COHERENCE, INTERFERENCE AND MANY BODY DYNAMICS IN QUANTUM HALL EDGE STATES

John Chalker

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Collaborators:

D. Kovrizhin: Phys. Rev. B (2009), (2010) and arXiv.

Y. Gefen and M. Veillette: Phys. Rev. B (2007).

Outline

Electron interference

In vacuum . . . . and in solids

Quantum Hall edge states

— as electron waveguides

Edge state interferometers

— surprises far from equilibrium

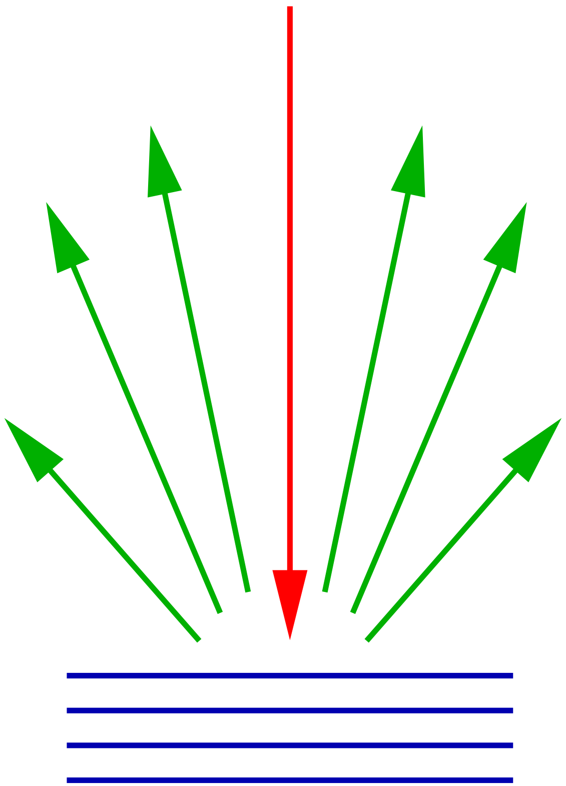
Edge states out of equilibrium

— understanding quantum relaxation

— consequences for interferometers

Electron Diffraction in Vacuum

Davisson and Germer, 1927



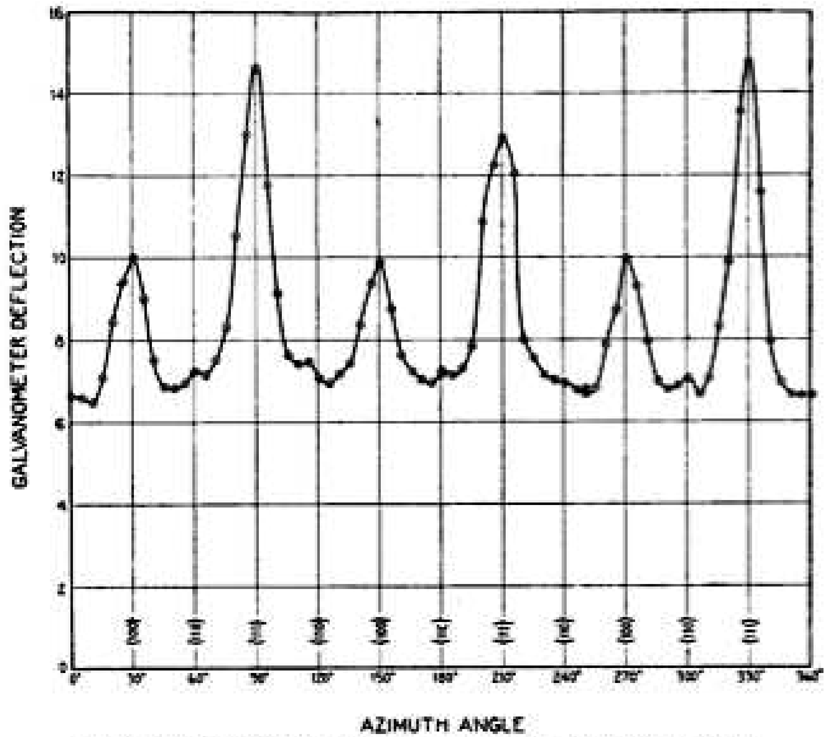


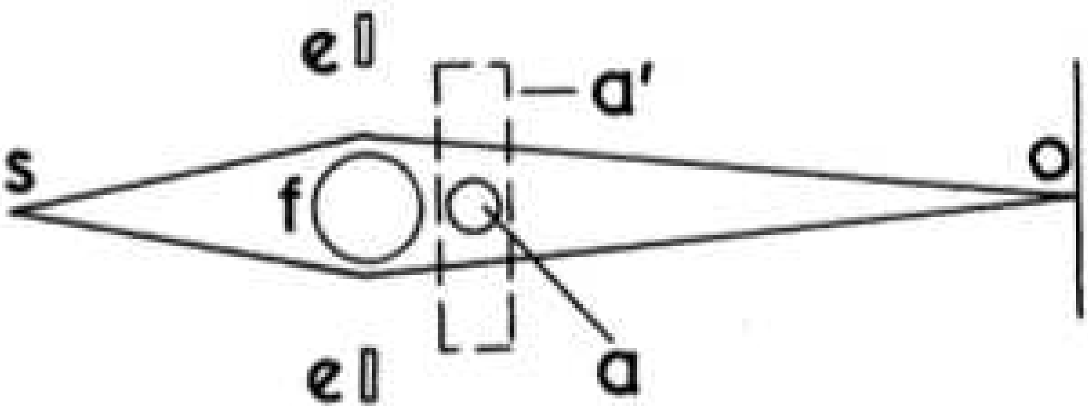
FIG. 2.—Intensity of electron scattering vs. azimuth angle—54 volts, co-latitude 50°.

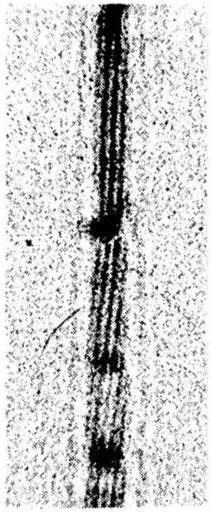
NATURE [APRIL 16, 1927

The Aharonov Bohm Effect

Chambers, 1960 **Phase Φ/Φ_0 from encircling flux Φ**

Flux quantum: $\Phi_0 = h/e$





SHIFT OF AN ELECTRON INTERFERENCE PATTERN BY ENCLOSED MAGNETIC FLUX

R. G. Chambers
H. H. Wills Physics Laboratory, University of Bristol, Bristol, England

(Received May 27, 1960)

Electron Interference in Conductors

Obstacle: Scattering

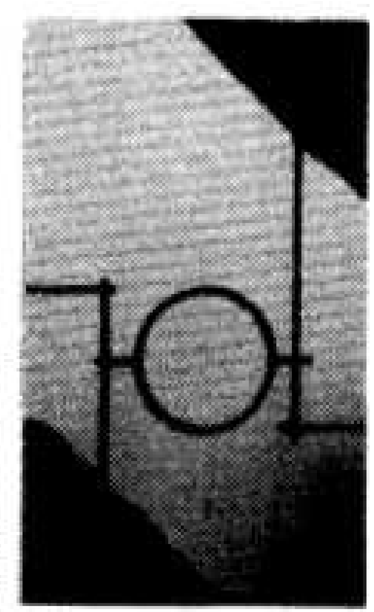
- by other electrons
- by impurities

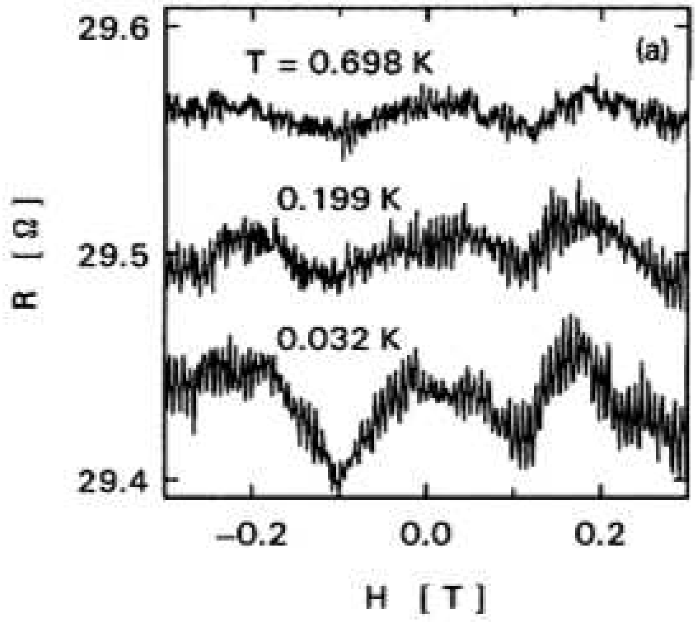
Solution: Work in the mesoscopic regime

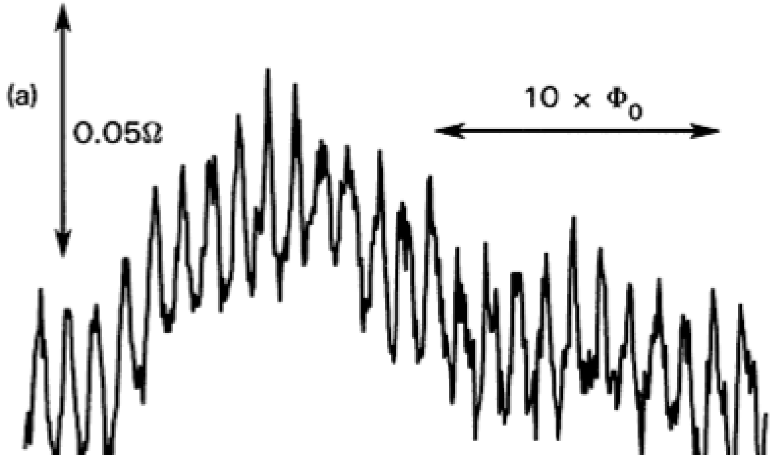
- small samples
- low temperatures

Aharonov Bohm Effect in Gold Rings

Measure resistance to probe interference







Diameter 800nm

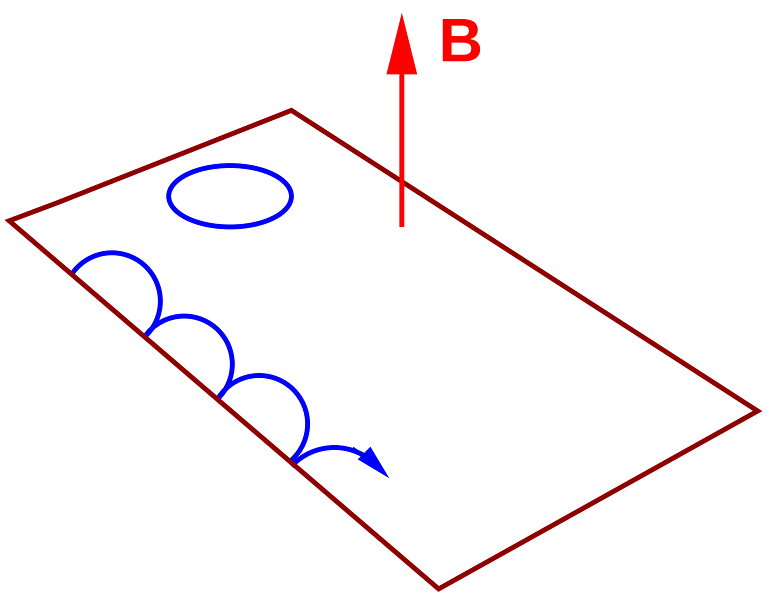
Webb *et al* Phys. Rev. Lett. (1985)

Many channels and impurities reduce fringe visibility

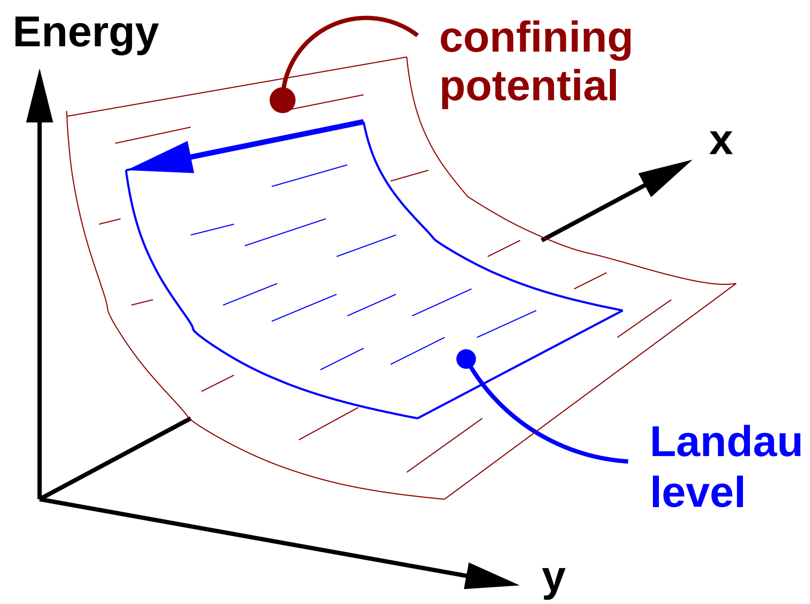
Quantum Hall Edge States

Two-dimensional electron gas in magnetic field

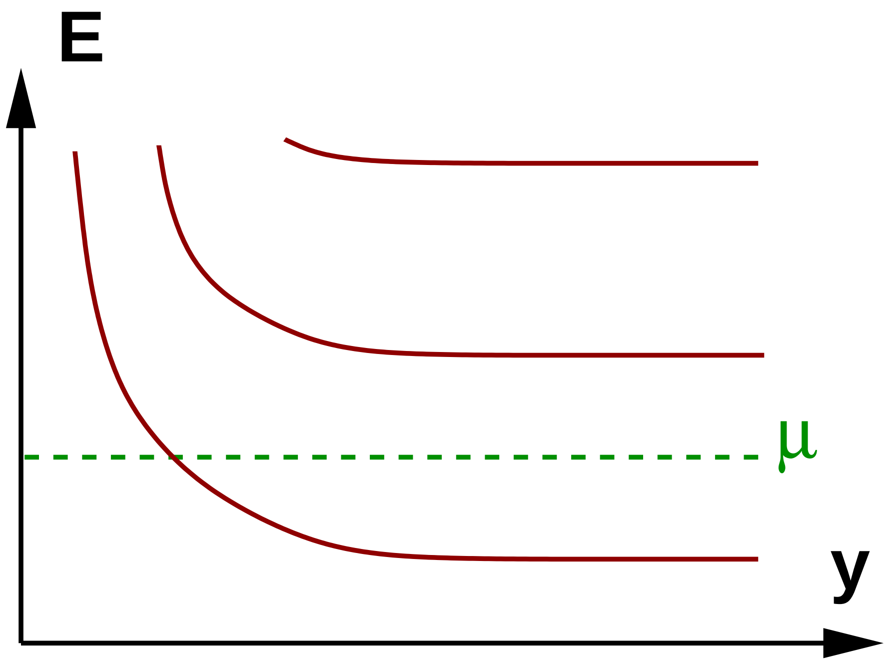
Classical skipping orbits



Quantum edge states



Edge states as Ideal Waveguides



Chiral motion

Only possible scattering is in forward direction

Theoretical Description of Edge States

Project from 2D to 1D

Classical Hamiltonian:

drift at constant speed

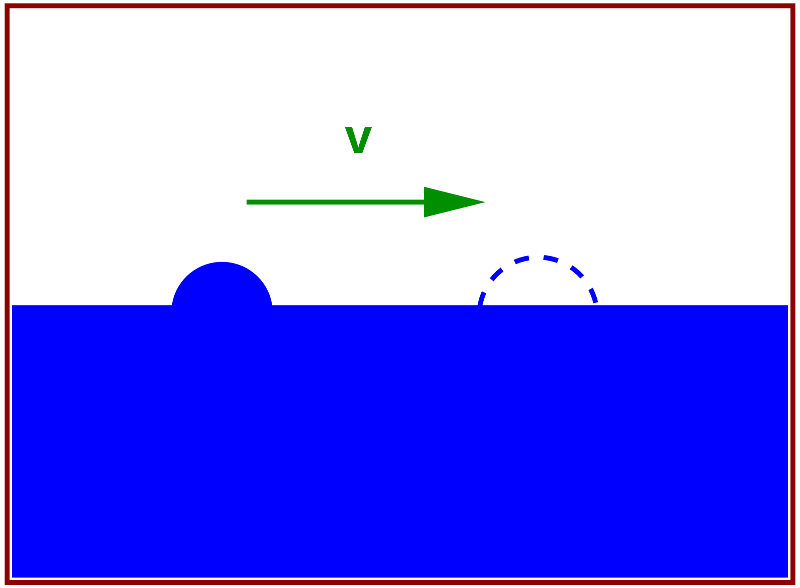
$$\mathcal{H} = v p_x \qquad \dot{x} = \partial_p \mathcal{H} = v$$

Single-particle quantum Hamiltonian:

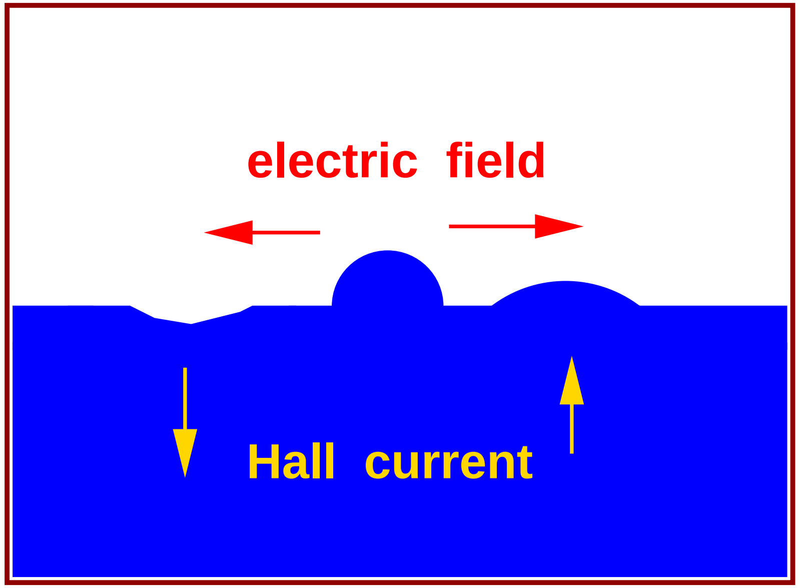
$$\mathcal{H} = \int \psi^\dagger(x)(-i\hbar v \partial_x)\psi(x) dx$$

Edge state dynamics with interactions

Free propagation



Charge flow in and out of bulk



Interactions make collective modes dispersive

Two alternative descriptions

— related via bosonization

As electrons:

$$H = -i\hbar v \int dx \psi^\dagger(x)\partial_x\psi(x) + \int dx \int dx'\ U(x-x')\rho(x)\rho(x')$$

$$\rho(x) = \psi^\dagger(x)\psi(x)$$

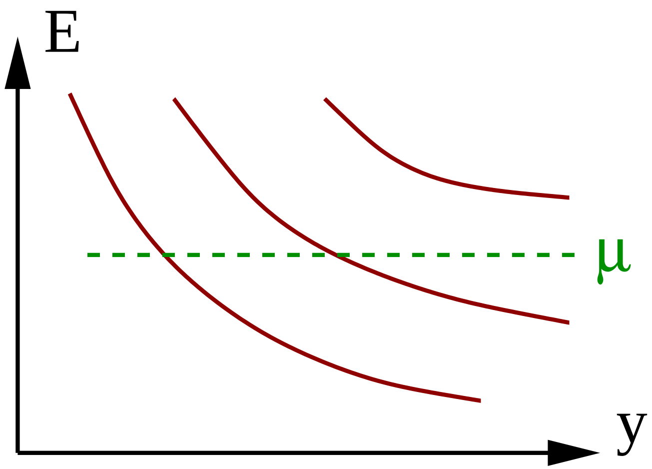
As collective modes:

$$H = \sum_q \hbar\omega(q) b_q^\dagger b_q$$

$$\omega(q) = [v + u(q)]\, q \qquad u(q) = (2\pi\hbar)^{-1} \int dx\ e^{iqx} U(x)$$

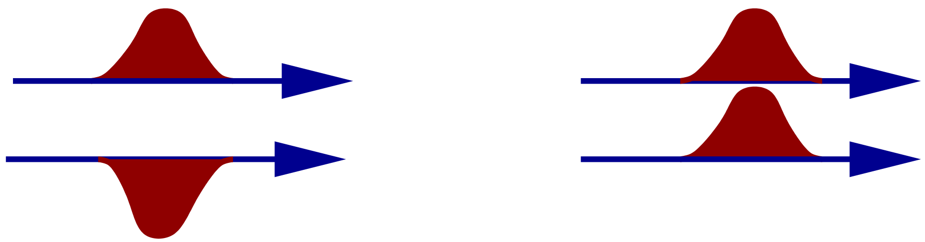
Consequences of interactions

Example: two filled Landau levels



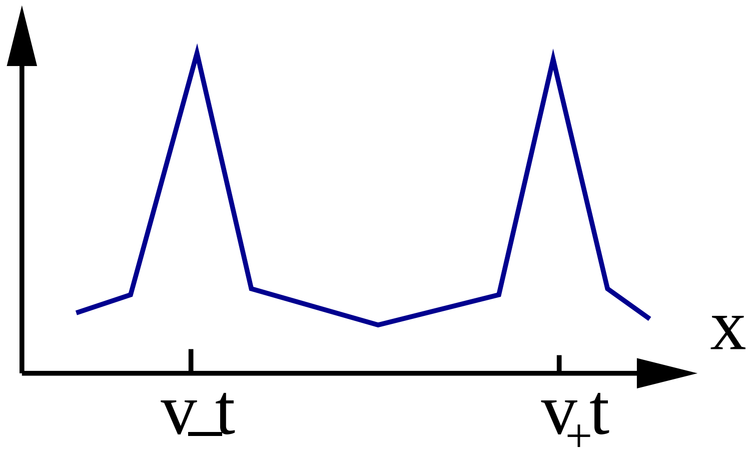
Modes mixed by interaction g

$$v_{\pm} = v \pm g$$



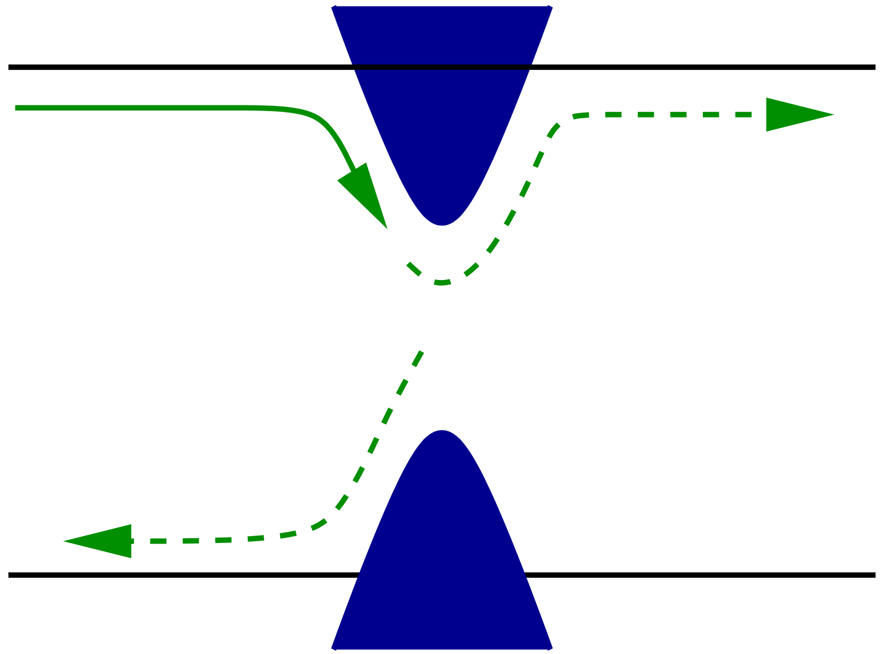
Injected electron fractionalises

$$|\langle \psi_1^{\dagger}(x,t)\psi_1(0,0)\rangle|^2 \sim$$



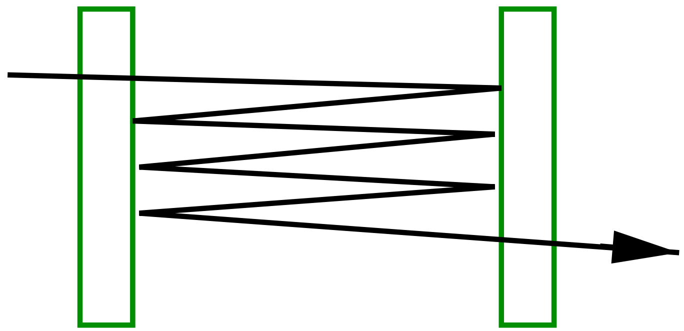
Manipulating Edge States

Quantum point contacts as beam splitters

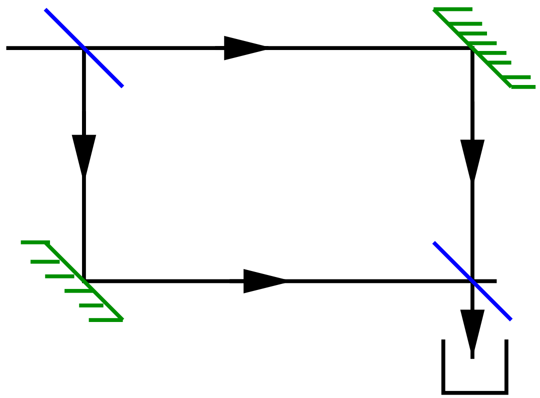


Edge State Interferometer Design

Fabry-Perot



Mach-Zehnder



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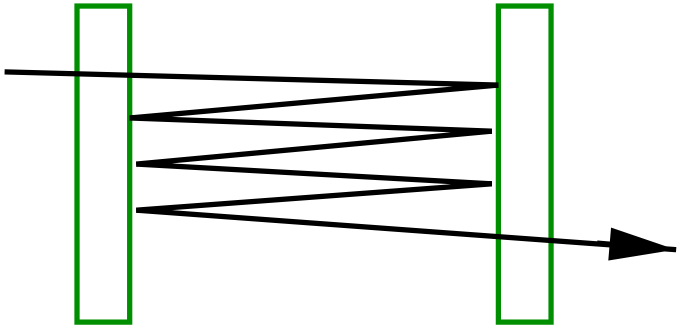
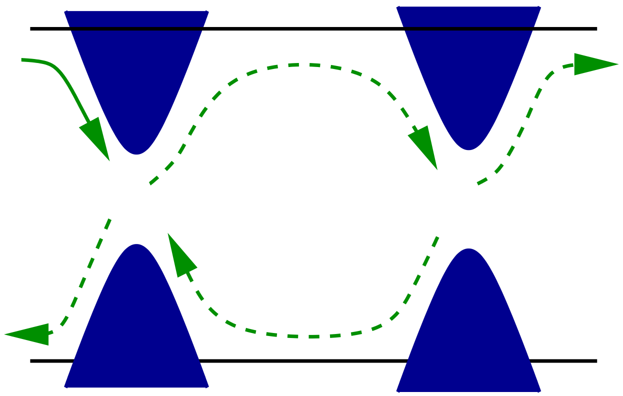
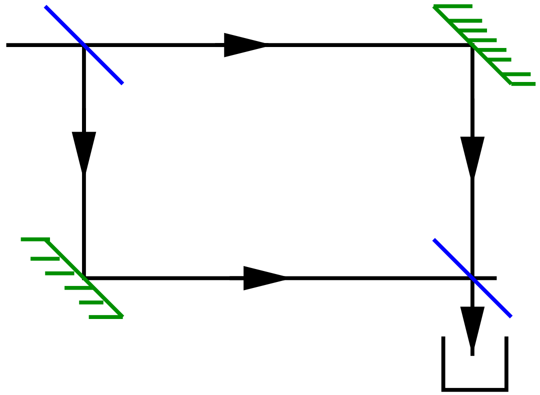


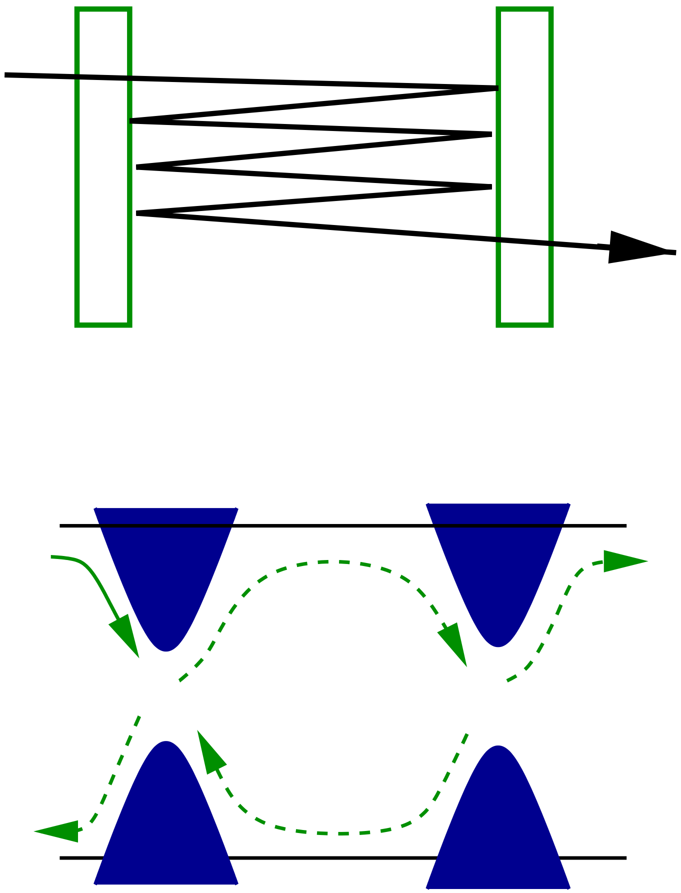


Mach-Zehnder

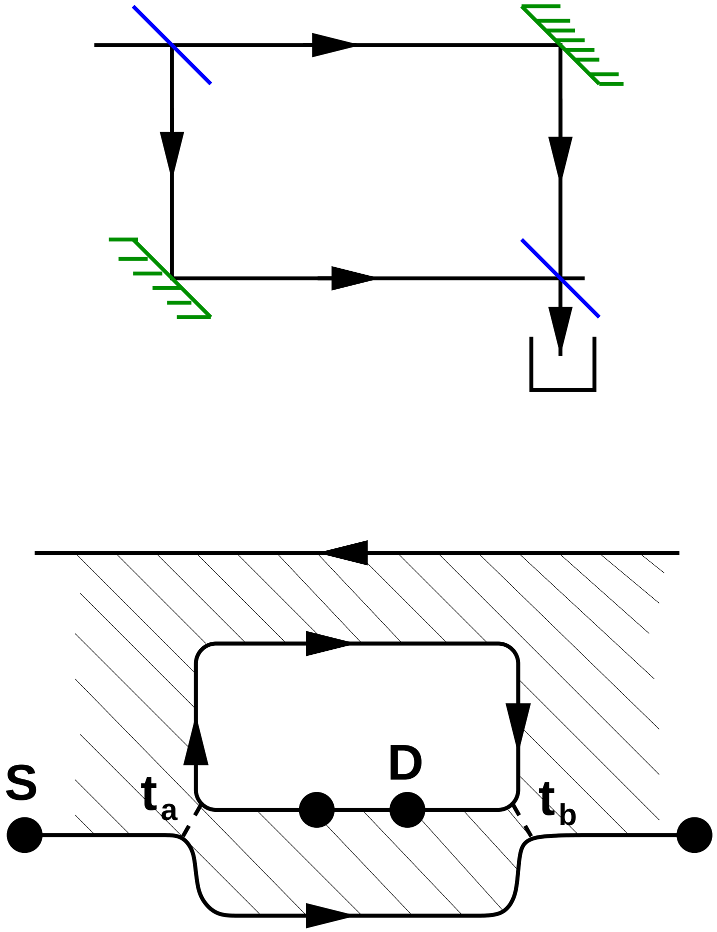


Edge State Interferometer Design

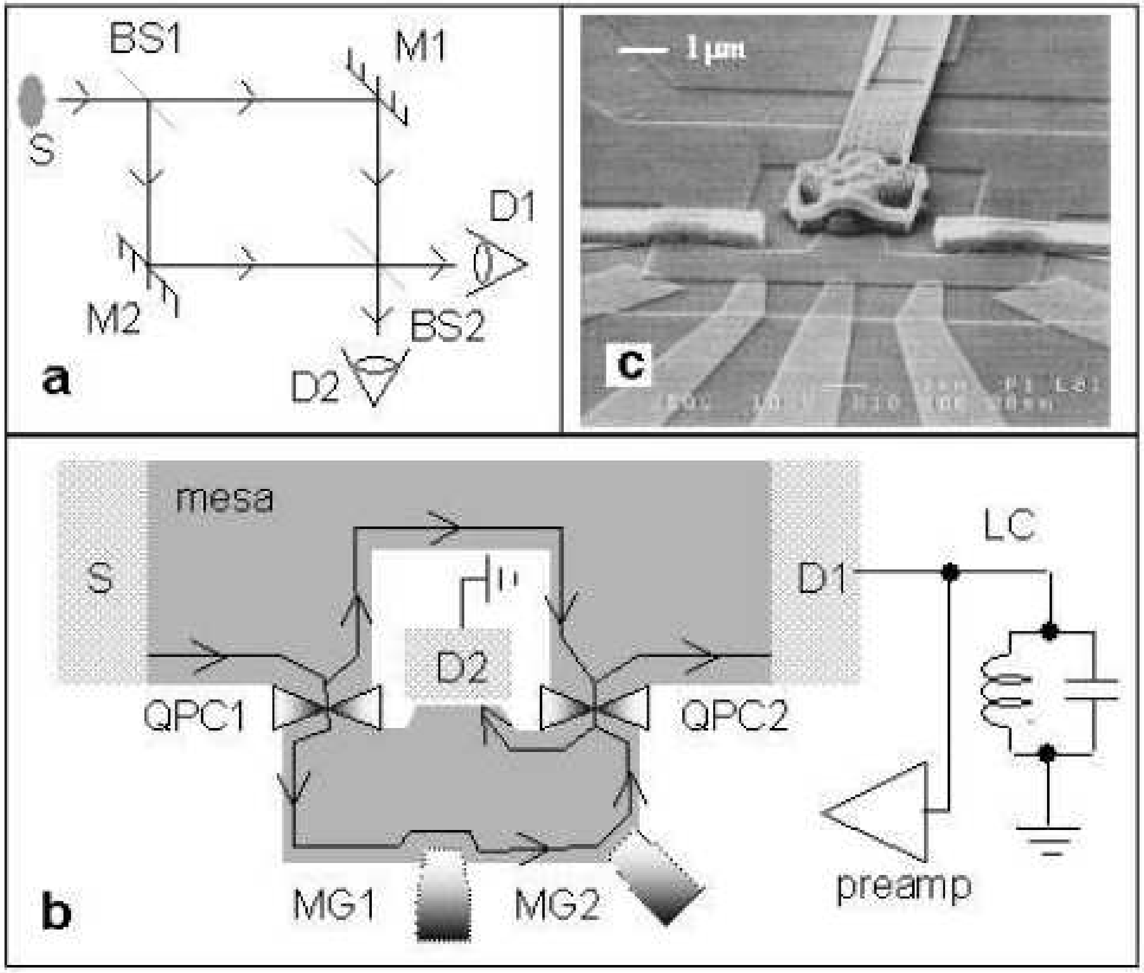
Fabry-Perot



Mach-Zehnder



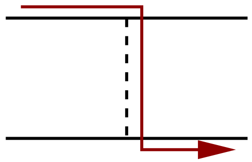
Experimental system



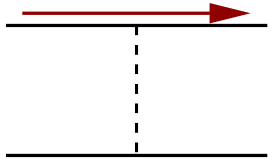
Heiblum Group, Weizmann Institute

Elementary theory

Scattering amplitudes



$$\frac{1}{\sqrt{2}}$$



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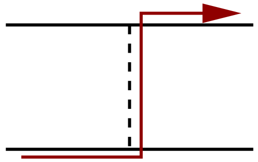
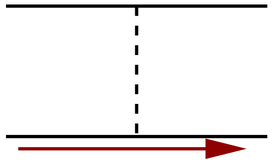
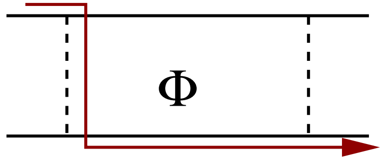


$$-\frac{1}{\sqrt{2}}$$

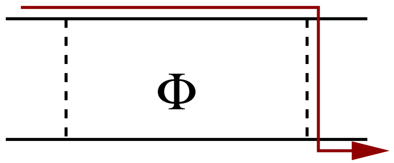


$$\frac{1}{\sqrt{2}}$$

Paths through interferometer



$$\frac{1}{2}\mathrm{e}^{\mathrm{i}\Phi/2}$$

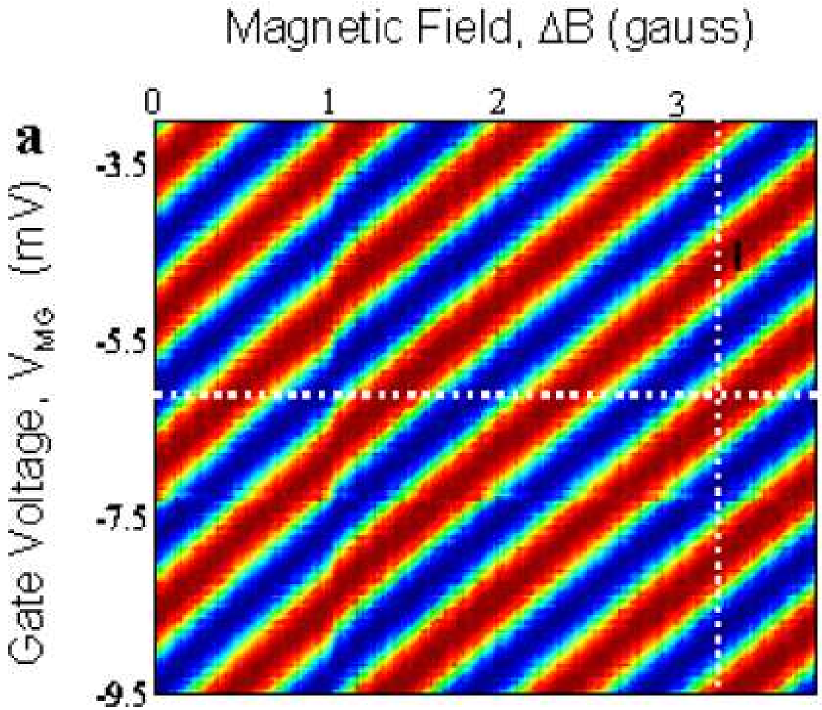


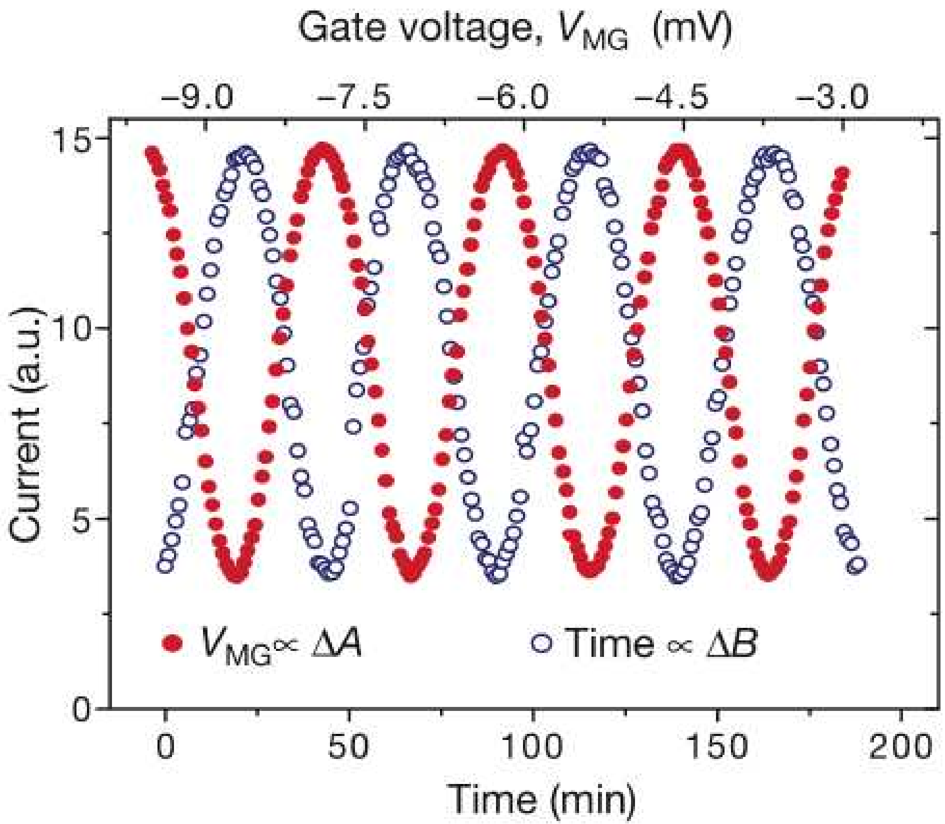
$$\frac{1}{2}\mathrm{e}^{-\mathrm{i}\Phi/2}$$

Combined amplitude $A = \cos(\Phi/2)$

Current $I \propto |A|^2 = \frac{1}{2}[1 + \cos(\Phi)]$

Fringes in Edge State Interferometer

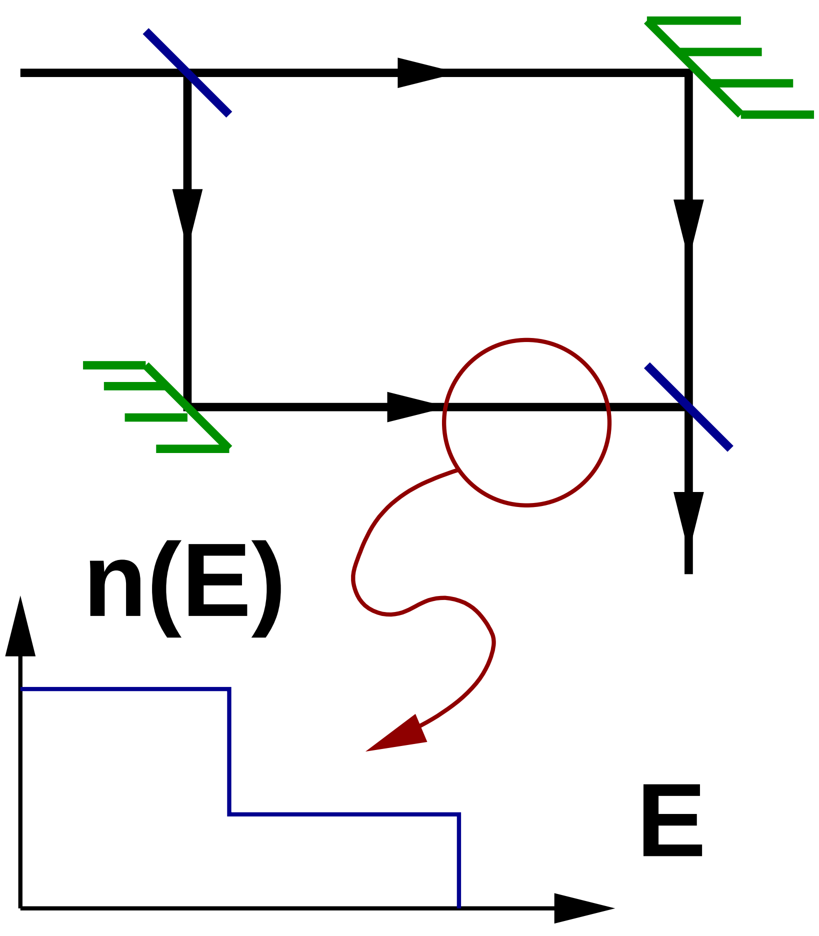




G_{SD} **vs** $Flux\ density$ **and** $Area$

Interferometer out of equilibrium

Decoherence from inelastic scattering

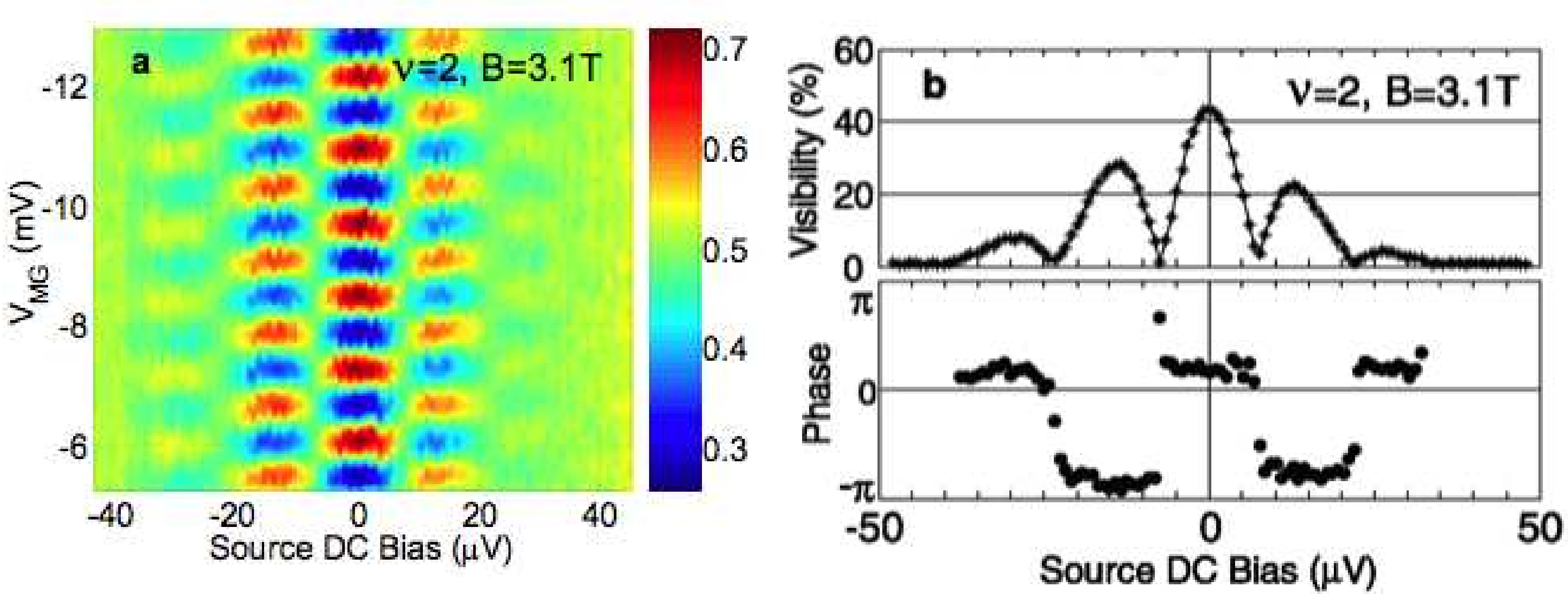


Surprises from experiment

Oscillatory dependence of visibility on bias

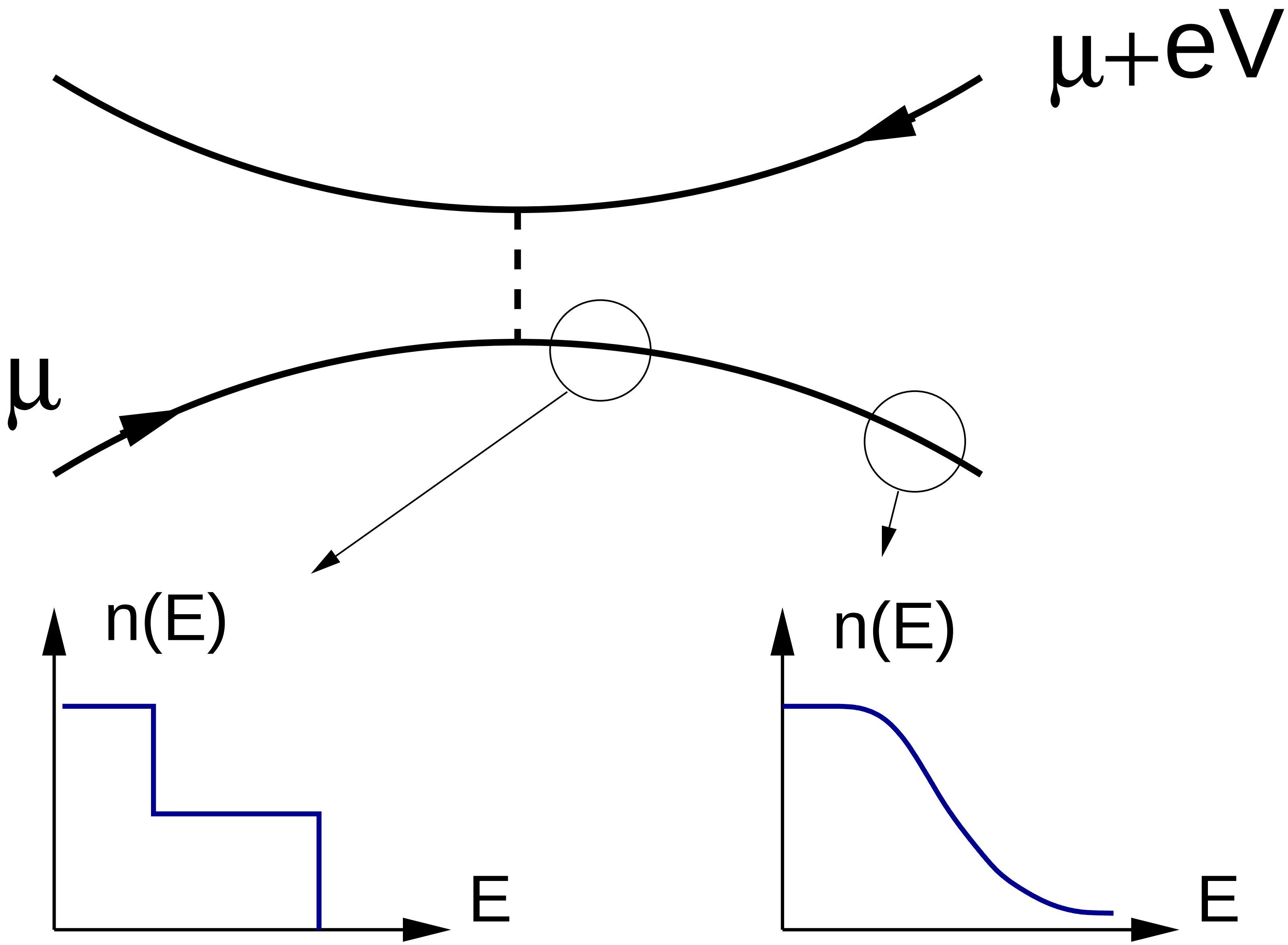
Differential conductance $G(\Phi_{AB}) = G_0 + G_1 \cos(\Phi_{AB})$

Fringe visibility $\mathcal{V} = |G_1|/G_0$



Neder *et al.*, PRL (2006) **Also Regensburg, Basel and Saclay groups**

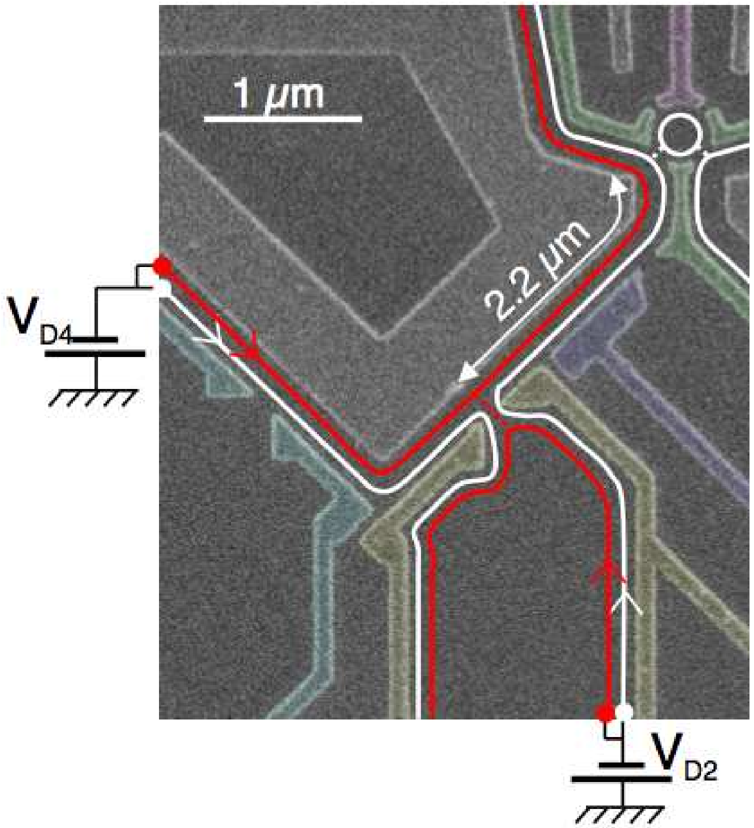
Focussing on non-equilibrium aspects



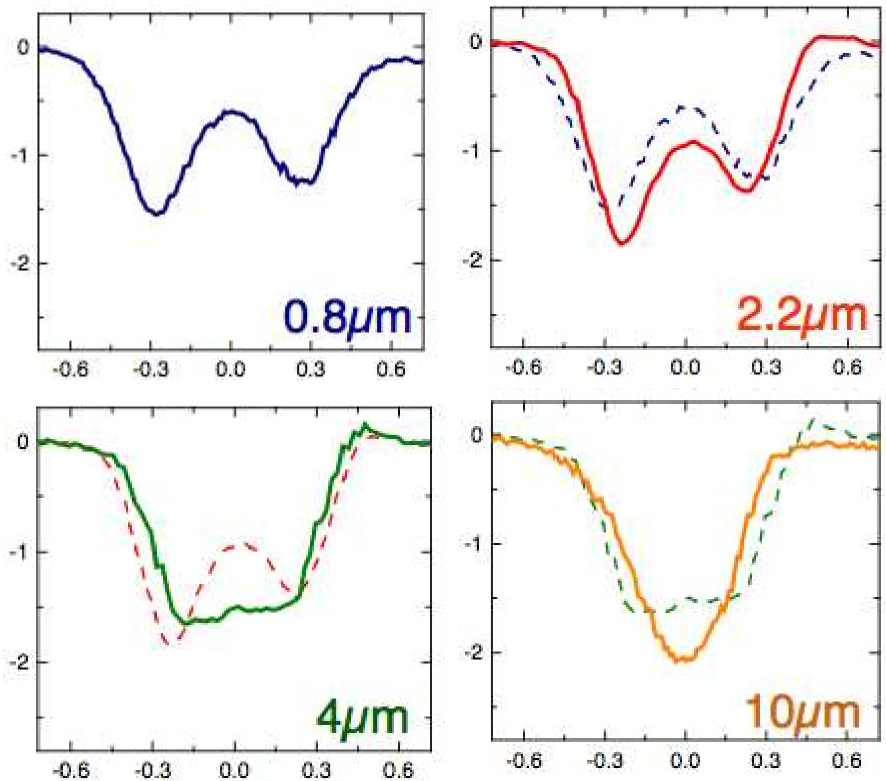
Experiment – Actual

le Sueur, Altimiras, Gennser, Cavanna, Mailly & Pierre, PRL (2010)

Sample Design



Evolution of Distribution



$\partial n(E)/\partial E$ **vs.** E

Theoretical Idealisation

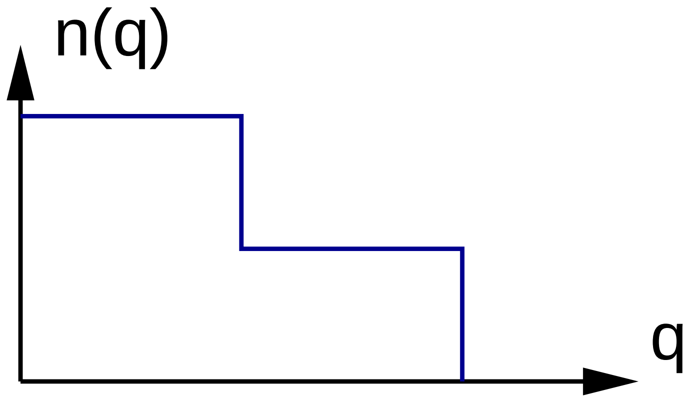
Evade treatment of point contact - treat quantum quench

Study time evolution in translationally-invariant edge

For approx theory with QPC see: Lunde *et al*, (2010) & Degiovanni *et al* (2010)

Initial state

$$|\Psi_0\rangle$$ **with**



Time evolution

$$|\Psi(t)\rangle = e^{i\mathcal{H}t}|\Psi_0\rangle$$

Properties of $|\Psi(t)\rangle$ **?**

Energies of collective modes conserved

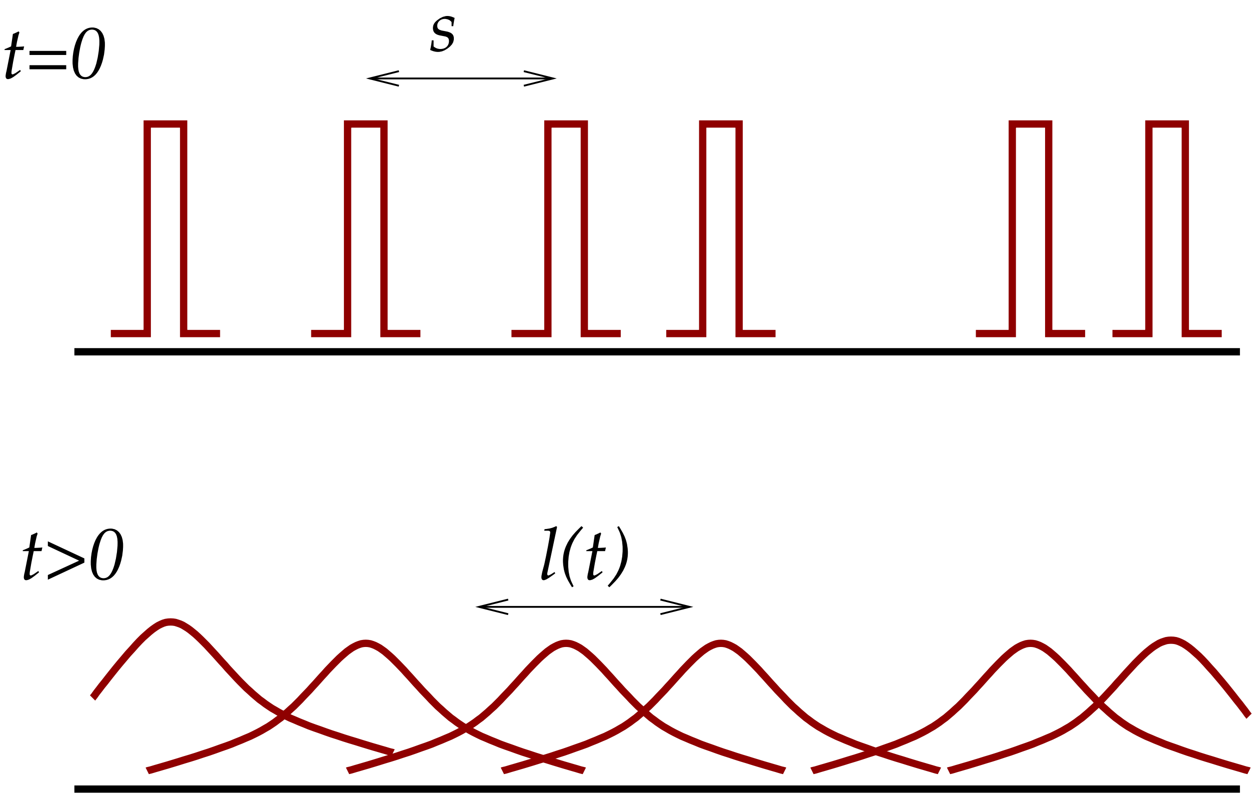
— consequences for equilibration?

Physical picture of equilibration

Collective mode Hamiltonian $\mathcal{H} = \sum_{nq} \hbar\omega_n(q) b^{\dagger}_{nq} b_{nq}$

Edge magnetoplasmon dispersion $\longrightarrow$ electron equilibration?

Initial quasi-particle separation $s = \hbar v / eV$



Equilibration when wavepacket spread $l(t) \gtrsim s$

Equilibration from two mode velocities

Two edge modes with short-range interactions

Two linearly dispersing modes $\omega_1(q) = v_+ q$ **&** $\omega_2(q) = v_- q$

Initial quasi-particle separation $s = \hbar v / eV$

Equilibration when wavepacket spread $l(t) \gtrsim s$

Spread $l(t) = [v_+ - v_-]t$

Equilibration time: $$t_{\text{eq}} \sim \frac{\hbar}{eV} \cdot \frac{v_+ + v_-}{v_+ - v_-}$$

Cf. Pascal Degiovanni, Charles Grenier *et al* (2010)

Nature of long-time state?

Simplest expectation: thermal with T fixed by energy density

Not so — energies in each collective mode conserved

No equipartition:

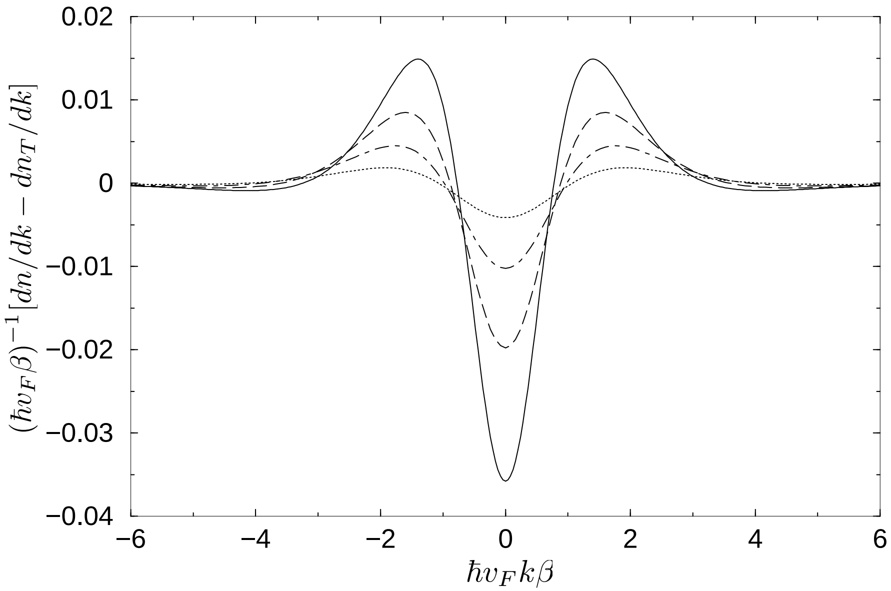
$$T_{\mathrm{final}\,1} = \left[f\,T^2_{\mathrm{initial}\,1} + (1-f)T^2_{\mathrm{initial}\,2}\right]^{1/2}$$

with f dependent on interactions

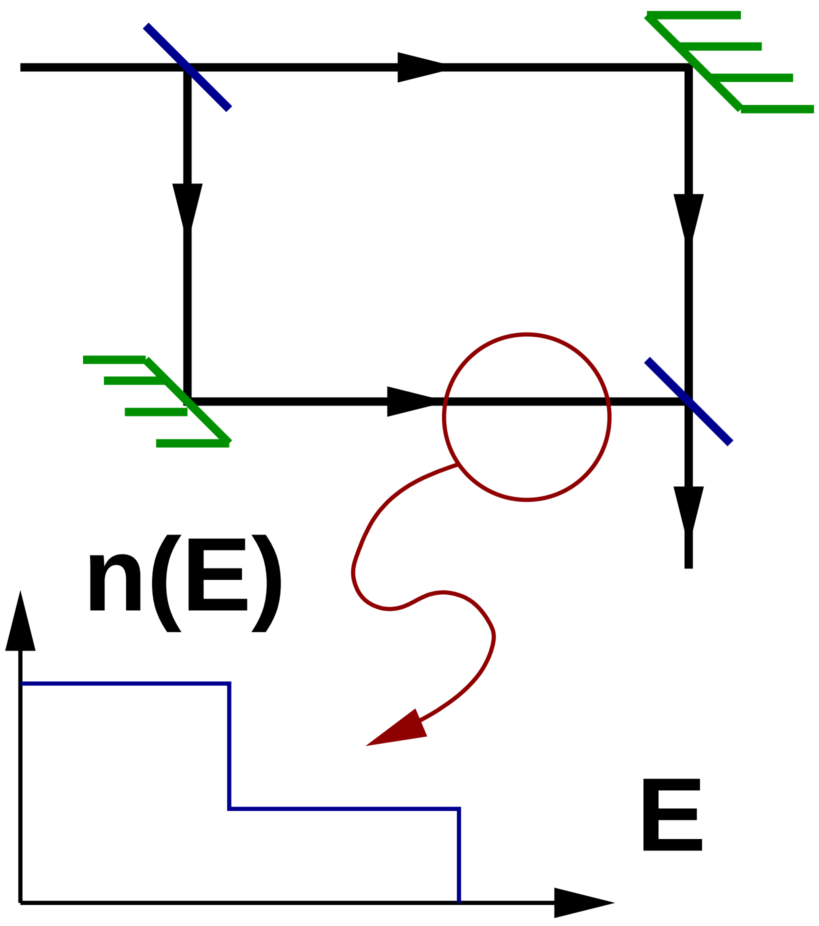
Momentum distribution:

difference between

long-time and thermal states

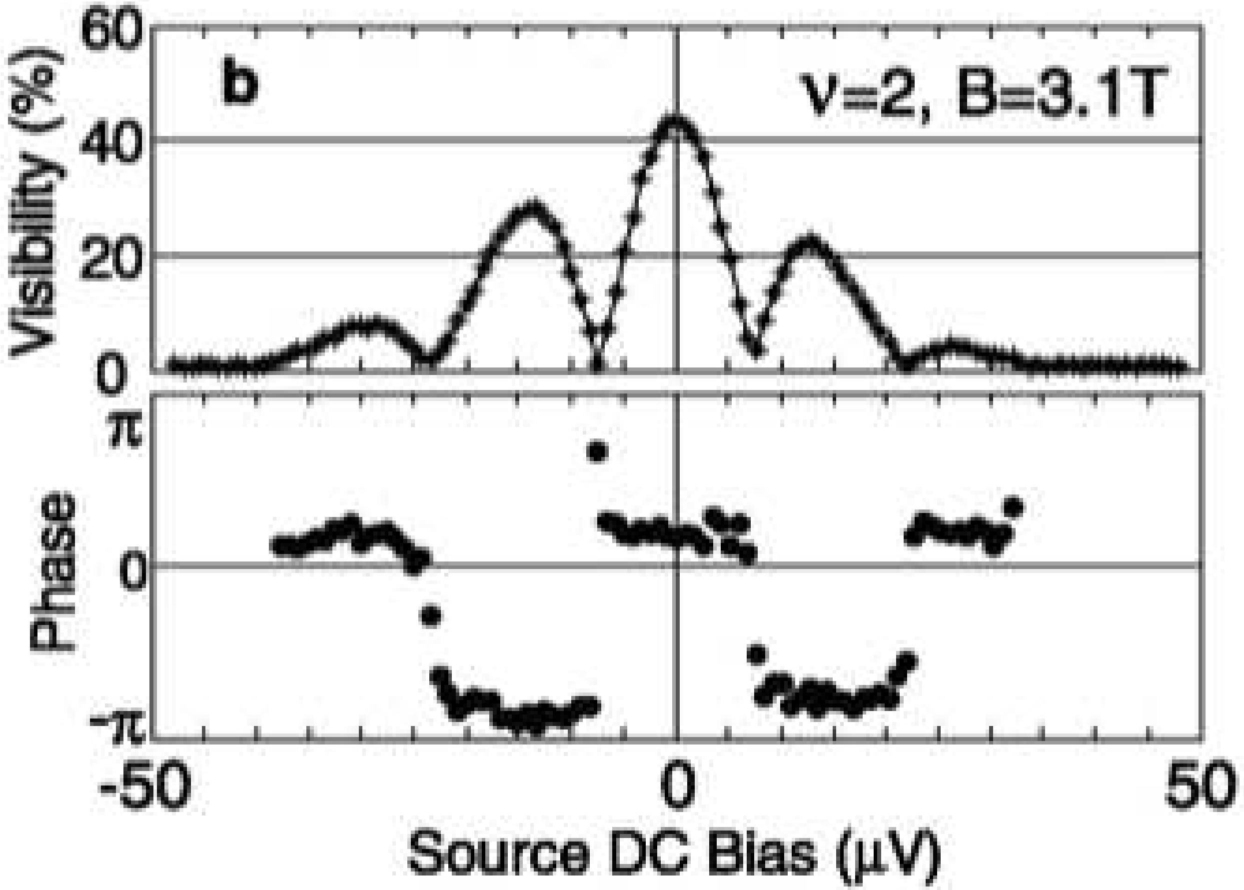


Back to non-equilibrium interferometer



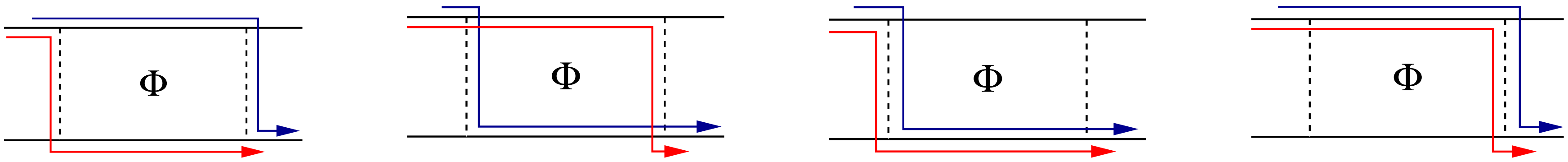
Surprises from experiment

Oscillatory dependence of visibility on bias



Clues from two-particle problem

Two-particle paths: both transmitted $A_2 = \frac{1}{2}\left(1 + \mathrm{e}^{-\mathrm{i}U\tau/\hbar}\cos\Phi\right)$



$$A_2 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\mathrm{e}^{\mathrm{i}\Phi}\mathrm{e}^{-\mathrm{i}U\tau/\hbar} + \frac{1}{4}\mathrm{e}^{-\mathrm{i}\Phi}\mathrm{e}^{-\mathrm{i}U\tau/\hbar}$$

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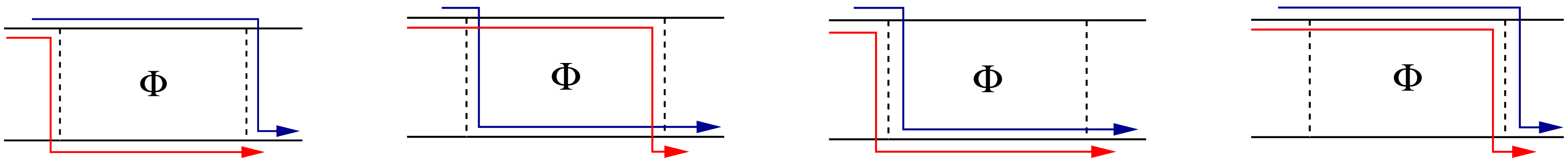
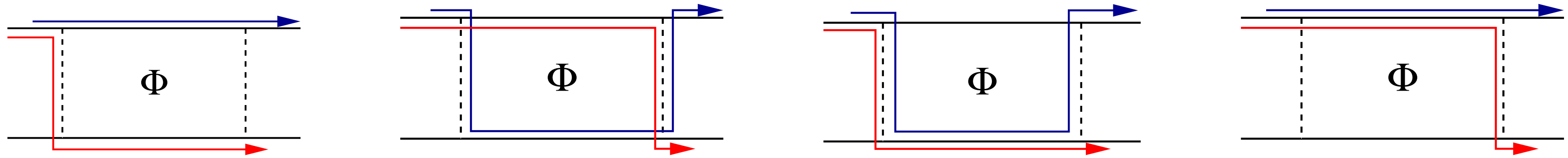


$$A_2 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\mathrm{e}^{\mathrm{i}\Phi}\mathrm{e}^{-\mathrm{i}U\tau/\hbar} + \frac{1}{4}\mathrm{e}^{-\mathrm{i}\Phi}\mathrm{e}^{-\mathrm{i}U\tau/\hbar}$$

Two-particle paths: first transmitted $A_1 = -\frac{\mathrm{i}}{2}\mathrm{e}^{-\mathrm{i}U\tau/\hbar} \sin\Phi$



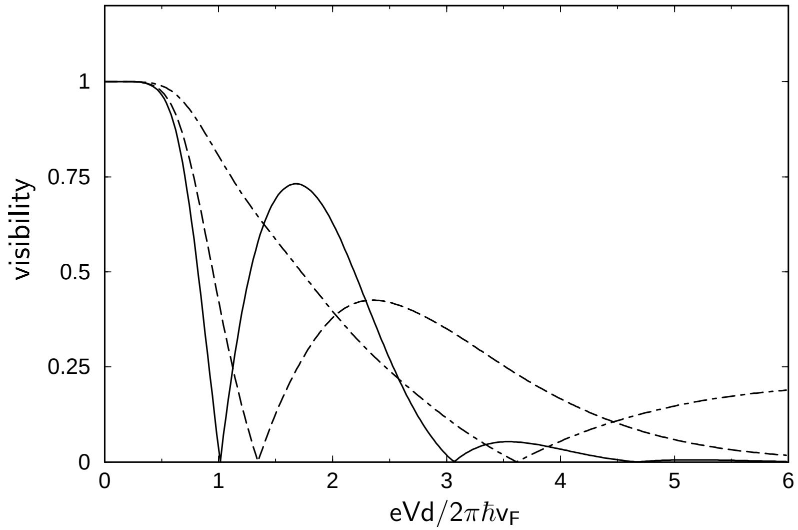
$$A_1 = \frac{1}{4} - \frac{1}{4} - \frac{1}{4}\mathrm{e}^{\mathrm{i}\Phi}\mathrm{e}^{-\mathrm{i}U\tau/\hbar} + \frac{1}{4}\mathrm{e}^{-\mathrm{i}\Phi}\mathrm{e}^{-\mathrm{i}U\tau/\hbar}$$

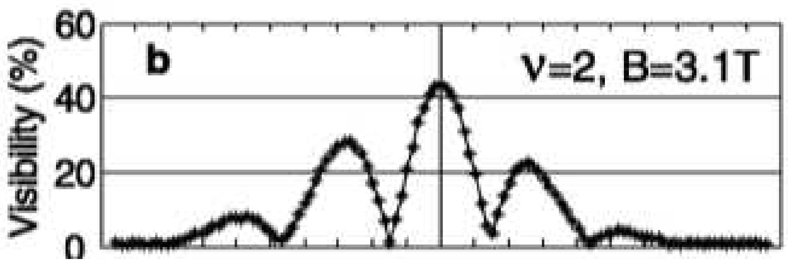
Current

$$\boxed{I \propto 2|A_2|^2 + 2|A_1|^2 = 1 + \cos\Phi\cos(U\tau/\hbar)}$$

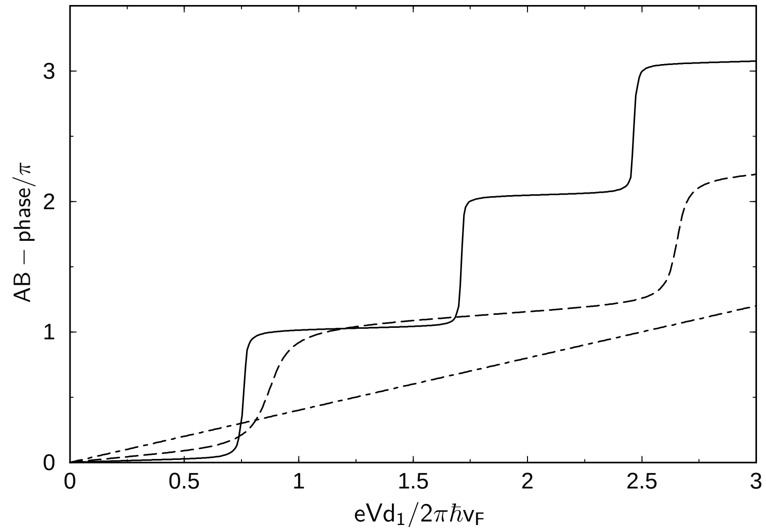
Results from full theory

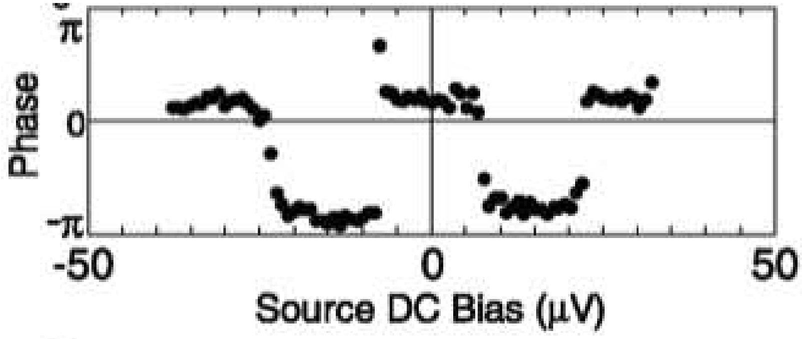
Visibility





Phase





Related calcs: Neder and Ginossar; alternative theory: Levkivskyi and Sukhorukov

Summary

Coherent many-body quantum dynamics

observed in QH edge states

Coherence far from equilibrium

probed in interferometer

Interactions bring edge into steady state

but this state is not thermal