

LECTURE

RETURN TO OUR BASIC CONSERVATION EQN. (1)

$$\frac{d}{dt} \langle \psi | Q | \psi \rangle = \frac{i}{\hbar} \langle \psi | [H, Q] | \psi \rangle$$

SO FAR STUDIED CONSEQUENCES FOR SIMPLEST CASE $Q = 1$

• NOW EXPLORE MORE GENERALLY

$$\left(\text{AS USUAL TAKE } H = \frac{p^2}{2m} + V(x) \right)$$

i) TOTAL ENERGY IS CONSERVED BECAUSE

$$[H, H] = HH - HH = 0$$

$$\text{SO } \frac{d}{dt} \langle \psi | H | \psi \rangle = 0 \text{ FOR ALL } |\psi\rangle$$

ii) NOW CONSIDER MOM'UM p

$$[H, p] = \left[\frac{p^2}{2m} + V(x), p \right]$$

$$= \left[\frac{p^2}{2m}, p \right] + [V(x), p]$$

$$= \frac{1}{2m} \underbrace{(p^2 p - p p^2)}_0 + [V(x), p]$$

$$= [V(x), p]$$

(2)

BUT $[V(x), p] = [V(x), -i\hbar \partial/\partial x]$

TO WORK THIS OUT PUT ARBITRARY FN ON RHS

$$\begin{aligned} [V(x), p] f(x) &= \left\{ V(x) \left(-i\hbar \frac{\partial}{\partial x} \right) + i\hbar \frac{\partial}{\partial x} V(x) \right\} f(x) \\ &= \underbrace{-i\hbar V(x) \frac{\partial f}{\partial x}} + i\hbar \frac{\partial V}{\partial x} f + \underbrace{i\hbar V \frac{\partial f}{\partial x}} \\ &= i\hbar \frac{\partial V}{\partial x} f \end{aligned}$$

$$\Rightarrow [V(x), p] = i\hbar \frac{\partial V}{\partial x} \quad \underline{\text{NOT ZERO}}$$

THEREFORE

$$[H, p] = i\hbar \frac{\partial V}{\partial x} \quad \text{AND}$$

$$\boxed{\frac{d\langle p \rangle}{dt} = -\left\langle \frac{\partial V}{\partial x} \right\rangle} \quad (*)$$

NOTE SIMILARITY OF (*) TO CLASSICAL
EQN OF MOTION

(*) SHOWS THAT $d\langle p \rangle/dt$ CANNOT BE ZERO
UNLESS $\partial V/\partial x = 0$. BUT $\partial V/\partial x = 0$
SAYS $V = \text{CONST}$, AND FIND LINEAR MOM'M