Syllabus for the M.Phys Major Option in Theoretical Physics (C6)

The mathematical description of systems with an infinite number of degrees of freedom: functionals, functional differentiation, and functional integrals. Multi-dimensional Gaussian integrals. Random fields: properties of a Gaussian field. Perturbation theory for non-Gaussian functional integrals. Path integrals and quantum mechanics. Treatment of free particle and of harmonic oscillator. [5 lectures]

Stochastic processes and path integrals: the Langevin and Fokker-Planck equation. Brownian motion of single particle. [4 lectures]

The link between quantum mechanics and the statistical mechanics of one-dimensional systems via Wick rotation. Transfer matrices for one-dimensional systems in statistical mechanics. [4 lectures]

Classical field theory: fields, Lagrangians and Hamiltonians. The least action principle and field equations. Space-time and internal symmetries: U(1) example, Noether current. The idea of an irreducible representation of a group. Irreducible representations of SU(2) and application to global internal symmetry. Simple representations of the Lorentz group via SU(2)xSU(2) without proof. U(1) gauge symmetry, action of scalar QED and derivation of Maxwell's eqns in covariant form. [5 lectures]

Canonical quantisation and connection to many body theory: quantised elastic waves; quantisation of free scalar field theory; many-particle quantum systems. [4 lectures]

Path integrals and quantum field theory: generating functional and free particle propagator for scalar and U(1) gauge fields (in Lorentz gauge). [5 lectures]

Perturbation theory at tree level for decay and scattering processes. Examples from pure scalar theories and scalar QED. Goldstone theorem. [4 lectures]

Canonical transformations in quantum field theory: Bogoliubov transformations applied to bose condensates, magnons in antiferromagnets, and to BCS theory. [4 lectures]

Landau theory and phase transitions: phase diagrams, first-order and continuous phase transitions. Landau-Ginsburg-Wilson free energy functionals. Examples including liquid crystals. Critical phenomena and scaling theory. [5 lectures]