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SUPERSYMMETRY: EXAMPLE SHEET 2

1. Show that

$$\begin{aligned} P_\mu &= -i\partial_\mu \\ Q_\alpha &= -i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu)_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu \\ \bar Q_{\dot\alpha} &= Li\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \theta^\gamma(\sigma^\mu)_{\gamma\dot\alpha}\partial_\mu \end{aligned}$$

satisfy the $\mathcal{N} = 1$ supersymmetry algebra.

2. Given the general scalar superfield

$$\begin{aligned} S(x,\theta,\bar\theta) &= \phi(x) + \theta\psi(x) + \bar\theta\bar\chi(x) + \theta\theta M(x) + \bar\theta\bar\theta N(x) \\ &\quad +(\theta\sigma^\mu\bar\theta)V_\mu(x) + (\theta\theta)\bar\theta\bar\lambda(x) + (\bar\theta\bar\theta)\theta\rho(x) + \theta\theta\bar\theta\bar\theta D(x), \end{aligned}$$

show that under a supersymmetry transformation

$$\delta S = i(\epsilon Q + \bar\epsilon\bar Q)S$$

the components of the superfield transform as

$$\begin{aligned} \delta\phi &= \epsilon\psi + \bar\epsilon\bar\chi & \delta\psi &= 2\epsilon M + (\sigma^\mu\bar\epsilon)(i\partial_\mu\phi + V_\mu) \\ \delta\bar\chi &= 2\bar\epsilon N - (\epsilon\sigma^\mu)(i\partial_\mu\phi - V_\mu) & \delta M &= \bar\epsilon\bar\lambda - \frac{i}{2}\partial_\mu\psi\sigma^\mu\bar\epsilon \\ \delta N &= \epsilon\rho + \frac{i}{2}\epsilon\sigma^\mu\partial_\mu\bar\chi & \delta V_\mu &= \epsilon\sigma_\mu\bar\lambda + \rho\sigma_\mu\bar\epsilon + \frac{i}{2}(\partial^\nu\psi\sigma_\mu\bar\sigma_\nu\epsilon - \bar\epsilon\bar\sigma_\nu\sigma_\mu\partial^\nu\bar\chi) \\ \delta\bar\lambda &= 2\bar\epsilon D + \frac{i}{2}(\bar\sigma^\nu\sigma^\mu\bar\epsilon)\partial_\mu V_\nu + i(\bar\sigma^\mu\epsilon)\partial_\mu M & \delta\rho &= 2\epsilon D - \frac{i}{2}(\sigma^\nu\bar\sigma^\mu\epsilon)\partial_\mu V_\nu + i(\sigma^\mu\bar\epsilon)\partial_\mu N \\ \delta D &= \frac{i}{2}\partial_\mu(\epsilon\sigma^\mu\bar\lambda - \rho\sigma^\mu\bar\epsilon) \end{aligned}$$

Note particularly that the transformation of the field D is a total derivative.

3. Verify that the superfield

$$\begin{aligned} \Phi(x,\theta,\bar\theta) &= \phi(x) + \sqrt{2}\theta\psi(x) + (\theta\theta)F(x) \\ &\quad +i(\theta\sigma^\mu\bar\theta)\partial_\mu\phi - \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\partial_\mu\partial^\mu\phi - \frac{i}{\sqrt{2}}(\theta\theta)\partial_\mu\psi(x)\sigma^\mu\bar\theta \end{aligned}$$

is a chiral superfield ($\bar{\mathcal{D}}\Phi = 0$). Find out how each of the components ϕ, ψ, F transform under supersymmetry.

4. Find the coefficient of $(\theta\theta)(\bar{\theta}\bar{\theta})$ of $\Phi^\dagger\Phi$ where Φ is a chiral superfield.

5. Find the coefficient of $(\theta\theta)$ of the combination $\frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3$, where Φ is a chiral superfield and m, g are constants.

6. In the Wess-Zumino model, find the minima of the scalar potential and expanding around one of the minima show explicitly that the mass of the scalar field equals the mass of the fermion field and that the quartic self-coupling of the scalar field equals the Yukawa coupling among this field and the fermion.

7. For an Abelian vector superfield V, show that the field strength superfield

$$W_\alpha(x,\theta,\bar{\theta}) = -\frac{1}{4}(\bar{\mathcal{D}}\bar{\mathcal{D}})\mathcal{D}_\alpha V(x,\theta,\bar{\theta})$$

is chiral ($\bar{\mathcal{D}}W_\alpha = 0$) and gauge invariant ($V \rightarrow V + i(\Lambda - \Lambda^\dagger), W_\alpha \rightarrow W_{alpha}$). Find the F-component of $\frac{1}{4}W^\alpha W_\alpha$ in terms of $F_{\mu\nu}, \lambda$ and D.

8. Prove that the Lagrangian

$$\mathcal{L} = (\partial_\mu\phi\partial^\mu\phi^*) - i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi + FF^* + m\left(\phi F - \frac{1}{2}\psi\psi + h.c.\right)$$

is invariant, up to a total derivative, under the supersymmetry transformations

$$\begin{aligned} \delta\phi &= \sqrt{2}\epsilon\psi \\ \delta\psi &= i\sqrt{2}\sigma^\mu\bar{\epsilon}\partial_\mu\phi + \sqrt{2}\epsilon F \\ \delta F &= \sqrt{2}i\bar{\epsilon}\bar{\sigma}^\mu\partial_\mu\psi \end{aligned}$$

where ϵ is an anticommuting parameter and m a constant (mass). Here $h.c.$ stands for hermitian conjugate.