Time dependence of correlation functions following a quantum quench

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Joint work with Pasquale Calabrese

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Statement of the problem

- quantum system extended in d dimensions, eg
  - quantum field theory
  - lattice of interacting quantum spins
- at $t = 0$ system is prepared in ground state $|\psi_0\rangle$ of hamiltonian H_0
- for $t > 0$ it evolves *unitarily* according to a *different* hamiltonian H

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How do the correlation functions $\langle \Phi(\mathbf{r}, t) \ldots \rangle$ of local operators depend on t?

- $\langle H \rangle$ is conserved so the system does not relax to the ground state of H
- up to now a rather academic question because for most condensed matter systems coupling to environment is unavoidable $\Rightarrow$ decoherence, dissipation
- but, in cold atoms in optical lattices this effect can be minimised

Outline

- path integral formulation, analytically continued from imaginary time
- initial state acts as a boundary condition
- boundary renormalisation group field theory
- $d = 1$, H critical: results from conformal field theory
- $d = 1$ results from simple exactly solvable models
- physical interpretation: semi-classical propagation of entangled pairs of particles
- higher dimensions and other open problems

Path integral formulation

$$\langle \mathcal{O}(t, \{\mathbf{r}_i\}) \rangle = Z^{-1} \langle \psi_0 | e^{iHt - \epsilon H} \mathcal{O}(\{\mathbf{r}_i\}) e^{-iHt - \epsilon H} | \psi_0 \rangle$$

where $Z = \langle \psi_0 | e^{-2\epsilon H} | \psi_0 \rangle$.

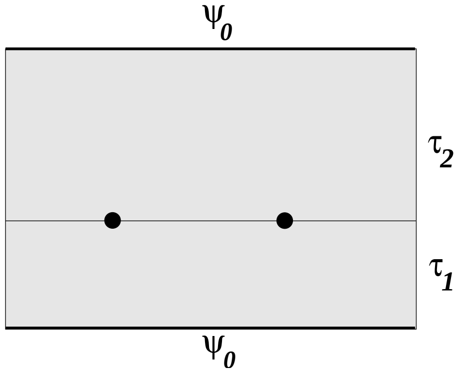
- path integral in imaginary time

$$\frac{1}{Z} \int [d\phi(\mathbf{r}, \tau)] \, e^{-S[\phi]} \, \langle \psi_0 | \phi(\mathbf{r}, \tau_1 + \tau_2) \rangle \, \mathcal{O}(\{\mathbf{r}_i\}, \tau_1) \, \langle \phi(\mathbf{r}, 0) | \psi_0 \rangle \, ,$$

continued to

$$\tau_1 = \epsilon + it \, , \qquad \tau_2 = \epsilon - it$$

- pictorially

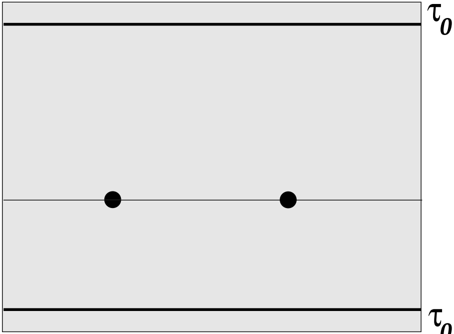


- slab of width 2ϵ with $\mathcal{O}$ inserted at $\tau = \epsilon + it$

Boundary renormalisation group theory

- wish to consider asymptotic limit when t and the separations $|\mathbf{r}_i - \mathbf{r}_j|$ are $\gg$ microscopic length and time scales
- if H is at (or close to) a quantum critical point, bulk properties of the critical theory are described by a bulk RG fixed point (or some relevant perturbation thereof)
- boundary conditions flow to one of a number of possible *boundary* fixed points
- replace $|\psi_0\rangle$ by the appropriate RG-invariant boundary state $|\psi_0^*\rangle$
- but this has problems, eg $|\psi_0^*\rangle$ is not normalisable

- solution: move the goalposts



- impose RG-invariant boundary conditions at $\tau = -\tau_0, 2\epsilon + \tau_0)$
- $\tau_0 \sim m_0^{-1}$ is the *extrapolation length*: characterizes the distance of the actual boundary state from the RG-invariant one
- effect is to replace $\epsilon \to \epsilon + \tau_0$
- limit $\epsilon \to 0+$ can now be taken, so width of the slab is $2\tau_0$

CFT results in 1+1 dimensions

- consider the case where H is described in the scaling limit by a 1+1-dimensional conformal field theory and $L \to \infty$
- conformally map half-plane $\to$ strip by $w = (2\tau_0/\pi)\log z$
- for *primary* operators such that $\langle \Phi_i \rangle = 0$ in the ground state $|0\rangle$ of H

$$\langle \prod_i \Phi_i(w_i) \rangle_{\text{strip}} = \prod_i |w'(z_i)|^{-x_i} \langle \prod_i \Phi_i(z_i) \rangle_{\text{UHP}}$$

One-point functions

$$\begin{array}{rrcl} \text{in half-plane} & \langle\Phi(z)\rangle_{\mathrm{UHP}} & \sim & (\mathrm{Im} z)^{-x_\Phi} \\ \text{in strip} & \langle\Phi(w)\rangle_{\mathrm{strip}} & \sim & \left[\frac{\pi}{4\tau_0}\frac{1}{\sin(\pi\tau/(2\tau_0))}\right]^{x_\Phi} \end{array}$$

- continuing $\tau \to \tau_0 + it$:

$$\langle\Phi(t)\rangle \sim \left[\frac{\pi}{4\tau_0}\frac{1}{\cosh(\pi t/(2\tau_0))}\right]^{x_\Phi} \sim e^{-\pi x_\Phi t/2\tau_0}$$

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- exponential relaxation to ground state value, lifetime $\propto \tau_0/x_\Phi$
- this does *not* apply to the energy density T_{tt} because it is not primary: under the conformal mapping it picks up a piece $\pi c/24(2\tau_0)^2$

Two-point functions

$$\langle\Phi(z_1)\Phi(z_2)\rangle_{\mathrm{UHP}} = \left(\frac{z_{1\bar{2}}z_{2\bar{1}}}{z_{12}z_{\bar{1}\bar{2}}z_{1\bar{1}}z_{2\bar{2}}}\right)^{x_\Phi} F(\eta)\,,$$

where $z_{ij} = z_i - z_j$, $z_{\bar{i}}$ is the image of z_i in the real axis, and $\eta \equiv z_{1\bar{1}}z_{2\bar{2}}/z_{1\bar{2}}z_{2\bar{1}}$

- F is universal but depends in detail on the particular BCFT
- mapping to strip and continuing as before:

$$\langle\Phi(r,t)\Phi(0,t)\rangle \sim \left(\frac{e^{\pi r/2\tau_0} + e^{\pi t/\tau_0}}{e^{\pi r/2\tau_0}\cdot e^{\pi t/\tau_0}}\right)^{x_\Phi} F(\eta)$$

where now

$$\eta \sim \frac{e^{\pi t/\tau_0}}{e^{\pi r/2\tau_0} + e^{\pi t/\tau_0}}$$

- for $r - 2t \gg \tau_0$, $\eta \to 0$, so $\langle\Phi(r,t)\Phi(0,t)\rangle \sim \langle\Phi(t)\rangle^2$
- for $2t - r \gg \tau_0$, $\eta \to 1$, so $\langle\Phi(r,t)\Phi(0,t)\rangle \sim e^{-\pi x_\Phi r/2\tau_0}$

- connected correlations vanish for $t < r/2$, and saturate to t-independent forms for $t > r/2$
- however they then decay exponentially in r, not as a power law (as if at finite temperature $T_{\rm eff} = 4\tau_0$)
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- above arguments rested on two main assumptions:
  - analytically continuing the imaginary time asymptotic behaviour given by CFT to real time
  - the extrapolation length
- important to check these in solvable models

Free field theory

- suppose

$$H=\frac{1}{2}\sum_r\left(\pi_r^2+m^2\phi_r^2+\sum_j\omega_j^2(\phi_{r+j}-\phi_r)^2\right)=\sum_k\Omega_k a_k^\dagger a_k$$

and similarly for H_0, with dispersion relation Ω_{0k}

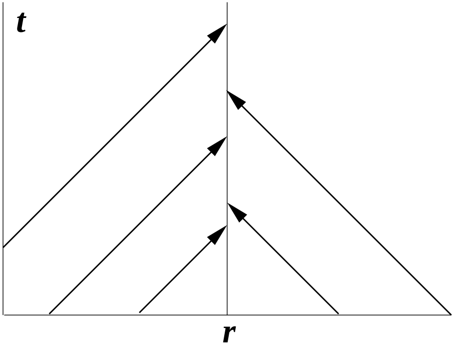
- integrating Heisenberg equations of motion (or otherwise):

$$\begin{aligned}\langle\phi_r(t)\phi_0(t)\rangle\quad &-\quad \langle\phi_r(0)\phi_0(0)\rangle\\ &=\quad \int_{\mathrm{BZ}}e^{ikr}\frac{(\Omega_{0k}^2-\Omega_k^2)(1-\cos(2\Omega_k t))}{\Omega_k^2\Omega_{0k}}dk\end{aligned}$$

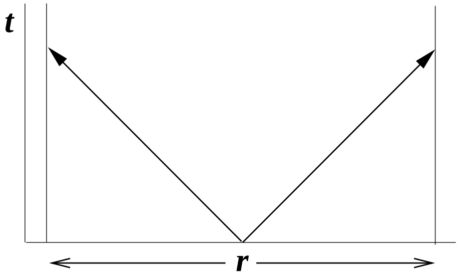
- for the CFT case, $\Omega_k\sim v|k|$ as $k\to 0$, this gives zero for $t<r/2v$ and $m_0(t-r/2v)$ for $t>r/2v$
- if we take $\Phi=\exp(iq\phi)$ we confirm all the CFT results with $\tau_0\sim m_0^{-1}$ and $x_\Phi\propto q^2$

Physical interpretation

- $|\psi_0\rangle$ has (extensively) higher energy than the ground state of H
- it acts as a source of particles at $t = 0$
- particles emitted from regions size $\sim m_0^{-1}$ are entangled
- subsequently they move classically (at velocity $\pm v$)
- incoherent particles arriving at r from well-separated initial points cause relaxation of local observables to their ground state values:



- horizon effect: local observables with separation r become correlated when left- and right-moving particles originating from the same spatial region $\sim m_0^{-1}$ can first reach them:



- if all particles move at unique speed v correlations are then frozen for $t > r/2v$

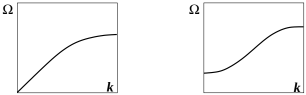
General dispersion relation

$$\partial_t \langle \phi_r(t) \phi_0(t) \rangle = 2 \int_{\mathrm{BZ}} e^{ikr} \frac{(\Omega_{0k}^2 - \Omega_k^2) \sin(2\Omega_k t)}{\Omega_k \Omega_{0k}} dk$$

- large r, t behaviour given by stationary phase approximation

$$2r/t = d\Omega_k/dk = \text{group velocity } v_k$$

- correlations begin to form at $t = r/2v_{\max}$
- large t behaviour dominated by slowest moving particles: eg lattice dispersion relation gives a power law approach to asymptotic limit



- agrees with exact results for Ising and XY spin chains `[Barouch and McCoy, 1970-71]`

Open questions

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- causality $\Rightarrow$ correlations do not change before $t \sim r/2v_{\text{max}}$, but what happens then?
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- what about non-integrable theories?
- higher dimensions
  - physical picture generalises straightforwardly, but is it correct?
  - very few exactly solvable examples beyond free field theory
  - large N: $\langle\phi^2\rangle$ shows exponential decay

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- correlations subsequently saturate to effective finite-temperature behaviour, at a rate dominated asymptotically by slowest moving particles
- lots of open questions!