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SUPERSYMMETRY: EXAMPLE SHEET 3

1. Consider the Wess-Zumino model consisting of one chiral superfield Φ with standard Kähler potential $K = \Phi^\dagger \Phi$ and tree-level superpotential.

$$W_{tree} = \frac{1}{2} m \Phi^2 + \frac{1}{3} g \Phi^3. \tag{1}$$

Consider the couplings g and m as spurion fields, and define two $U(1)$ symmetries of the tree-level superpotential, where both $U(1)$ factors act on Φ, m and g, and one of them also acts on θ (making it an R-symmetry).

Using these symmetries, infer the most general form that any loop corrected superpotential can take in terms of an arbitrary function $F(\Phi, m, g)$. Consider the weak coupling limit $g \to 0$ and $m \to 0$ to fix the form of this function and thereby show that the superpotential is not renormalised.

2. Consider a renormalisable $\mathcal{N} = 1$ susy Lagrangian for chiral superfields with F-term supersymmetry breaking. By analysing the scalar and fermion mass matrix, show that

$$\mathrm{STr}(M^2) \equiv \sum_j (-1)^{2j+1}(2j+1) m_j^2 = 0, \tag{2}$$

where j is particle spin. Verfiy that this relation holds for the O'Raifertaigh model, and explain why this forbids single-sector susy breaking models where the MSSM superfields are the dominant source of susy breaking.

3. This problem studies D-term susy breaking. Consider a chiral superfield Φ of charge q coupled to an Abelian vector superfield V. Show that a nonvanishing vev for D, the auxiliary field of V, cam break supersymmetry, and determine the goldstino in this case.

Write down the D-term scalar potential, and find the condition that the FI term and the charge q have to satisfy for supersymmetry to be broken. Find the spectrum of the model once supersymmetry is broken, and find the mass splitting of the multiplet.

4. Consider a renormalisable $\mathcal{N} = 1$ supersymmetric theory with chiral superfields $\Phi_i = (\phi_i, \psi_i, F_i)$ and vector superfields $V_a = (\lambda_a, A_a^\mu, D_a)$ with both D- and F-term supersymmetry breaking ($F_i \neq 0, D_a \neq 0$). Show that in the vacuum

$$\frac{\partial V}{\partial \phi^i} = F^j \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} - g^a D^a \phi_j^\dagger (T^a)_i^{\ j} = 0.$$

Here g^a and T^a are the different gauge couplings and generators. Also note that as the superpotential is gauge invariant, the gauge variation of W is

$$\delta^a_{gauge} = \frac{\partial W}{\partial \phi^i} \delta^{(a)}_{gauge} \phi^i - F_i^\dagger (T^a)_j^{\ i} \phi^j.$$

Write these two conditions in the form of a matrix M acting on a 'two-vector' with components $\langle F^j \rangle$ and $\langle D^a \rangle$. Identify this matrix and show that it is the same as the fermion mass matrix. Argue that it always has one zero eigenvalue, which can be identified as a goldstone fermion - the goldstino.

5. This problem studies the vacuum structure of $\mathcal{N}=4$ SYM written in $\mathcal{N}=1$ language.

Consider a $SU(2)\mathcal{N}=1$ supersymmetric theory with three chiral superfields in the adjoint representation Φ_1, Φ_2, Φ_3 with superpotential:

$$W = \epsilon_{ijk}\mathrm{Tr}\Phi_i[\Phi_j, \Phi_k].$$

Assuming a canonical Kähler potential, calculate the scalar potential and find the form of solutions that have vanishing F and D fields.

Find the flat scalar field directions which preserve supersymmetry and the parts of the gauge group that are broken along these directions.

The above is the $\mathcal{N}=4$ theory in $\mathcal{N}=1$ language. We can explicitly break the $\mathcal{N}=4$ theory down to either $\mathcal{N}=2$ or $\mathcal{N}=1$ supersymmetry by adding terms to the superpotential. Write $W_{new} = W + \Delta W$, where

$$\Delta W = m_1\mathrm{Tr}\Phi_1^2 + m_2\mathrm{Tr}\Phi_2^2 + m_3\mathrm{Tr}\Phi_3^2.$$

If all masses are non-zero, the resulting theory is $\mathcal{N}=1$ supersymmetric. Show that the field equations now become

$$[\Phi_i, Phi_j] = \epsilon_{ijk} m_l \Phi_k.$$

Which matrices Φ_i satisfy this equation?

6. This problem studies the MSSM. The field content of the MSSM under $SU(3)\times SU(2)\times U(1)_Y$ is

$$Q_i = (3, 2, -1/6), \qquad \bar{U}_{R,i} = (\bar{3}, 1, 2/3), \qquad \bar{D}_{R,i} = (\bar{3}, 1, -1/3)$$
$$L_i = (1, 2, 1/1), \qquad \bar{E}_R = (1, 1, -1), \qquad H_u = (1, 2, -1/2), \qquad H_d = (1, 2, 1/2).$$

Write down the most general gauge-invariant cubic superpotential for these fields, and group the terms into those that preserve baryon and lepton number and those that violate it. Show that if all such terms are present, combining two of the baryon/lepton-number violating terms gives proton decay,

$$p \to e^+ + \pi^0,$$

and estimate the rate of this process.

Experimentally

$$\tau_{proton} > 6\times 10^{32}\mathrm{years} = 2.4\times 10^{64}\mathrm{GeV}^{-1}.$$

Use this to bound the product of two 'Yukawa' couplings appearing in the above diagram.

Verify that R-parity forbids all dangerous B and L-violating couplings, and show that there is a quartic Higgs term in the resulting theory.

7. This problem examines no-scale supergravity. Consider $\mathcal{N}=1$ supergravity with three chiral superfields S, T and C. The Kähler potential iss ($M_P = 1$).

$$K = -\ln(S+\bar{S}) - 3\ln\left(T + \bar{T} - C\bar{C}\right)$$

The superpotential is

$$W = C^3 + ae^{-\alpha S} + b$$

Here a, b are arbitrary complex numbers and $\alpha > 0$. Compute the scalar potential. What is the vacuum energy in the minimum and are there flat directions in the potential?

In supergravity the auxiliary fields are proportional to the Kähler covariant derivatives $D_i W = \partial_i W + \partial_i K W$. Find the auxiliary fields for S, T, C and verify that susy is broken. What is the gravitino mass?

If the gauge kinetic function is $f = T$, what is the gaugino mass for this gauge group?