A Bluffers' Guide to Supersymmetry

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1 Introduction and References

These notes represent a quick and dirty introduction to supersymmetry and accompany the three lectures given as part of the supersymmetry course in Trinity term 2009. For further reading:

- *Weak Scale Supersymmetry*, CUP (2006), by Howard Baer and Xerxes Tata.

  Recommended. The focus of this book is really supersymmetry at the LHC, and so fits in well with the general structure of the whole course. If your interest in supersymmetry is primarily the MSSM and its phenomenology, this is the book to buy.

- *The Soft Supersymmetry Breaking Lagrangian: Theory and Applications*, hep-ph/0312378, Chung et al.

  This is a good review of the MSSM and supersymmetry breaking. It's phenomenologically minded and is reasonably easy to read and dip in and out of. Not deep, but a good first reference for TeV supersymmetry and its phenomenology.

- *Part III Supersymmetry Course Notes*, Fernando Quevedo and Oliver Schlotterer, http://www.damtp.cam.ac.uk/user/fq201/susynotes.pdf.

  These are the lecture notes for the 24 hour Part III supersymmetry course at Cambridge given by Fernando Quevedo. This is a full susy course aimed at theorists at roughly first year graduate level.

- *Supersymmetry and Supergravity*, Princeton University Press (1992), Julius Wess and Jonathan Bagger.

  Commonly known just as Wess and Bagger, this book is a classic but is not dip-in reading. Dry but correct. Best used as a reference book. Very good on supergravity and has the 4d N=1 supergravity action in explicit complete gory detail.

- *Quantum Theory of Fields, Volume III*, CUP(2000), Steven Weinberg.

  Everything you ever wanted to know about supersymmetry. The advantage and disadvantage of these books is that Weinberg is one of the great masters of quantum field theory. You learn a lot, but you have to work for it.

2 Motivations for Supersymmetry

Why is supersymmetry even worth considering as a symmetry of nature? We present three reasons here, starting in the more formal and ending in the more phenomenological.

2.1 Susy is Special

We know nature likes symmetries. Symmetry principles underlie most of the dynamics of the Standard Model.

We are familiar with two basic types of symmetry. The particles of the Standard Model are defined as representations of the $SO(3,1)$ symmetry of Minkowski space. The Minwoski space metric

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \quad (1)$$

is invariant under $SO(3,1)$ Lorentz transformations of the coordinates. These transformations are the Lorentz transformations and are the foundation of special relativity. For example, under a Lorentz transformation given by an $SO(3,1)$ matrix $M_{\mu\nu}$, a vector field A_μ transforms as

$$A_\mu^{new} = M_{\mu\nu} A_\nu^{old}. \quad (2)$$

Other representations are the spinor representation that describes fermions such as the quarks and leptons and the (trivial) scalar representation which describes the Higgs.

A second type of symmetry are internal symmetries. These may be either global symmetries (such as the approximate $SU(2)$ isospin symmetry of the strong interactions) or local (gauge) symmetries. An example of these is the $SU(3)$ gauge symmetry of the strong interactions which transforms quark states of different colour into one another. Internal symmetries do not change the Lorentz indices of a particle: an $SU(3)_c$ rotation changes the colour of a quark, but not its spin.

It seems like a cute idea to try and combine these two types of symmetries into one big symmetry group, which would contain spacetime $SO(3,1)$ in one part and internal symmetries such as $SU(3)$ in another. In fact this was an active area of research in prehistoric times (the 1960s), when there were attempts to combine all known symmetries into a single group such as $SU(6)$. However this program was killed by a series of no-go results, culminating in the Coleman-Mandula theorem (1967):

Coleman-Mandula Theorem: *The most general bosonic symmetry of scattering amplitudes (i.e. of the S-matrix) is a direct product of the Poincare and internal symmetries.*

$$G = G_{Poincare} \times G_{internal}$$

In other words, program over. There is no non-trivial way to combine space-time and internal symmetries.

However, like all good no-go theorems, the Coleman-Mandula theorem has a loophole. Not knowing better, Coleman and Mandula only considered bosonic symmetry generators. In the early 1970s supersymmetry began to appear on the scene, and in 1975 Haag, Lopuskanski and Sohnius extended the Coleman-Mandula theorem to include the case of fermionic symmetry generators, which relate particles of different spins.

The result?

Haag, Lopuskanski and Sohnius: *The most general symmetry of the S-matrix is a direct product of super-Poincare and internal symmetries.*

$$G = G_{super-Poincare} \times G_{internal}$$

The super-Poincare algebra is the extension of the Poincare group to include transformations that turn bosons into fermions and fermions into bosons: *supersymmetry* transformations. So Coleman-Mandula were almost right, but not quite: there is one non-trivial way to extend the spacetime symmetries, and that is to incorporate supersymmetry.

The upshot of this is that supersymmetry appears in a special way: it is the unique extension of the Lorentz group as a symmetry of scattering amplitudes. This represents one reason to take supersymmetry seriously as a possible new symmetry of nature.

2.2 Strings and Quantum Gravity

Einstein's theory of general relativity is described, just like other theories, by a Lagrangian

$$\mathcal{L}_{GR} = 16\pi^2 M_P^2 \int d^4x \sqrt{g}\mathcal{R}.$$

Unlike the Standard Model, general relativity is a non-renormalisable theory: all interactions are suppressed by a scale $M_P = 2.4 \times 10^{18}\text{GeV} = 1.7 \times 10^{14} E_{LHC}$. This is somewhat analogous to the Fermi theory of weak interactions, which is also non-renormalisable, and where interactions are suppressed by $1/M_W^2 \sim 1/(100\text{GeV})^2$).

At energy scales below M_P, general relativity works just fine. However at energy scales above M_P, just as for Fermi theory above M_W, general relativity loses predictive power. There are an infinite number of counterterms to be included $(\mathcal{R}^2, \mathcal{R}^3, \ldots)$ and no way to determine their coefficients. This is the problem of quantum gravity.

Theorists rush where experimenters are too sensible to tread, and despite (or in same cases because of) the absence of data lots of theorists have spent lots of time thinking about quantum gravity and how to formulate a consistent theory of it. To make a molehill out a mountain, the most attractive approach is that of string theory. String theory succeeds, often by surprising ways, in taming the divergences of quantum gravity and giving finite answers. It turns out supersymmetry plays a central and essential role in this: in any string theory, nature always looks supersymmetric at sufficiently high energy scales.

If string theory is telling us something about nature, nature is supersymmetric at some energy scale, giving us good reason to regard supersymmetry as a genuine symmetry of nature. The only problem is that it gives us no reason why this scale should be any smaller than the quantum gravity scale, $M_P = 2.4 \times 10^{18}$GeV.

2.3 The Hierarchy Problem

The hierarchy problem is the one really good reason why there is a fair[1] chance supersymmetry will show up at the TeV energy scale.

The hierarchy problem is the problem of why the weak scale is so much smaller than the Planck scale. It is also the most serious theoretical problem of the Standard Model. To recall, in the Standard Model the electroweak symmetry is broken by a vacuum expectation value for the Higgs boson, $\langle v\rangle = 246$GeV. The size of this vev determines the masses of the Z and W bosons, $m_W = \frac{g_w}{\sqrt{2}}v$. The vev of the Higgs is determined by the parameters μ and λ in the Higgs potential,

$$V_H = -\mu^2|\phi| + \lambda|\phi|^4. \tag{3}$$

As in any quantum theory, the parameters in this Lagrangian are subject to quantum corrections. The actual value of the Higgs vev is determined not by the classical values of μ and λ, but by those after quantum corrections. If the Standard Model is valid up to a scale Λ (i.e. no new particles appear until we reach the scale Λ) then we can estimate the size of quantum corrections to the term μ^2. The three largest contributions come from the Higgs self-loop, the W loop and the top quark loop. Let's focus on the top quark loop, shown in figure 2.3.

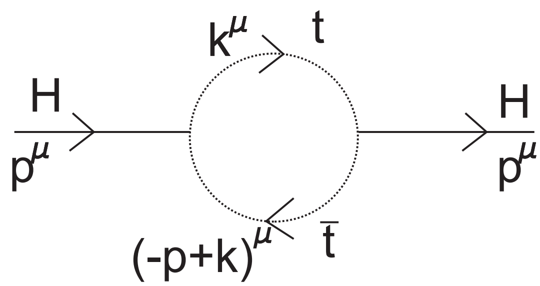

Figure 1: Quantum Corrections to the Higgs Mass.

This diagram gives a correction to the mass term in (3). This amplitude behaves as

$$\begin{aligned}\mathcal{M} &\sim -i\int \frac{d^4k}{(2\pi)^4}\frac{-iy_t}{\not k - m_t}\frac{-iy_t}{\not k + \not p - m_t} \\ &\to_{|k|\to\infty} y_t^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2} \\ &\sim \frac{y_t^2}{16\pi^2}\int_{m_t}^{\Lambda}|k|dk \sim \frac{y_t^2}{16\pi^2}\Lambda^2. \end{aligned}\tag{4}$$

[1]For pub entertainment, quantify 'fair'!

If the Standard Model is valid up to the GUT scale ($\Lambda \sim 10^{16}$GeV), the quantum correction to the Higgs mass term of size $\delta\mu^2 \sim \frac{y_t^2}{16\pi^2}\Lambda^2 \gg m_W^2$. Quantum corrections to the Higgs potential are then enormous: we would expect the quantum-corrected Higgs vev to be around the GUT scale, not the weak scale. However, we know the masses of the W and Z bosons are $\mathcal{O}(10^2\text{GeV})$ and not $\mathcal{O}(10^{16})$GeV.

This is the hierarchy problem: why is the weak scale so much less than the Planck scale? There are two options. Either there is an enormous fluky cancellation going on, or the Standard Model is not valid up to the GUT scale and new physics comes in at the TeV scale, cancelling the divergent terms in the Higgs potential.

As we will see, TeV supersymmetry has the attractive feature of automatically taming the Higgs divergence. This finally makes it reasonable that supersymmetry is not just a symmetry of nature, but a symmetry that could appear at the next major collider, the LHC.

3 Constructing Supersymmetric Theories

Supersymmetric Lagrangians are in many senses easier to write down and easier to deal with than ordinary Lagrangians. Supersymmetric field theories are just special cases of ordinary field theories, but they have many nice properties. It is useful to know at least something about how to construct them, as it underlies the really important physics that supersymmetry is responsible for.

In ordinary quantum field theory, the basic unit of a Lagrangian is a *field*. A field can be *scalar*, *spinor* or *vector*, depending on which type of particle it describes. Different elements of a field (i.e. the different polarisation states of the photon) have essentially the same interactions. Supersymmetry is a symmetry that relates different particles of different spin, and so we to combine fields of different spin into one object. The basic object that does this is called a *superfield*, and as with fields it comes in different types. The different fields in this have related couplings.

The most common type of superfield is called a *chiral superfield*, which we denote by $\Phi = (\phi, \psi_L, F)$. Φ brings together a complex scalar ϕ, a left-handed fermion ψ[2] and a non-dynamical complex scalar F.[3] An example of a superfield would be right-handed top quark superfield $T_R = (\tilde{t}_R, t_R, F_{t_R})$. The physical fields this brings together are the top quark and the top squark.

The other main type of superfield is called a *vector superfield*, denoted by $V = (A_\mu, \lambda, D)$. This brings together a gauge boson A_μ, a gaugino λ and another non-dynamical scalar D. As with F for chiral superfields, D parametrises supersymmetry breaking, and the expression 'D-term supersymmetry breaking' corresponds to this D. An example of a vector superfield is the gluon superfield $G = (g, \tilde{g}, D_g)$, that brings together the gluon and the gluino.

[2]often called a Weyl fermion

[3]F is non-dynamical and so in principal doesn't need to be present. However it makes the equations look much nicer and also encodes the form of supersymmetry breaking. If a theorist talks about F-term breaking, it is this F they are referring to.

3.1 Theories with Chiral Superfields

We will focus on chiral superfields. This will include most of the interactions present in the MSSM. We suppose we have a bunch of chiral superfields Φ_i. To specify the Lagrangian, we now *only* need to specify one function, called the *superpotential*, written $W(\Phi_i)$.[4] $W(\Phi)$ has to be holomorphic - this just means there is no complex conjugation, so Φ^2 is OK but $\Phi\Phi^*$ is not. The nice thing about the superpotential is that it completely specifies the theory, and all the interactions.

Some examples of superpotentials for a theory of two holomorphic superfields Φ_1 and Φ_2:

$$W = m\Phi_1\Phi_2 + \lambda\Phi_1^3$$
$$W = \alpha\Phi_1^2\Phi_2 + \beta\Phi_1\Phi_2^2$$

An inadmissible superpotential (not holomorphic) would be

$$W = m\Phi_1\Phi_2^* + y\Phi_2^3.$$

Lets now write down without proof the general expression for the Lagrangian for chiral superfields. Equations (5) and (6) are here for completeness. The F field plays an important role in the more formal aspects of supersymmetric theories, but here it seems unmotivated: why introduce a field to integrate it out? Feel free to skip to equation (7).

$$\begin{aligned}\mathcal{L} &= \sum_i \left[\partial_\mu\phi_i^*\partial^\mu\phi_i - \frac{1}{2}\sum_i \bar{\psi}_{L,i}\partial\!\!\!/\psi_{L,i} + F_i^*F_i\right] \\ &\quad - \sum_i \left(\left(\frac{\partial W}{\partial\Phi_i}|_{\Phi_i=\phi_i}F_i\right) + \left(\frac{\partial W^*}{\partial\Phi_i^*}|_{\Phi_i=\phi_i}F_i^*\right)\right) \\ &\quad - \frac{1}{2}\sum_{i,j}\left(\left.\frac{\partial^2 W}{\partial\Phi_i\partial\Phi_j}\right|_{\Phi_i=\phi_i}\bar{\psi}_{L,i}\psi_{L,j} + c.c\right). \end{aligned} \tag{5}$$

What we see here are kinetic terms for the scalar and fermion in the superfields, and then a specific set of interactions. As F is non-dynamical, we can solve for its equations of motions to get

$$\begin{aligned}\frac{\partial L}{\partial F_i} &= 0 \longrightarrow F_i^* = \left(\left.\frac{\partial W}{\partial\Phi_i}\right|_{\Phi_i=\phi_i}\right) \\ \frac{\partial L}{\partial F_i^*} &= 0 \longrightarrow F_i = \left(\left.\frac{\partial W^*}{\partial\Phi_i^*}\right|_{\Phi_i=\phi_i}\right)\end{aligned} \tag{6}$$

Integrating out F we therefore obtain the general supersymmetric Lagrangian

[4]In general, we also need a non-holomorphic object called the *Kähler potential* $K(\Phi_i, \bar{\Phi}_j)$, but this is only for non canonically-normalised fields.

for a theory of chiral superfields:

$$\mathcal{L} = \sum_i \left[\partial_\mu \phi_i^* \partial^\mu \phi_i - \frac{1}{2}\sum_i \bar{\psi}_{L,i} \partial\!\!\!/ \psi_{L,i}\right] - \left|\frac{\partial W}{\partial \Phi_i}\right|^2_{\Phi_i=\phi_i} - \frac{1}{2}\sum_{i,j}\left(\left.\frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j}\right|_{\Phi_i=\phi_i} \bar{\psi}_{L,i}\psi_{L,j} + c.c\right). \quad (7)$$

Let's use this. Consider a theory with two superfields H_1 and H_2 and lets consider a superpotential

$$W = \mu H_1 H_2. \quad (8)$$

(This theory of course has absolutely nothing to do with the Higgs sector of the MSSM, and any resemblance is entirely coincidental!) This superpotential looks a bit like a mass term as it couples two superfields. The Lagrangian is

$$\mathcal{L} = \partial_\mu H_1^* \partial_\mu H_1 + \partial_\mu H_2^* \partial_\mu H_2 - \frac{1}{2}\bar{\tilde{H}}_1 \partial\!\!\!/ \tilde{H}_1 - \frac{1}{2}\bar{\tilde{H}}_2 \partial\!\!\!/ \tilde{H}_2 - \mu^2 |H_1|^2 - \mu^2 |H_2|^2 - \mu(\bar{\tilde{H}}_1 \tilde{H}_2 + c.c) \quad (9)$$

The superpotential is indeed revealed to be a mass term. We have a theory with identical masses (μ) for both the scalar fields H_1 and H_2 and their fermionic partners $\tilde{H}_1$ and $\tilde{H}_2$. This is absolutely typical of supersymmetry: in an exactly supersymmetric theory, all components of a superfield have the same mass: the masses of particles and their superpartners are identical.

Now consider a slightly more complicated example. This time we take three superfields, $H_u = (h_u, \tilde{h}_u)$, $T_L = (\tilde{t}_L, t_L)$ and $T_R = (\tilde{t}_R, t_R)$. Again, this is a simplified version of the top quark sector of the MSSM. The superpotential is

$$W = y_t H_u T_L T_R \quad (10)$$

The kinetic terms are all standard. The interaction terms of the Lagrangian are

$$\mathcal{L}_{int} = -y_t^2 (\tilde{t}_L \tilde{t}_L^*)(\tilde{t}_R \tilde{t}_R^*) - y_t^2 (\tilde{t}_L \tilde{t}_L^*)(h_u h_U^*) - y_t^2 (\tilde{t}_R \tilde{t}_R^*)(h_u^* h_u) - y_t\left(\left(h_u \bar{t}_L t_R + \tilde{t}_l \bar{\tilde{h}}_u t_R + \tilde{t}_R \bar{\tilde{h}}_u t_L\right) + c.c\right) \quad (11)$$

Note that we have two types of interaction: quartic scalar interactions and cubic Yukawa couplings. Looking at the Yukawa couplings, we can see among them the familar top quark Yukawa coupling of the Standard Model. Remember, this is the interaction that gave the hierarchy problem. But as well as this we also see quartic scalar interactions that couple the Higgs boson to the top squarks. Furthermore, the strength of this quartic coupling (y_t^2) is *precisely* the square of the strength of the cubic Yukawa coupling.

This is really the single most important point about supersymmetry. Why? It is this feature that leads to the cancellation of quadratic divergences and the taming of the hierarchy problem.

3.2 Formal Information about Susy Field Theories

Supersymmetric quantum field theories have several nice properties that make means theorists spend a lot of time studying them.

For reference let us enumerate some of these

- The potential energy is always positive, $V > 0$. This follows from

$$V_F = \sum_i |\frac{\partial W}{\partial \phi_i}|^2 \tag{12}$$

  Supersymmetry is exact for vanishing potential energy, and is broken for any positive value of the vacuum energy.[5]

- Supersymmetric theories only undergo wavefunction renormalisation. A solution that is supersymmetric at tree level remains supersymmetric to all orders in perturbation theory.

- (more advanced) The best way to state the previous point is that the superpotential is not renormalised in perturbation theory. In contrast, the Kähler potential does get renormalised at all orders in pertubation theory. These non-renormalisation results allow for powerful results regarding susy theories and their behaviour at strong coupling.

4 The MSSM

The MSSM (Minimal Supersymmetric Standard Model) is the reason this course exists. As experimenters, the MSSM in one of its various incarnations is what many of you will spend your PhDs looking for. However, MSSM is really a bad name because all of the interest in the MSSM lies in the fact that the MSSM is not supersymmetric. Before we get to the MSSM, we ought to start with a theory that really is supersymmetric, which we will call the SSM (supersymmetric standard model).

The SSM is a fully supersymmetric theory and is constructed according to the rules we learn above. The SSM consists of three vector superfields, corresponding to the gauge bosons for the $SU(3)_c$, $SU(2)_w$ and $U(1)_Y$ forces. There are also a bunch of chiral superfields which also carry gauge charges. These are shown in table 1. The field content of the Supersymmetric Standard Model should be recognisable as the same as that of the Standard Model, with one exception. As with the Standard Model there are three generations of quarks and leptons, extended of course to include the scalar partners, the squarks and the sleptons. However, whereas in the Standard Model there is only *one* Higgs doublet, in the SSM we see that there are two sets of Higgs doublets.

[5] At first sight this appears to rule out supersymmetry as a symmetry of nature, as the measured vacuum energy $(10^{-3}eV)^4$ is so close to zero and much lower than the susy breaking scale. However gravity saves the day: when you incorporate gravitational effects, the vacuum energy in exactly supersymmetric theories can be negative. A supersymmetry-breaking theory can then have vanishing cosmological constant without an inconsistency arising.

Table 1: Field content of the SSM/MSSM

Field	$SU(3)$	$SU(2)_W$	$U(1)_Y$
$Q_{L,i=1,2,3}$	$\mathbf{3}$	$\mathbf{2}$	1/3
$\bar{U}_{R,i=1,2,3}$	$\bar{\mathbf{3}}$	1	-4/3
$\bar{D}_{R,i=1,2,3}$	$\bar{\mathbf{3}}$	1	2/3
$L_{i=1,2,3}$	1	$\mathbf{2}$	-1
$E_{R,i=1,2,3}$	1	1	2
$H_u = \begin{pmatrix} \hat{h}_u^+ \\ \hat{h}_u^0 \end{pmatrix}$	1	$\mathbf{2}$	1
$H_d = \begin{pmatrix} \hat{h}_d^- \\ \hat{h}_d^0 \end{pmatrix}$	1	$\mathbf{2}$	-1

We will see in a moment why this is required in a supersymmetric theory. However, this immediately tells us one way in which the MSSM differs from the Standard Model: the MSSM has more physical Higgs fields. Electroweak symmetry breaking causes the three $SU(2)$ gauge bosons to acquire a mass, in the process eating three of the scalar Higgs degrees of freedom. The number of physical Higgs particles left over in the MSSM is then $8 - 3 = 5$ compared to the $4 - 3 = 1$ Higgs particle left in the Standard Model.

The superpotential of the SSM is

$$W_{MSSM} = \mu H_u H_d + \sum_{ij} (Y_u)_{ij} H_u Q_{L,i} \bar{U}_{R,j} + \sum_{ij} Y_d)_{ij} (H_d)(Q_L)_i \bar{D}_{R,j} + \sum_{ij} (Y_L)_{ij} H_d L_i \bar{E}_{R,j} \tag{13}$$

Y_u, Y_d and Y_L are the respective Yukawa couplings for the up-type quarks, the down-type quarks and the leptons. At first it seems odd that the superfields are $\bar{E}_R$ and $\bar{U}_R$ - why don't we just use E_R and U_R? However this is necessary in order to have a supersymmetric theory. A supersymmetric Lagrangian must be holomorphic in the superfields, and W_{MSSM} is holomorphic in $\bar{E}_R$ but not in E_R (as complex conjugation breaks holomorphy properties).[6]

We can now see why the SSM requires two Higgs doublets: its necessary for holomorphy purposes. Let's write the superpotential we would need if we only had one Higgs doublet:

$$W_{MSSM} = \mu H_u H_u^* + \sum_{ij} (Y_u)_{ij} H_u Q_{L,i} \bar{U}_{R,j} + \sum_{ij} Y_d)_{ij} (H_u^*)(Q_L)_i \bar{D}_{R,j} + \sum_{ij} (Y_L)_{ij} H_u^* L_i \bar{E}_{R,j} \tag{14}$$

This isn't holomorphic in H_u, as we have both H_u and H_u^* present in the superpotential. So the only way to incorporate all the Yukawa couplings in a supersymmetric theory is to have two Higgs doublet rather than the single Higgs doublet of the Standard Model.

There is a second reason why we need two Higgs doublets in a supersymmetric theory. The whole point of supersymmetry is that bosons come with

[6] An alternative way to say this is that chiral superfields by convention involve left-handed fermions. The right-handed electron is not left-handed, but its physically equivalent antiparticle ($\bar{e}_R$) is.

fermionic partners and fermions come with bosonic partners. Now, in the presence of chiral fermions it is possible for gauge theories to be anomalous. If a gauge symmetry is anomalous, it means it is not preserved at the quantum level. However, the gauge symmetry was the principle we used to build the theory in the first place. So a theory with an anomalous gauge symmetry is actually deeply sick, and is in fact inconsistent. In principle the Standard Model could have been anomalous: but when you work out the anomalies coming from the Standard Model fermions you find that they all cancel - which is good because we know the Standard Model describes nature.[7]

The Higgs of the Standard Model is a scalar field, and does not contribute to anomalies. However in a susy theory the Higgs has fermionic Higgsino partners, which do contribute to anomalies and would make the theory inconsistent. Supersymmetrising the Higgs doublet of the Standard Model actually leads to an inconsistent theory! However if we have two Higgs doublets, then the anomaly from one Higgsino can be cancelled by that of the other, keeping the theory quantum mechanically consistent.

4.1 R Parity

Astute observers such as yourselves will have noticed that the MSSM superpotential (14) is not the most general superpotential of the MSSM fields that is consistent with all the gauge symmetries. There are several other operators that can be added to the MSSM. For example consider the following extension of the superpotential (14).

$$W = W_{MSSM} + \alpha \bar{U}_R \bar{D}_R \bar{D}_R + \beta Q_L L \bar{D}_R \tag{15}$$

Why have we focused on these operators? It is because these operators are literally fatal: the proton becomes unstable to the decay $p \to \pi_0 e^+$. To see how this comes about, remember that we derive fermion-fermion-scalar couplings from a superpotential by differentiating twice: $(\partial^2 W/\partial\Phi_i \partial\Phi_j)|_{\Phi_k \to \phi_k} \psi_i \psi_j$. The superpotential (15) therefore gives the interactions (suppressing spinor and colour indices)

$$\mathcal{L} = \alpha u_R d_R \tilde{d}_r + \beta q_l e_R \tilde{d}_R \tag{16}$$

These two vertices can be combined to generate the decay $p \to \pi_0 e^+$. The lifetime of this decay can be estimated from dimensional analysis to be

$$\tau \sim \alpha^2 \beta^2 \frac{m^5_{\tilde{d}_R}}{m_p^4} \sim \left(\frac{m_{\tilde{d}_R}}{1\text{TeV}}\right)^5 10^{-10} s.$$

This suggests that we do not want the operators (15) in the superpotential.

R-parity is the standard way to forbid the operators (15). It is not the only such way, but it is the most common approach. R Parity is the imposition of a discrete $\mathbb{Z}_2$ symmetry

$$(-1)^{3(B-L)+2S}$$

[7] Anomalies are the reason the top quark had to exist - it was only ever a question of what its mass was. The Standard Model without the top quark is an anomalous theory and not consistent at a quantum level. So after the bottom quark was discovered, the top also had to exist, it was just a question of what its mass was. In due course the top was indeed discovered at the TeVatron in 1995.

on the MSSM. Here B is baryon number (1/3 for quarks, -1/3 for antiquarks), L is lepton number and S is the spin of the field. Only interactions with $R = +1$ are kept. Note that Standard Model fields and their superpartners have different charges under R parity.[8]

It can now be verified (exercise!) that the only operators allowed in the superpotential are those already in W_{MSSM}: R parity ditches the junk that leads to phenomena such as proton decay and keeps the good stuff. For example, the interaction $u_R d_R \tilde{d}_R$ that proved dangerous for proton decay has R-parity -1. The easiest way to remember the R-charge of each field is to remember that $R = +1$ for Standard Model fields and $R = -1$ for all superpartners.

This last comment gives another important phenomenological implication of R-parity: the lightest superpartner is stable, as it has R-charge -1 and there are no lighter fields for it to decay into. This fact is important both for collider searches for supersymmetry (LSP does not decay and escapes the detector) and for cosmology (LSP can be stable on cosmological timescales).

4.2 The MSSM and Soft Breaking

The SSM was the supersymmetrisation of the Standard Model. As an exactly supersymmetric theory, this means that all superpartners have the same masses as their ordinary counterparts. For example, the slepton would have a mass of 0.5 MeV. No superpartners have yet been seen, so this theory clearly does not describe nature.

The MSSM (Minimal Supersymmetric Standard Model) is the extension of the SSM to include supersymmetry breaking (this is why MSSM is a misleading name). In fact, all of the interest in the MSSM lies in the terms, called *soft terms*, that break supersymmetry. The expression *soft* has a particular meaning. We originally said that supersymmetry is attractive because it cancels quadratic divergences. A general non-supersymmetric theory does not cancel quadratic divergences. *Soft terms* are those particular non-supersymmetric terms that can be added to a supersymmetric Lagrangian while preserving the property that all quadratic divergences cancel.[9]

The full set of soft terms were classified by Girardello and Grisaru back in 1982. The soft terms consist of

- Scalar masses: $m^2 \tilde{q}_L^* \tilde{q}_L$, $m^2 H_u^* H_u$

  These are mass terms for squarks, sleptons and Higgs fields.

- Gaugino masses: $M \lambda^a \lambda_a$

  These are masses for the the gluino, the wino, the zino and the bino.

[8] In supersymmetric field theories, any symmetries under which the different-spin components of a supermultiplet have different charges is called an R symmetry. This is the origin of the R in R parity.

[9] The appearance of soft terms are not *ad hoc*. They are generated when you have a fundamental theory that is supersymmetric, but in which supersymmetry is spontaneously broken at low energies. The underlying supersymmetry implies that the only non-supersymmetric terms that arise are the soft terms.

- Trilinear scalar A-terms $A_{\alpha\beta\gamma}\phi^{\alpha}\phi^{\beta}\phi^{\gamma}$
- B term BH_uH_d.

  This term only exists for the Higgs doublets, as it is a holomorphic mass term.

Although it is not a soft term (as it is present in the supersymmetric theory), it is helpful to regard the μ term μH_1H_2 in a similar way, as its value in the MSSM is not specified by knowledge of the Standard Model.

The MSSM (Minimal Supersymmetric Standard Model) is just the supersymmetric Standard Model extended by the soft terms. The phenomenology of the MSSM is completely determined by the value of the soft terms and μ term.

4.2.1 MSUGRA

One canonical choice of soft terms is given by mSUGRA (short for minimal supergravity, also called the CMSSM (constrained MSSM)). It is unlikely (but not impossible) that nature is described by mSUGRA. However mSUGRA provides the straw man against which other theories can be compared, and also provides a benchmark for experiments to use.

mSUGRA is defined by the following choice of soft terms:[10]

1. Universal scalar masses $m_\phi^2 = m_0^2$
2. Universal gaugino masses $M_a = M_{1/2}$
3. Universal A-terms $A_{\alpha\beta\gamma} = AY_{\alpha\beta\gamma}$
4. B-term B and μ term μ.

The universality conditions are defined at the GUT scale $M_{GUT} = 2\times10^{16}$GeV, as they are assumed to be generated by string/GUT scale physics. The actual physical Lagrangian is obtained by using the renormalisation group to evolve these soft terms down to the TeV scale. There are several programs that do this evolution; one of the best known is SoftSUSY by Ben Allanach. A typical evolution of soft terms is shown in figure 2.

The physical mass spectrum is determined by the details of the renormalisation group flow. Many of the typical properties of supersymmetric spectra follow from these renormalisation group equations. For example, it is the RGEs that imply the colored particles are typically much heavier than the sleptons, that the stau is the lightest slepton and so on.

After electroweak symmetry breaking, several of the MSSM particles are neutral under the vacuum $SU(3)\times U(1)_{em}$ gauge symmetry. The neutral fermions are called the *neutralinos*. The neutralinos, normally denoted $\chi_1, \chi_2, \chi_3, \chi_4$ are the physical mass states formed from the photino, the Zino and two Higgsinos. That is, the superpartners of the hypercharge gauge boson, the neutral $SU(2)$

[10] It is not obvious that universality for the A terms should mean a universal constant *multiplied by the Yukawa couplings*. Please accept this!

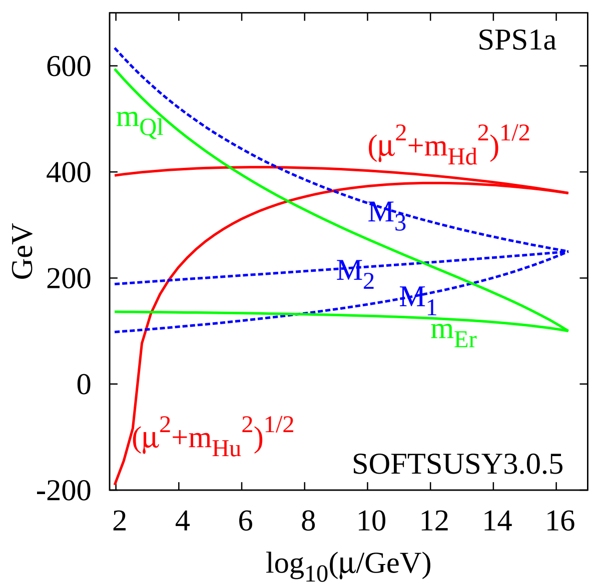

Figure 2: The evolution of the soft parameters in the MSSM. Note how the coloured particles (M_3 and Q_L rapidly grow in mass whereas the wino (M_2) and bino (M_1) decreases slightly. Also note that crucially the up-type Higgs mass becomes negative at small energies, inducing radiative electroweak symmetry breaking.

gauge boson and the two neutral Higgses are all neutral and mix to create the physical neutralino states.

$$\begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} \begin{pmatrix} \tilde{B} \\ \tilde{Z} \\ \tilde{H}_u \\ \tilde{H}_d \end{pmatrix} \tag{17}$$

Across a large area of MSUGRA parameter space the lightest neutralino is both mostly Bino and also the lightest supersymmetric particle. This lightest neutralino is a WIMP and a good candidate for cold dark matter.

The parameters B and μ relate to the Higgs potential. At low energy the Higgs potential determines the mass of the Z boson and also the way the physical Higgs field is made up of the up-type Higgs and down-type Higgs that are present in the MSSM. It is customary to trade the parameters B and μ for M_Z (which has been measured) and the ratio

$$\tan\beta = \frac{\langle h_u^0 \rangle}{\langle h_d^0 \rangle}$$

$\tan\beta$ is the ratio of the up-type Higgs vev and down-type Higgs vev, and describes the composition of the physical Higgs (large tan β implies the physical Higgs is mostly up-type Higgs). In practice the parameters of mSUGRA are therefore

1. Universal scalar masses $m_\phi^2 = m_0^2$
2. Universal gaugino masses $M_a = M_{1/2}$
3. Universal A-terms $A_{\alpha\beta\gamma} = AY_{\alpha\beta\gamma}$
4. $\tan\beta = \frac{\langle h_u^0 \rangle}{\langle h_d^0 \rangle}$.
5. sgn(μ), the sign of the μ term.

Various groups have scanned over the parameter space of MSUGRA, studying the spectrum generated at each point and the phenomenological constraints on the parameter space. Various experimental constraints - for example $BR(b \to s\gamma)/$ or the LEP constraints on the Higgs mass - constrain the data. The (strong) assumption that the dark matter consists of the lightest supersymmetric particle as a thermal WIMP further restricts the parameter space, and the allowed allowed regions in this parameter space go by the name of 'focus point region', 'stau coannihilation region' and so forth.

Let us end on an important note: MSUGRA is a toy model. The world is not MSUGRA, even if supersymmetry is realised in nature. Be prepared!