Exercises 3

These problems will not be marked. Solutions will be posted on my web page in due course. Any questions or clarifications before then can be addressed to me at
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1. Show that, just as the form of the bulk 3-point function in a CFT is fixed by conformal covariance, so also is the 2-point function between a bulk operator and one on the boundary, in the upper half plane. What about the 2-point function of two bulk operators in the UHP?

2. By considering the partition function on an annulus of circumference β and length $L \gg \beta$, show that the total free energy of a 1+1-dimensional CFT at finite temperature has, in addition to the extensive term $\propto L$, a universal $O(1)$ term coming from the boundaries. Show that, for a minimal model, this term can be related to certain elements of the modular $\mathbf{S}$-matrix.

3. Consider an SLE curve γ in the upper half plane from 0 to ∞. Let $P(z, \epsilon)$ be the probability that γ passes through a small disc of radius $\epsilon \ll |z|$ centered at z. Write down a second order differential equation satisfied by P as a function of z, one the basis of (a) SLE; (b) that P may be written in terms of a CFT correlation function $\langle \phi_{2,1}(0) \cdots \rangle$. Show that this has a simple solution which has a fixed power law behaviour in ϵ, times a function of z. Use this to argue a value for the fractal dimension of the SLE curve.

4. In the previous solutions it was pointed out that the CFT corresponding to the 2d Lee-Yang edge singularity is the $(2, 5)$ minimal model with $c = -\frac{22}{5}$. Use the differential equation and the bootstrap to work out the 4-point function $\langle \phi_{1,2}\phi_{1,2}\phi_{1,2}\phi_{1,2} \rangle$, and hence the OPE coefficient b in $\phi_{1,2} \cdot \phi_{1,2} = 1 + b\phi_{1,2}$.