EFFECT OF QUENCHED IMPURITIES on FIRST-ORDER TRANSITIONS

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Outline.

- random fields *vs.* 'random bonds'
- Imry-Ma-Imbrie argument for random fields
- analog argument for random bonds (Aizenman-Wehr)
- what happens beyond the crossover length?
- numerical work on Q-state Potts model
- a more-or-less solvable example:
  N coupled Ising models
- experiment

These transparencies available at
`http://www-thphys.physics.ox.ac.uk/users/JohnCardy/seminars/rfo.ps`

The Problem

- pure system with a unique high-T phase and $Q \geq 2$ equivalent low-T phases;
- transition at T_c in pure system is *first-order*:
  - coexistence at T_c of $Q+1$ phases with same free energy at T_c
  - discontinuities in internal energy U (latent heat L) and order parameter M
  - finite correlation length ξ_0 at T_c
  - finite interfacial tensions at T_c
- add quenched uncorrelated random impurities, either:
  - random fields, coupling to local order parameter $M(r)$

$$\mathcal{H} = \mathcal{H}_{\text{pure}} + \sum_r h(r)M(r); \qquad h(r) = \pm h$$

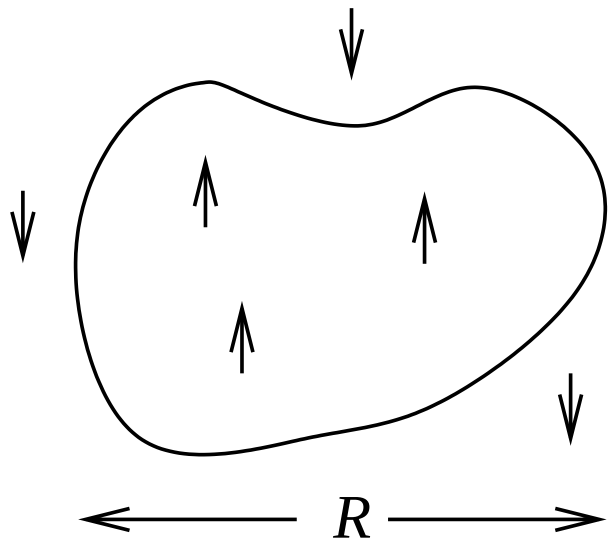
  - or 'random T_c', coupling only to local energy density $U(r)$

$$\mathcal{H} = \mathcal{H}_{\text{pure}} + \sum_r \Delta t_c(r)U(r); \qquad \Delta t_c(r) = \pm\Delta$$

- Random field Ising model and random T_c are physically very different but mathematically analogous

Random Field Ising Model – Imry-Ma argument

- look at low temperature (ignore thermal fluctuations)
- consider stability of ordered phase against formation of region of oppositely ordered phase, size $O(R)$



- by choosing region appropriately, reduce energy by $\sim hM_0\sqrt{R^d}$
- increase of energy due to domain wall $\sim JR^{d-1}$
- $\Rightarrow$ for $d > 2$, ordered phase is stable at low T against weak random fields

- $d = 2$ more subtle: in fact ordered phase is unstable [`Imbrie`]
- RG argument [`Bray-Moore`] based on looking at a *single* interface gives

$$
\begin{aligned}
dh/d\ell &= (d/2)h \\
dJ/d\ell &= J((d-1) - (h/J)^2 + \cdots)
\end{aligned}
$$

so that

$$d(h/J)/d\ell = (1 - d/2)(h/J) + (h/J)^3 + \cdots$$

– critical value $h^* = O(J)$ for $d > 2$

– finite correlation length $\xi \sim e^{\mathrm{const}(J/h)^2}$ for $d = 2$

Random T_c Analogy

$$
\begin{aligned}
\text{magnetization } M &\leftrightarrow \text{internal energy } U \\
\text{random field } h &\leftrightarrow \text{random } T_c \sim \Delta \\
\text{ignore thermal fluctuations} &\leftrightarrow \text{strong first-order} \\
\text{interfacial tension } J &\leftrightarrow \text{interfacial tension } L\xi_0
\end{aligned}
$$

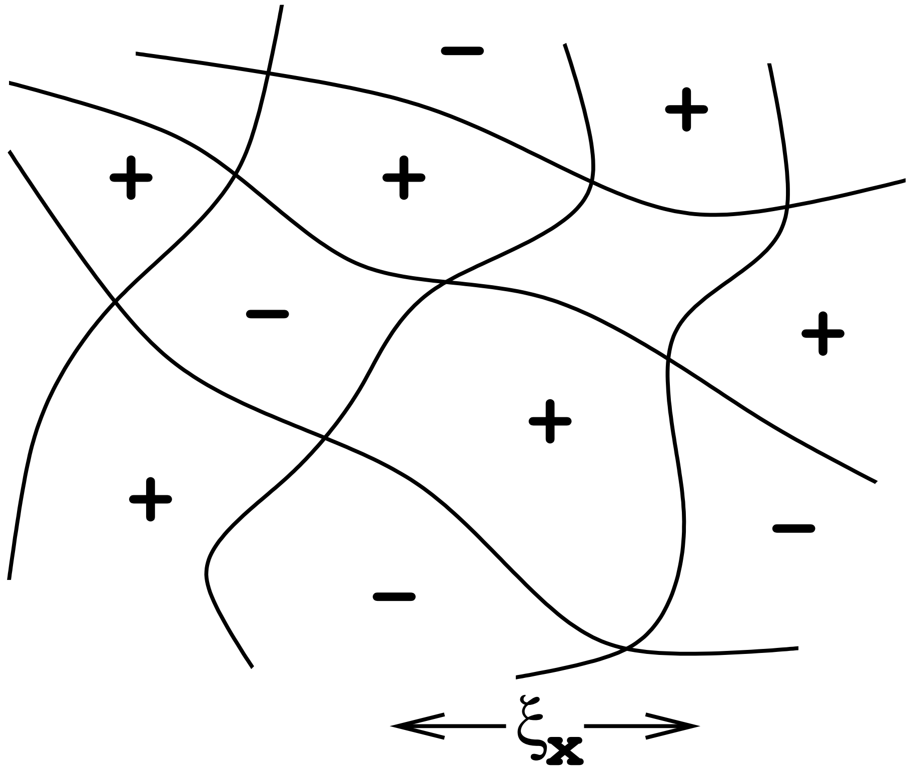
Then:

- for $d > 2$ there is critical $\Delta^* = O(L\xi_0)$ above which phase coexistence cannot occur
- thermal exponents at this tricritical point are related to the magnetic exponents of the random field Ising model [`Jacobsen & Cardy`]
- for $d = 2$ any amount of randomness destroys phase coexistence [`Aizenman-Wehr`] and there is a *cross-over* length scale $\xi_\times \sim \xi_0 e^{\mathrm{const}(L\xi_0/\Delta)^2}$

BUT:

- what is the nature of the critical state for $\Delta > \Delta^*$ for $d > 2$, and on length scales $\gg \xi_\times$ in $d = 2$?
- why is the random T_c case different from the random field Ising model on length scales $\gg \xi_\times$?

System breaks up into domains size $O(\xi_\times)$:



For $Q \geq 2$ the ordered regions can still fluctuate - but what is the resultant critical behaviour?

- does the correlation length diverge?
- what are the critical exponents?

Numerical Results for the Q-state Potts Model

`Chen, Ferrenberg, Landau (1995), JC & Jacobsen (1997), ..., Jacobsen & Picco (1999)`

Pure system:

- 'spins' $s(r)$ taking values $1, 2, \ldots, Q$
- $\mathcal{H} = -J \, \Sigma_{\text{n.n.}} \, \delta_{s(r),s(r')}$
- Q ordered states for $T < T_c$
- first-order for $Q > Q_c = 4$ in $d = 2$
- strongly first-order for large Q: $L \sim \ln Q$, $\xi_0 = O(1)$
- T_c known exactly by duality for $d = 2$

Now add randomness to J. Results ($d = 2$):

- correlation length appears to diverge
- ν numerically consistent with 1
- β increases steadily with Q, asymptoting to $\beta \approx 0.19$
- effective central charge $c \approx \frac{1}{2} \ln_2 Q$

Comment: c is a measure of the entropy of the fluctuating large-scale degrees of freedom $\Rightarrow c \propto \ln Q$. But why the coefficient?

A Solvable Case: N coupled Ising models

`JC (1995 & unpublished)`

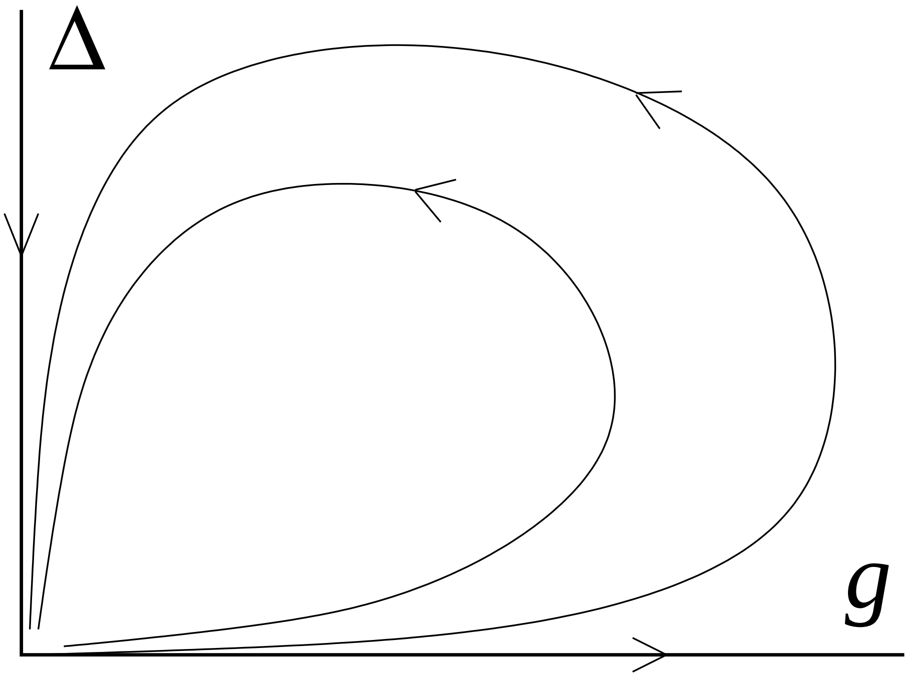
Pure system:

- Ising spins $s_j(r) = \pm 1$ ($j = 1, \ldots, N$)

$$\mathcal{H} = -J \sum_j \sum_{\text{n.n.}} s_j(r) s_j(r') - g \sum_{jk} \sum_{\text{nn}} ((s_j(r)s_j(r'))(s_k(r)s_k(r'))$$

- for $g > 0$ model undergoes a first-order transition if $N > 2$ (see below)

Now add quenched randomness in J: RG flows



- At large enough length scales,
  N strongly coupled Ising models with weak disorder
  $\approx N$ weakly coupled Ising models with strong disorder
- if so, and such transitions are 'superuniversal', this explains
  - why $\nu \approx 1$
  - why $c \approx \frac{1}{2}\ln_2 Q = \frac{1}{2}N$ where $2^N = Q$
- but β is apparently not superuniversal

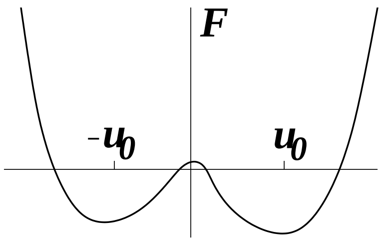
Light from the fermion description of the 2d Ising model

- continuum limit Ising model $\leftrightarrow$ free fermion
  $\mathcal{H} = \int (\overline{\psi}\sigma \cdot \partial\psi + m\overline{\psi}\psi) d^2r$ where $m \propto T - T_c$

- N coupled Ising models

$$\begin{aligned} Z &= \int [d\psi] e^{-\int \sum_j (\overline{\psi}_j \sigma\cdot\partial\psi_j + m\overline{\psi}_j\psi_j) - g(\sum_j \overline{\psi}_j\psi_j)^2) d^2r} \\ &= \int [d\psi][du] e^{-\int (u^2/g + \sum_j (\overline{\psi}_j\sigma\cdot\partial\psi_j + (m+u)\overline{\psi}_j\psi_j) d^2r} \\ &= \int [du] e^{-\int (u^2/g) d^2r} \left(e^{-F_{\mathrm{Ising}}(m+u)}\right)^N \end{aligned}$$

- rescale $g \to g/N$: large N saddle-point found by minimizing
  $F/N = F_{\mathrm{Ising}}(m+u) + u^2/g \sim (m+u)^2 \ln |m+u| + u^2/g$



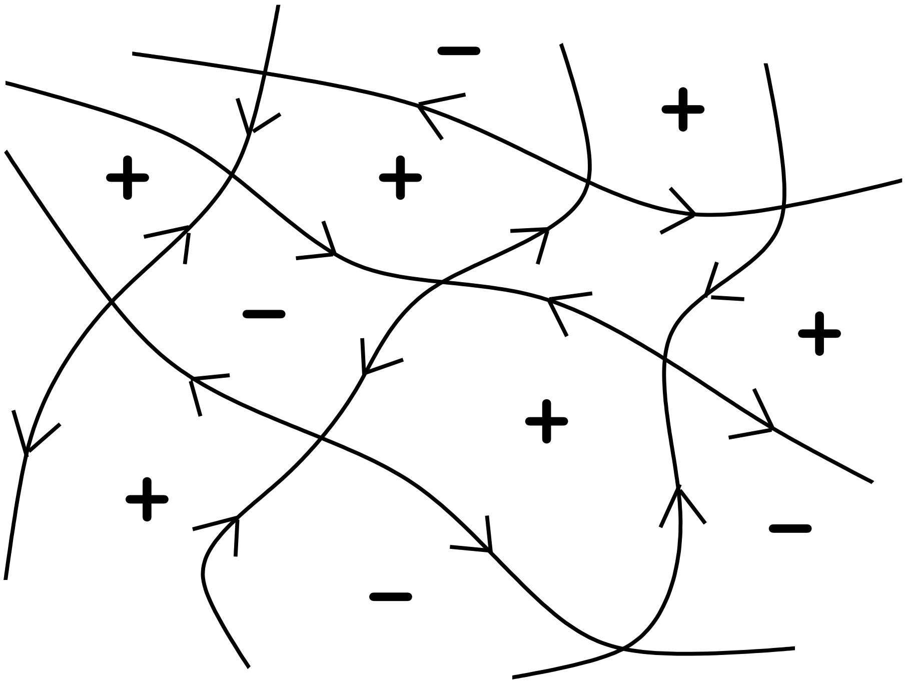
- for $m \to 0+$, $u \sim u_0 \sim e^{-1/g}$
  for $m \to 0-$, $u \sim -u_0 \Rightarrow$ first-order transition!

- now add randomness $m \to m(r)$:

$$\begin{aligned} m(r) \text{ predominantly } &> 0 \;\Rightarrow\; u \sim u_0 \\ m(r) \text{ predominantly } &< 0 \;\Rightarrow\; u \sim -u_0 \end{aligned}$$

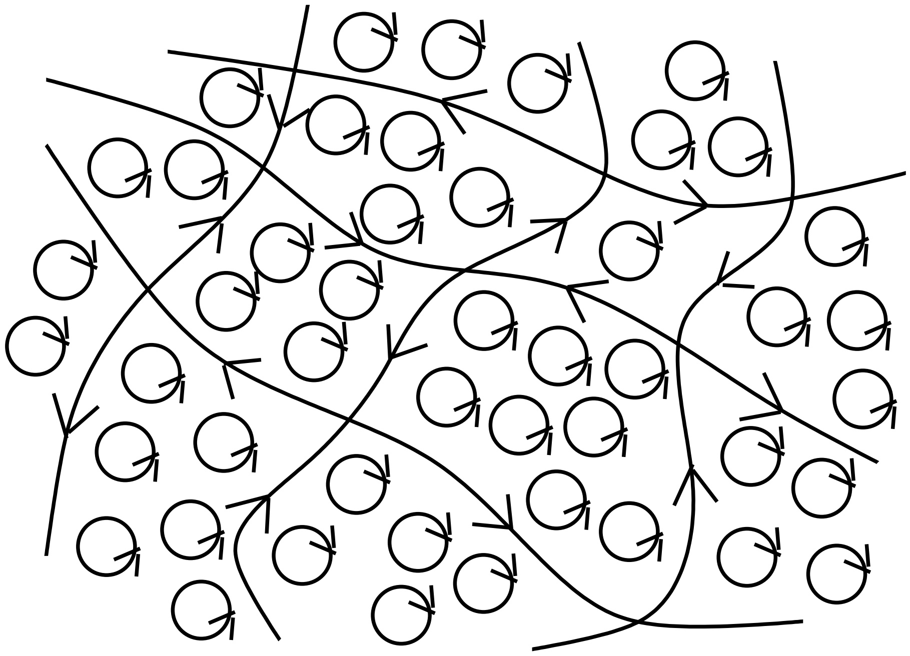
  – the Imry-Ma domains, smoothed out on scale ξ_0

- but along lines where $u = 0$ there are gapless fermions!



- N decoupled fermions ($= N$ decoupled Ising models) in disorder correlated over regions $\sim \xi_\times$

- picture much clearer in network model description of random Ising models [`Chalker and Merz, 2001`]



- surviving fermions completely analogous to *edge states* in quantum Hall physics!

Summary & Open Questions

- strong analogy between random field Ising model and random T_c at a first-order transition $\Rightarrow$ Imry-Ma domains
- for $d = 2$, one model for which nature of large scale critical state understood, analogy to quantum Hall physics
- superuniversality of energy fluctuations: $\nu \sim 1$, $c \sim \frac{1}{2} \ln_2 Q$
- why are order parameter fluctuations not superuniversal?
- experimental realizations:
  - difficult to realize random T_c without random fields
  - very large length scales necessary, need good samples to get sharp transition