_\nu &= 0.1\text{eV} \left(\sin^2\beta \times \frac{3 \times 10^{14}\text{GeV}}{\Lambda} \right). \end{aligned}$$

Neutrino Masses





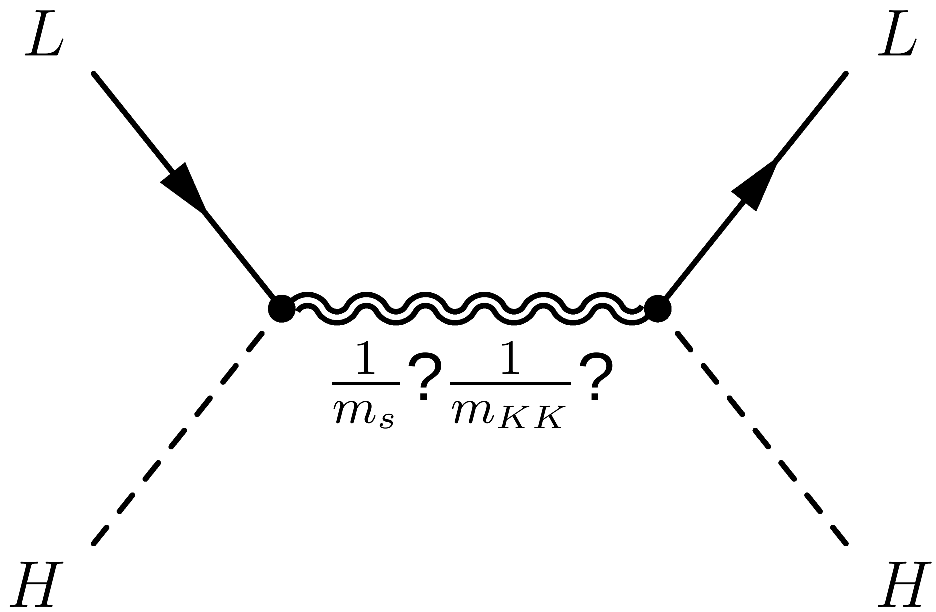
Neutrino Masses

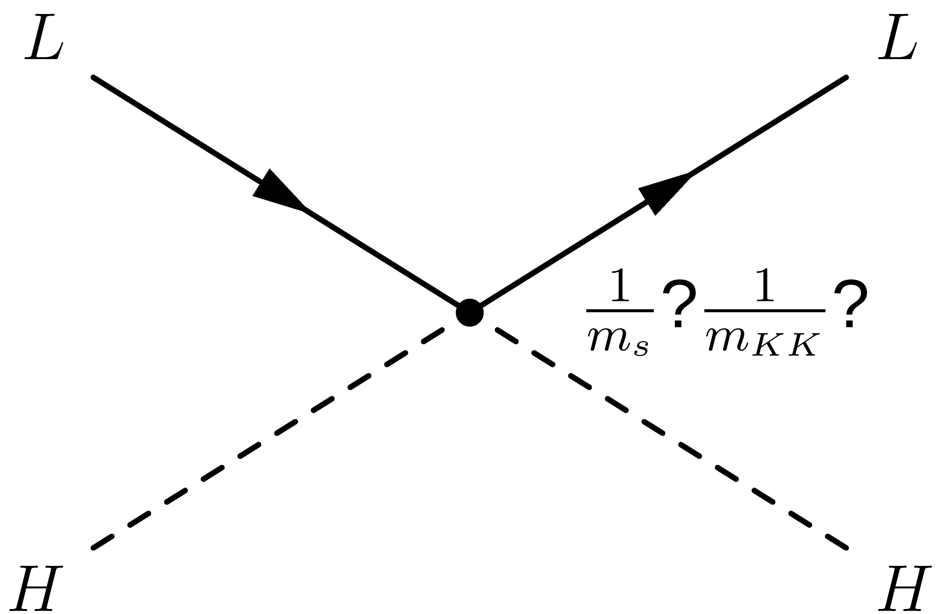
Neutrino masses imply a scale $\Lambda \sim 3 \times 10^{14}$GeV which is

- not the Planck scale 10^{18}GeV
- not the GUT scale 10^{16}GeV
- not the intermediate scale 10^{11}GeV
- not the TeV scale 10^{3}GeV

Can the intermediate-scale string give a quantitative understanding of this scale?

Neutrino Masses





Neutrino Masses

How to describe a Kaluza-Klein or string state in 4d supergravity?

$$m_{heavy} = \frac{m_s}{\mathcal{V}^{\alpha}} = \frac{M_P}{\mathcal{V}^{1/2+\alpha}},$$

- WRONG:

$$K = \Phi\bar{\Phi}$$

$$W = M_{heavy}\Phi^2 = \frac{M_P}{\mathcal{V}^{1/2+\alpha}}\Phi^2 = \frac{M_P}{(T+\bar{T})^{\frac{3+6\alpha}{4}}}\Phi^2.$$

- RIGHT:

$$K = \frac{1}{\mathcal{V}^{1/2-\alpha}}\Phi\bar{\Phi}$$

$$W = M_P\Phi^2$$

Neutrino Masses

The Lagrangian is

$$\begin{aligned}\mathcal{L} &= K_{\Phi\bar{\Phi}}\partial_\mu\Phi\partial^\mu\bar{\Phi} + e^K\left(K^{i\bar{j}}D_iWD_jW - 3|W|^2\right) \\ &= K_{\Phi\bar{\Phi}}\partial_\mu\Phi\partial^\mu\bar{\Phi} + \frac{M_P^2}{\mathcal{V}^2}K^{\Phi\bar{\Phi}}\Phi\bar{\Phi}.\end{aligned}$$

- For KK states,

$$K = \frac{1}{\mathcal{V}^{1/3}}\Phi\bar{\Phi} \text{ gives } m_{KK} = \frac{M_P}{\mathcal{V}^{2/3}}.$$

- For stringy states,

$$K = \frac{1}{\mathcal{V}^{1/2}}\Phi\bar{\Phi} \text{ gives } m_s = \frac{M_P}{\mathcal{V}^{\frac{1}{2}}}.$$

Neutrino Masses

Consider a KK state as a right-handed neutrino

$$
\begin{aligned}
W &= M_P \Phi^2 + Y_{\Phi H L} \Phi H L \\
K &= \frac{1}{\mathcal{V}^{1/3}} \Phi \bar{\Phi} + \frac{1}{\mathcal{V}^{2/3}} H \bar{H} + \frac{1}{\mathcal{V}^{2/3}} L \bar{L}
\end{aligned}
$$

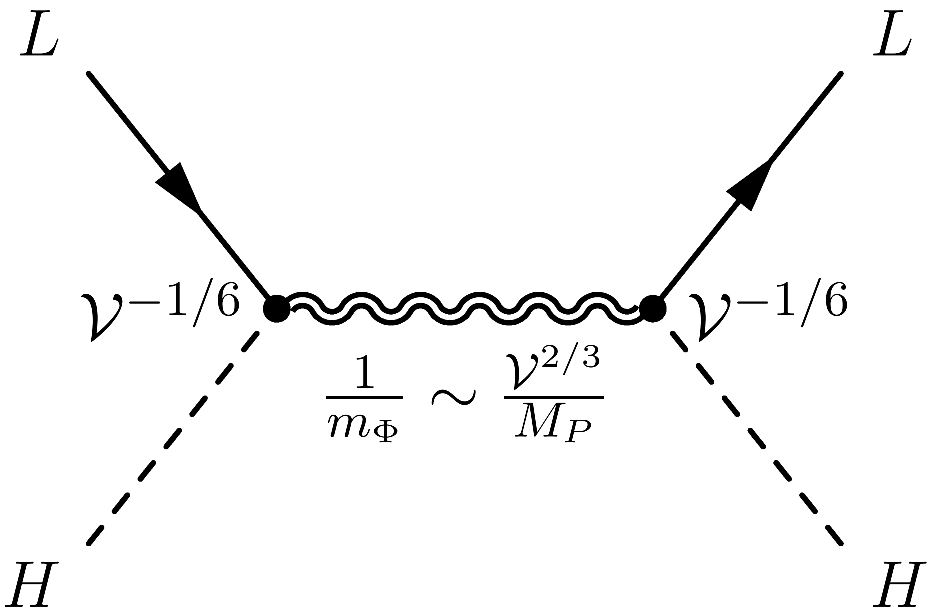
($K_{H\bar{H}}, K_{L\bar{L}} \sim \mathcal{V}^{-2/3}$ follows from locality)

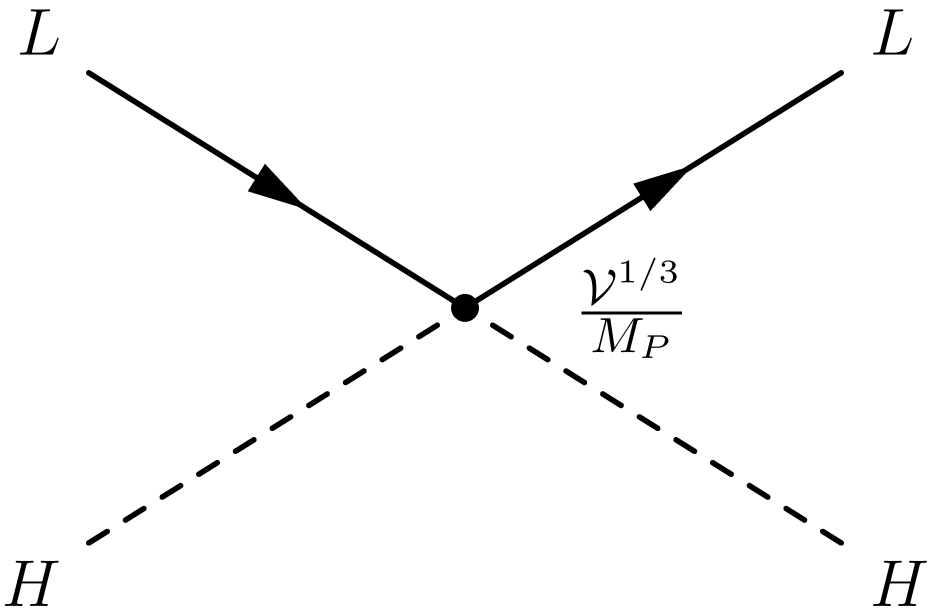
The physical Yukawa is

$$
\begin{aligned}
\hat{Y}_{\Phi H L} &= e^{\hat{K}/2} \frac{Y_{\Phi H L}}{\sqrt{K_{\Phi\bar{\Phi}} K_{L\bar{L}} K_{H\bar{H}}}} \\
&= \mathcal{V}^{-\frac{1}{6}} Y_{\Phi H L}.
\end{aligned}
$$

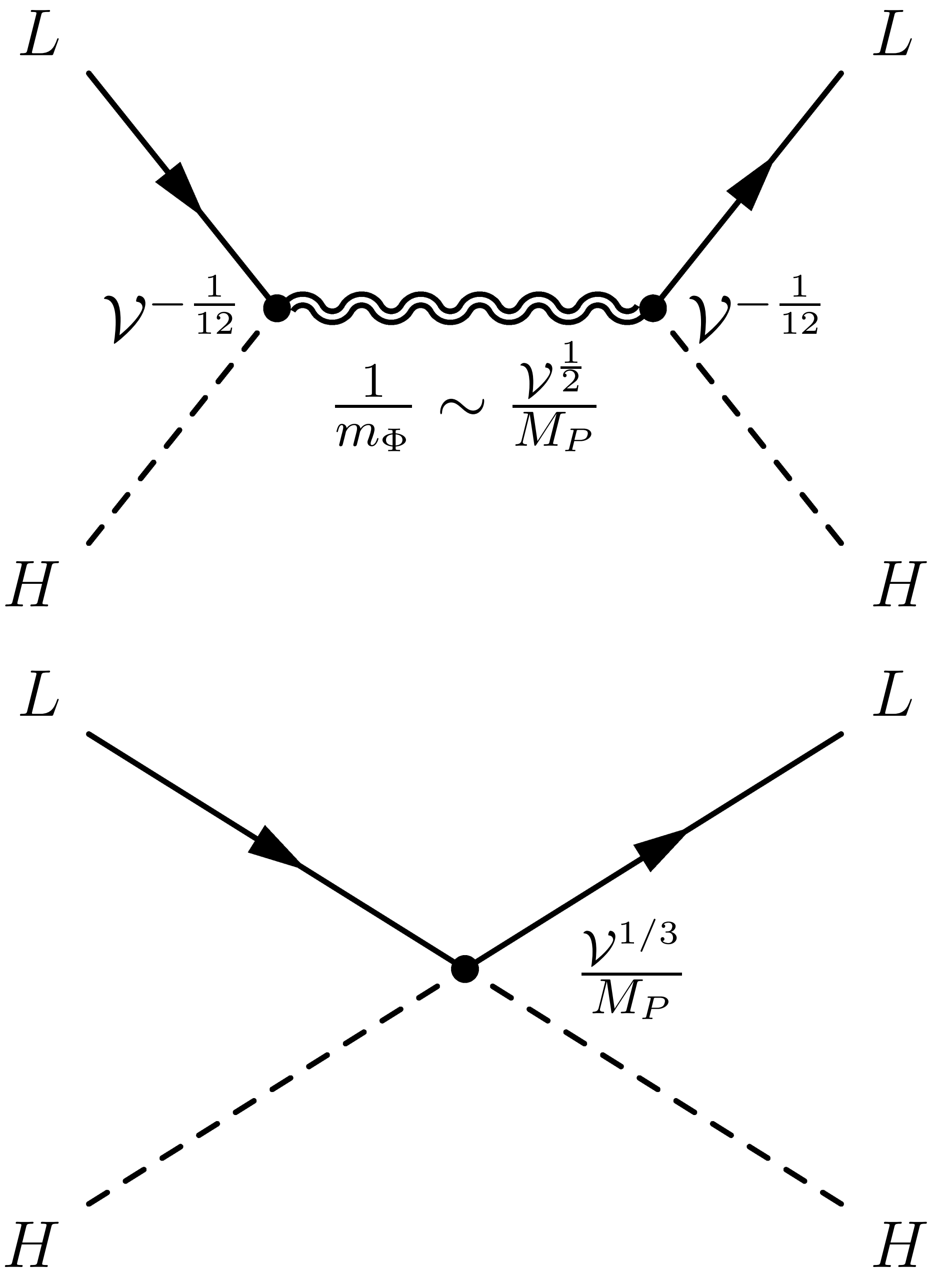
For string states, $\hat{Y}_{\Phi H L} = \mathcal{V}^{-\frac{1}{12}} Y_{\Phi H L}$.

Neutrino Masses

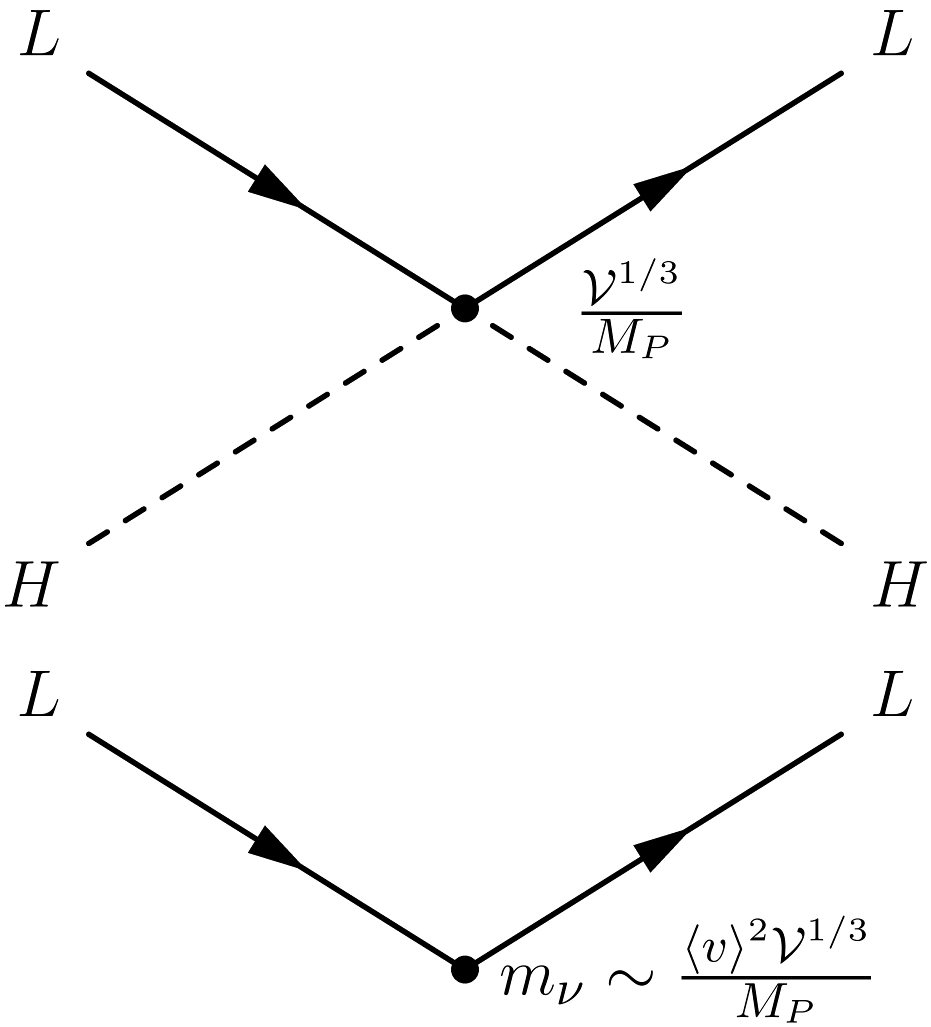




Neutrino Masses



Neutrino Masses



Neutrino Masses

Integrating out string / KK states generates a dimension-five operator suppressed by

$$\text{(string)} \qquad \mathcal{V}^{-1/12} \times \frac{\mathcal{V}^{\frac{1}{2}}}{M_P} \times \mathcal{V}^{-1/12} \sim \frac{\mathcal{V}^{1/3}}{M_P}$$

$$\text{(KK)} \qquad \mathcal{V}^{-1/6} \times \frac{\mathcal{V}^{2/3}}{M_P} \times \mathcal{V}^{-1/6} \sim \frac{\mathcal{V}^{1/3}}{M_P}$$

- Integrating out heavy states of mass M does not produce operators suppressed by M^{-1}.
- The dimension-five suppression scale is independent of the masses of the heavy states integrated out.

Neutrino Masses

Fully,

$$K_{H\bar{H}} \sim K_{L\bar{L}} \sim \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}}.$$

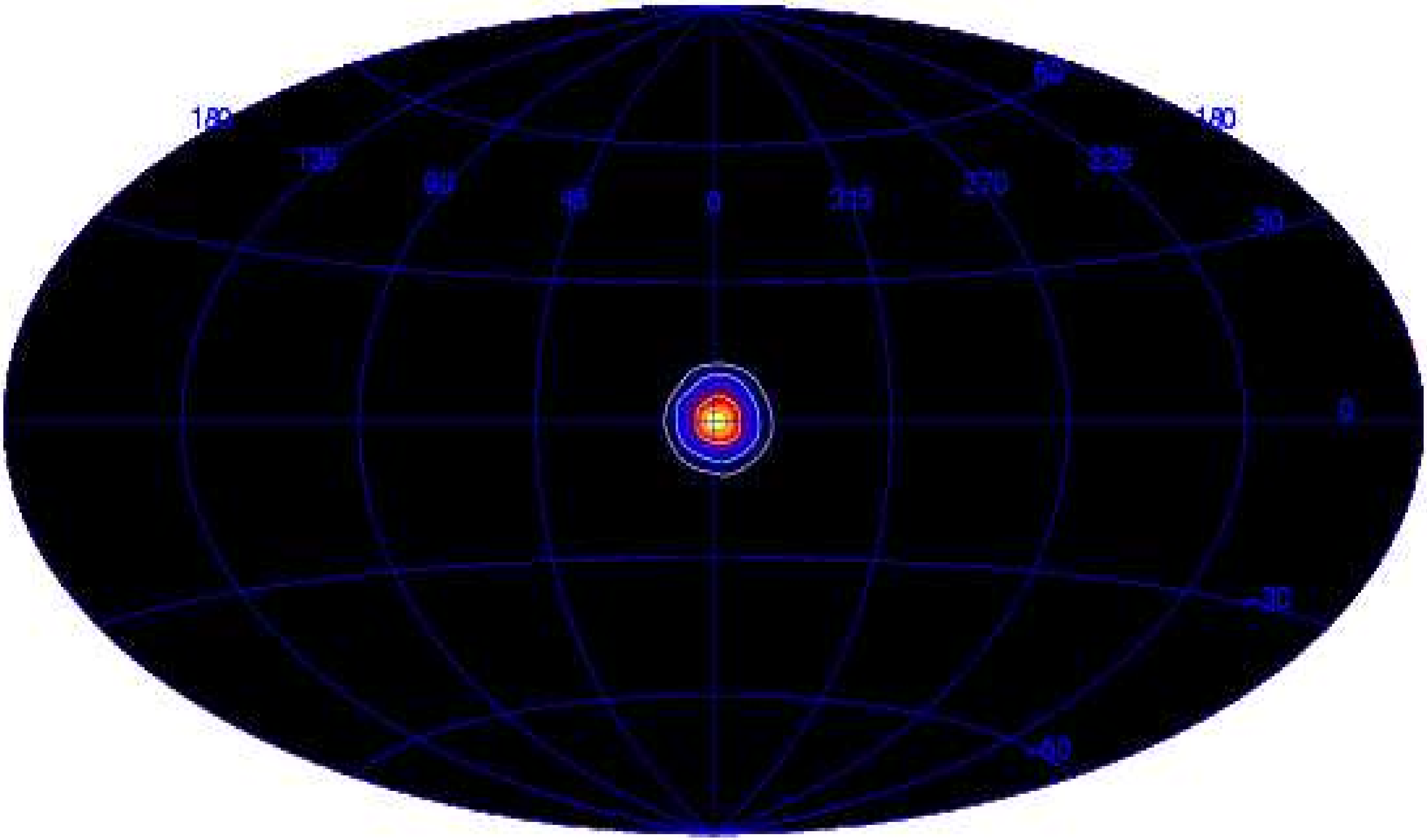
As $\tau_s \sim \alpha_{SM}^{-1}(m_s)$, we have

$$\begin{aligned} m_\nu &\simeq \frac{\langle v \rangle^2 \sin^2\beta \, (\alpha_{SM}(m_s))^{2/3}}{2M_P^{2/3} m_{3/2}^{1/3}} \\ &\simeq 0.09\text{eV} \left(\sin^2\beta \times \left(\frac{20\text{TeV}}{m_{3/2}} \right)^{1/3} \right) \end{aligned}$$

This works remarkably well!

A new scale...

The sky at 511keV



(INTEGRAL/SPI, ESA)

A new scale...

- The galactic centre has an excess of positrons.
- There hints at new physics around 1 MeV.
- This can arise from dark matter annihilating or decaying in the galactic centre.

A new scale....

- The volume modulus χ always has a mass

$$m_\chi \sim \frac{m_{3/2}^{3/2}}{M_P^{\frac{1}{2}}} \sim 1\text{MeV}.$$

- This particle can decay via $\chi \to 2\gamma$ and $\chi \to e^+e^-$.
- One can show

$$Br(\chi \to e^+e^-) \sim \frac{\ln(M_P/m_{3/2})^2}{20} Br(\chi \to 2\gamma)$$

- The e^+e^- decay is strongly preferred.

Large Volumes are Power-ful

In large-volume models, an exponentially large volume naturally appears ($\mathcal{V} \sim e^{\frac{c}{g_s}}$). This generates scales

- Susy-breaking: $m_{soft} \sim \frac{M_P}{\mathcal{V}} \sim 10^3$GeV
- Axions: $f_a \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11}$GeV
- Neutrinos/dim-5 operators: $\Lambda \sim \frac{M_P}{\mathcal{V}^{1/3}} \sim 10^{14}$GeV
- A new scale at $m \sim \frac{M_P}{\mathcal{V}^{3/2}} \sim 1$MeV.
- All four scales (plus flavour universality) are yoked in an attractive fashion.
- The origin of all four scales is the exponentially large volume.