

**Table 2.1** Formulae for the dimensionless quantities  $I \equiv a_2 a_3 a_1^{-1} \int_0^\infty d\tau \Delta^{-1}$  and  $A_i \equiv a_1 a_2 a_3 \int_0^\infty d\tau \Delta^{-1} (a_i^2 + \tau)^{-1}$  that occur in equations (2.128) and Table 2.2.  $\Delta^2(\tau) \equiv \prod_{i=1}^3 (a_i^2 + \tau)$ . The functions  $F(\theta, k)$  and  $E(\theta, k)$  are elliptic integrals (Appendix C.4).

	$a_1 = a_2 > a_3$ (oblate)	$a_1 = a_2 < a_3$ (prolate)	$a_1 > a_2 > a_3$ (triaxial)
	$e \equiv \sqrt{1 - a_3^2/a_1^2}$	$e \equiv \sqrt{1 - a_1^2/a_3^2}$	$k \equiv \sqrt{\frac{a_1^2 - a_2^2}{a_1^2 - a_3^2}} ; k'^2 \equiv 1 - k^2 ; \theta \equiv \cos^{-1} \left( \frac{a_3}{a_1} \right)$
$I$	$2 \frac{\sqrt{1-e^2}}{e} \sin^{-1} e$	$\frac{1}{e} \ln \left( \frac{1+e}{1-e} \right)$	$2 \frac{a_2 a_3}{a_1^2} \frac{F(\theta, k)}{\sin \theta}$
$A_1$	$\frac{\sqrt{1-e^2}}{e^2} \left[ \frac{\sin^{-1} e}{e} - \sqrt{1-e^2} \right]$	$\frac{1-e^2}{e^2} \left[ \frac{1}{1-e^2} - \frac{1}{2e} \ln \left( \frac{1+e}{1-e} \right) \right]$	$2 \frac{a_2 a_3}{a_1^2} \frac{F(\theta, k) - E(\theta, k)}{k^2 \sin^3 \theta}$
$A_2$	$= A_1$	$= A_1$	$2 \frac{a_2 a_3}{a_1^2} \frac{E(\theta, k) - k'^2 F(\theta, k) - (a_3/a_2) k^2 \sin \theta}{k^2 k'^2 \sin^3 \theta}$
$A_3$	$2 \frac{\sqrt{1-e^2}}{e^2} \left[ \frac{1}{\sqrt{1-e^2}} - \frac{\sin^{-1} e}{e} \right]$	$2 \frac{1-e^2}{e^2} \left[ \frac{1}{2e} \ln \left( \frac{1+e}{1-e} \right) - 1 \right]$	$2 \frac{a_2 a_3}{a_1^2} \frac{(a_2/a_3) \sin \theta - E(\theta, k)}{k'^2 \sin^3 \theta}$

**Table 2.2** Potentials and potential-energy tensors of ellipsoidal bodies

Thin shell	$\Phi(\mathbf{x}_{\text{int}}) = -\frac{G a_1}{2 a_2 a_3} I(\mathbf{a}) M_{\text{shell}}$	$\Phi(\mathbf{x}_{\text{ext}}) = -\frac{G a'_1}{2 a'_2 a'_3} I(\mathbf{a}') M_{\text{shell}}$
Homogeneous	$\Phi(\mathbf{x}_{\text{int}}) = -\pi G \rho [I(\mathbf{a}) a_1^2 - \sum_{i=1}^3 A_i(\mathbf{a}) x_i^2]$	$W_{ij} = -\frac{8}{15} \pi^2 G \rho^2 a_1 a_2 a_3 A_i a_i^2 \delta_{ij}$
	$\Phi(\mathbf{x}_{\text{ext}}) = -\pi G \rho \frac{a_1 a_2 a_3}{a'_1 a'_2 a'_3} [I(\mathbf{a}') a'_1^2 - \sum_{i=1}^3 A_i(\mathbf{a}') x_i^2]$	$W = -\frac{8}{15} \pi^2 G \rho^2 a_1^3 a_2 a_3 I$
Inhomogeneous	$\Phi(\mathbf{x}) = -\pi G \frac{a_2 a_3}{a_1} \int_0^\infty \frac{d\tau}{\Delta} \{ \psi(\infty) - \psi[m(\tau, \mathbf{x})] \}$	$W_{ij} = -2 \pi^2 G \frac{a_2 a_3}{a_1^4} \mathcal{S} A_i a_i^2 \delta_{ij} ; W = -2 \pi^2 G \frac{a_2 a_3}{a_1^2} \mathcal{S}$

NOTES:  $I$  and  $A_i$  as in Table 2.1.  $\mathbf{x}_{\text{int}}$  and  $\mathbf{x}_{\text{ext}}$  denote points on the interior or exterior of the ellipsoidal shell or body. If  $\sum_{i=1}^3 x_i^2 / [a_i^2 + \lambda(\mathbf{x})] = 1$ , then  $a'_i{}^2 \equiv a_i^2 + \lambda(\mathbf{x})$ ;  $\Delta^2(\tau) \equiv \prod_{i=1}^3 (a_i^2 + \tau)$ ;  $m^2(\tau, \mathbf{x}) \equiv a_1^2 \sum_{i=1}^3 x_i^2 / (a_i^2 + \tau)$ ;  $\psi(m) \equiv \int_0^{m^2} \rho(\mathbf{x}) dm$ ;  $\mathcal{S} \equiv \int_0^\infty dm^2 \rho(m^2) \int_0^{m^2} dm'^2 m' \rho(m'^2) = \frac{1}{2} \int_0^\infty dm [\psi(\infty) - \psi(m)]^2$ .