Dynamics of stellar discs

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Story so far

- Fluctuations in Φ cause stars to diffuse through action space
- Flux is
$$\begin{aligned}\mathbf{F}(\mathbf{J}) &= \mathbf{F}_1(\mathbf{J}) + \mathbf{F}_2(\mathbf{J}) \\ &= -\mathbf{D}_1(\mathbf{J}) f_0 - \mathbf{D}_2(\mathbf{J}) \cdot \frac{\partial f_0}{\partial \mathbf{J}}\end{aligned}$$

$$\mathbf{D}_1(\mathbf{J}) = -\tfrac{1}{2}(2\pi)^4 m \sum_{\mathbf{n}\mathbf{n}'} \int \mathrm{d}^3\mathbf{J}' \left|E_{\mathbf{n}\mathbf{n}'}(\mathbf{J},\mathbf{J}',-\mathrm{i}\mathbf{n}\cdot\mathbf{\Omega}_0)\right|^2 \mathbf{n}' \cdot \frac{\partial f_0}{\partial \mathbf{J}'} \delta(\mathbf{n}'\cdot\mathbf{\Omega}_0' - \mathbf{n}\cdot\mathbf{\Omega}_0)\,\mathbf{n}$$

$$\mathbf{D}_2(\mathbf{J}) = \tfrac{1}{2}(2\pi)^4 m \sum_{\mathbf{n}\mathbf{n}'} \int \mathrm{d}^3\mathbf{J}' \left|E_{\mathbf{n}\mathbf{n}'}(\mathbf{J},\mathbf{J}',-\mathrm{i}\mathbf{n}\cdot\mathbf{\Omega}_0)\right|^2 f_0(\mathbf{J}')\delta(\mathbf{n}'\cdot\mathbf{\Omega}_0' - \mathbf{n}\cdot\mathbf{\Omega}_0)\,\mathbf{n}\otimes\mathbf{n}$$

- So star at J is disturbed by stars at J’ that resonate with it
- D_1 drives stars back towards low J and low E while D_2 causes them to diffuse to high J and high E
- Einstein first recognised the need for the “dynamical friction term” D_1 in his model of Brownian motion

Thermal equilibrium

- By the Principle of Detailed Balance F has to vanish in thermal equilibrium
- The $f_0 = \exp(-\beta H)$
- Plugging this in to our expression for F we find

$$\mathbf{D}_1(\mathbf{J}) - \beta\mathbf{D}_2(\mathbf{J}) \cdot \mathbf{\Omega}_0(\mathbf{J}) = 0$$

- This equation provides a useful check on formulae for the D_i

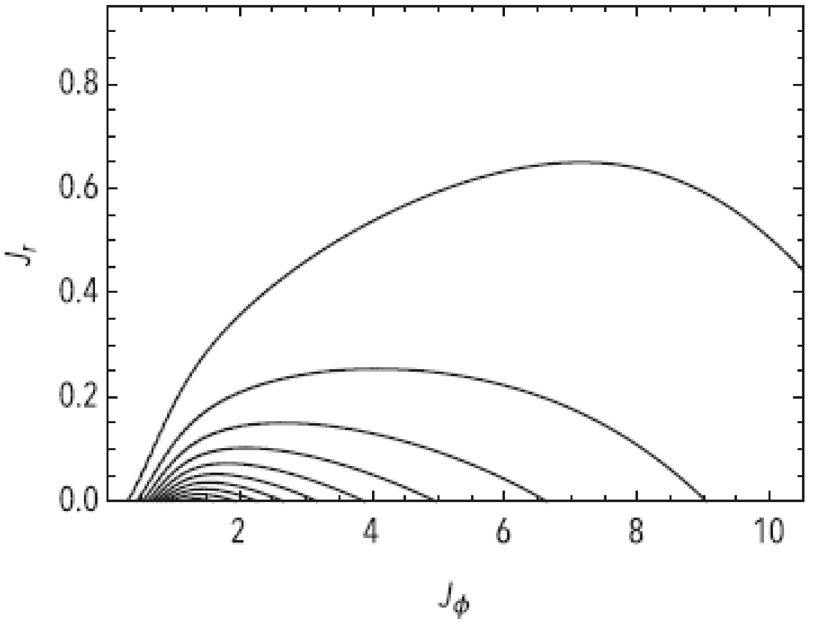
Disc dynamics

Fouvry et al 2015 A&A 584, A129

- Our treatment goes beyond Spitzer-Chandrasekhar 2-body work in "dressing" particles: $E_{nn'}$ takes fully into account the "polarisation" of the system induced by each particle: in the language of QFT particles are "dressed".
  - This is a huge effect in cool stellar disc
  - It may be a significant effect in a globular cluster with a radially biased DF, but this has yet to be investigated
- We shall see that in a disc major consequences follow from the resonant aspect of diffusion
- We consider only strictly planar discs – currently technically too difficult to treat the 3d case

Model disc

- Assume unperturbed Φ = v_0^2 ln(R)
  - generates constant circular speed v_0
- A DF $f_0 = C\, J_\phi^{\,q} \exp(-E/\sigma^2)$ with $q = v_0^2/\sigma^2 - 1$ exactly generates the required surface density
- But we taper this DF at small R (gravity of the bulge) and at large R (gravity of the dark halo)
- Also in the middle we use only ξ times the full DF because the dark halo contributes significant gravity at all R



$$T_{\rm inner}(J_\phi) = \frac{J_\phi^{\nu_{\rm i}}}{(R_{\rm i}V_0)^{\nu_{\rm i}} + J_\phi^{\nu_{\rm i}}},$$

$$T_{\rm outer}(J_\phi) = \left[1 + \left[\frac{J_\phi}{R_0 V_0}\right]^{\mu_{\rm t}}\right]^{-1}$$

Implementation

- We compute AA cords for disc
- Choose a system of orthogonal potential-density pairs

$$\Phi^{\alpha}(r,\phi) = \mathrm{e}^{\mathrm{i}l\phi}\,\Phi^{l}_{n}(r) \qquad \rho^{\alpha}(r,\phi) = \mathrm{e}^{\mathrm{i}l\phi}\,\rho^{l}_{n}(r).$$

- Φ^{l}_{n} a polynomial and ρ^{l}_{n} a polynomial times half power of 1 - r^2/r_0^2
- Compute their form in AA coordinates

$$\hat{\Phi}^{(\alpha)}(\mathbf{n},\mathbf{J}) = \delta_{\alpha_2,n_2}\frac{1}{\pi}\int_{r_\mathrm{p}}^{r_\mathrm{a}} \mathrm{d}r\,\Phi^{l}_{n}(r)\cos[n_1\theta_1 + n_2(\theta_2-\phi)].$$

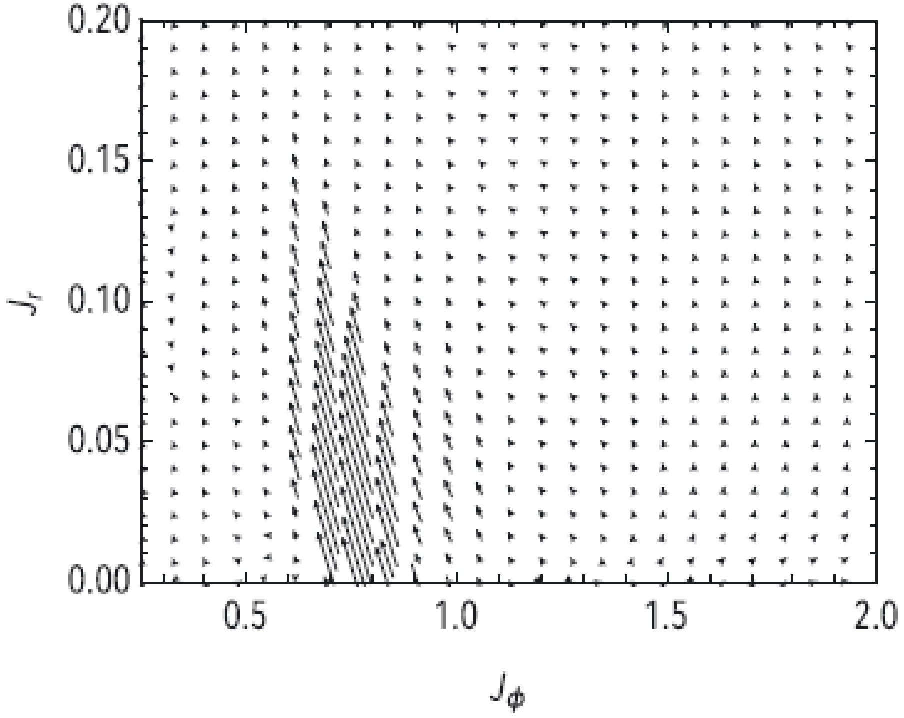
- Compute $$\epsilon_{\alpha\alpha'}(p) \equiv \delta_{\alpha\alpha'} + \frac{(2\pi)^3}{\mathcal{E}}\mathrm{i}\int \mathrm{d}^3\mathbf{J}\,\sum_{\mathbf{n}} \frac{\mathbf{n}\cdot\frac{\partial f_0}{\partial \mathbf{J}}}{p+\mathrm{i}\mathbf{n}\cdot\mathbf{\Omega}_0}[\hat{\Phi}^{(\alpha)}(\mathbf{n},\mathbf{J})]^{*}\hat{\Phi}^{(\alpha')}(\mathbf{n},\mathbf{J})$$

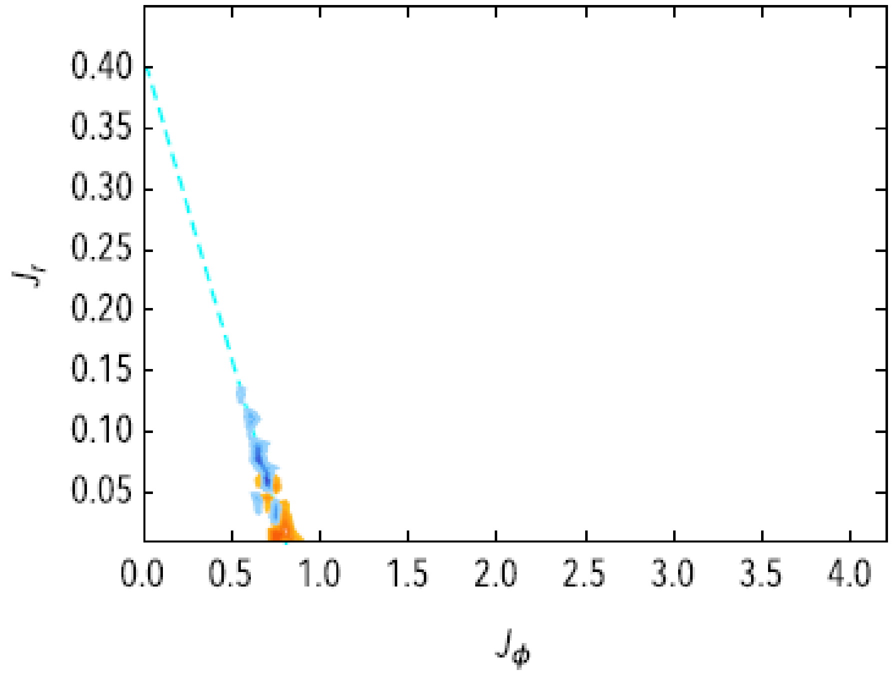
- Hence compute $E_{nn'}$

Results

- Obtain F and div F strongly concentrated along sloping line

$$2\Omega_\phi - \Omega_r = \text{constant} \equiv 2\omega_\mathrm{p}$$



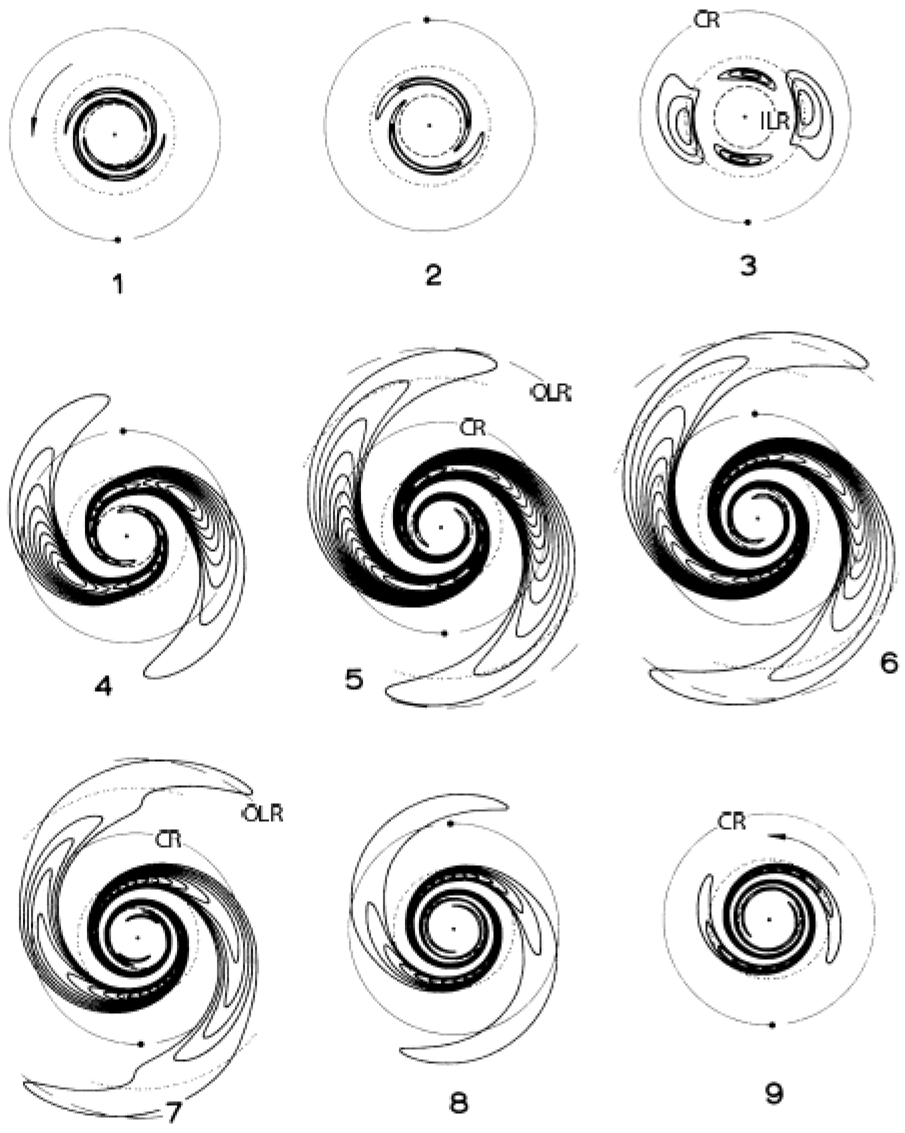


Explanation

- As a leading-spiral wave, moving outwards, approaches the co-rotation radius, differential rotation shears it into a trailing spiral
- Gravity amplifies the wave as it unwinds
- Factor by which it amplifies diverges as Toomre Q to 1

$$Q \equiv \frac{\sigma_R \kappa}{3.36 G \Sigma}$$

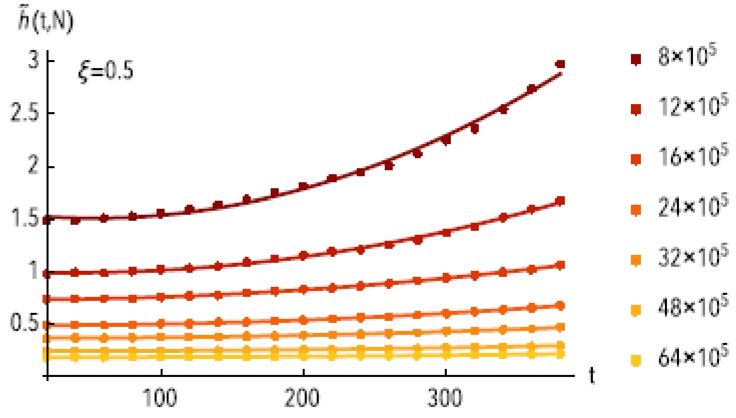
- The trailing wave then moves to inner Lindblad resonance n=(2,-1) where it is Landau damped
- The tapers favour waves with ILR and OLR in regions of high active density
- Hence narrow range in J space impacted by Landau damping

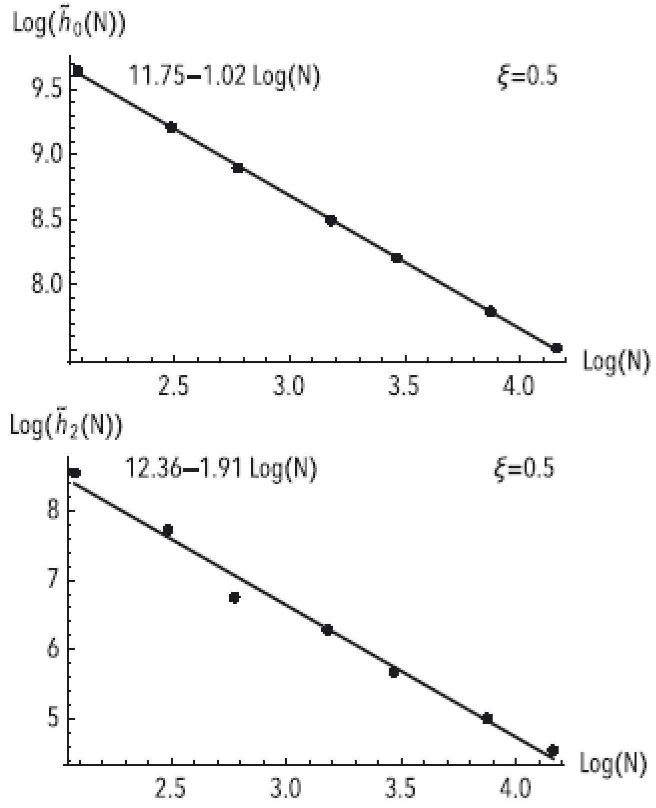


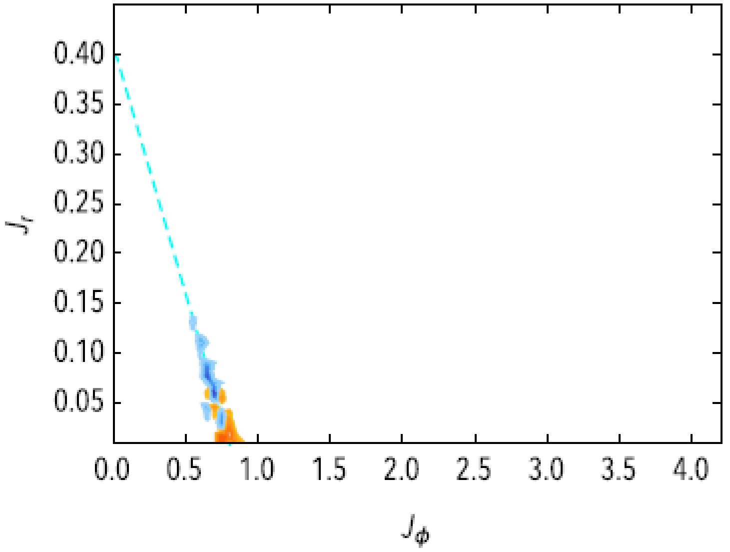
Toomre 1981

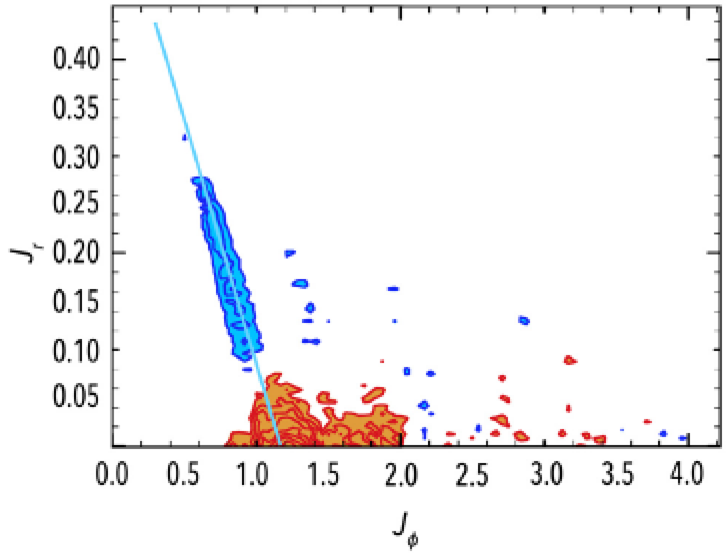
Comparison with N bodies

- Compute $h(t) = \int d^2J\,[f(J,t) - f(J,0)]^2$
- Fit to quadratic in t: $h(t) = h_0 + h_1 t + h_2 t^2/2$
- Explore dependence on N and ξ
- $h(\xi{=}0.6)/h(\xi{=}0.5)$ = 29(NB) or 42(BL)
- N-body noise >1000 times as loud as Spitzer-Chandrasekhar predicted because particles dressec



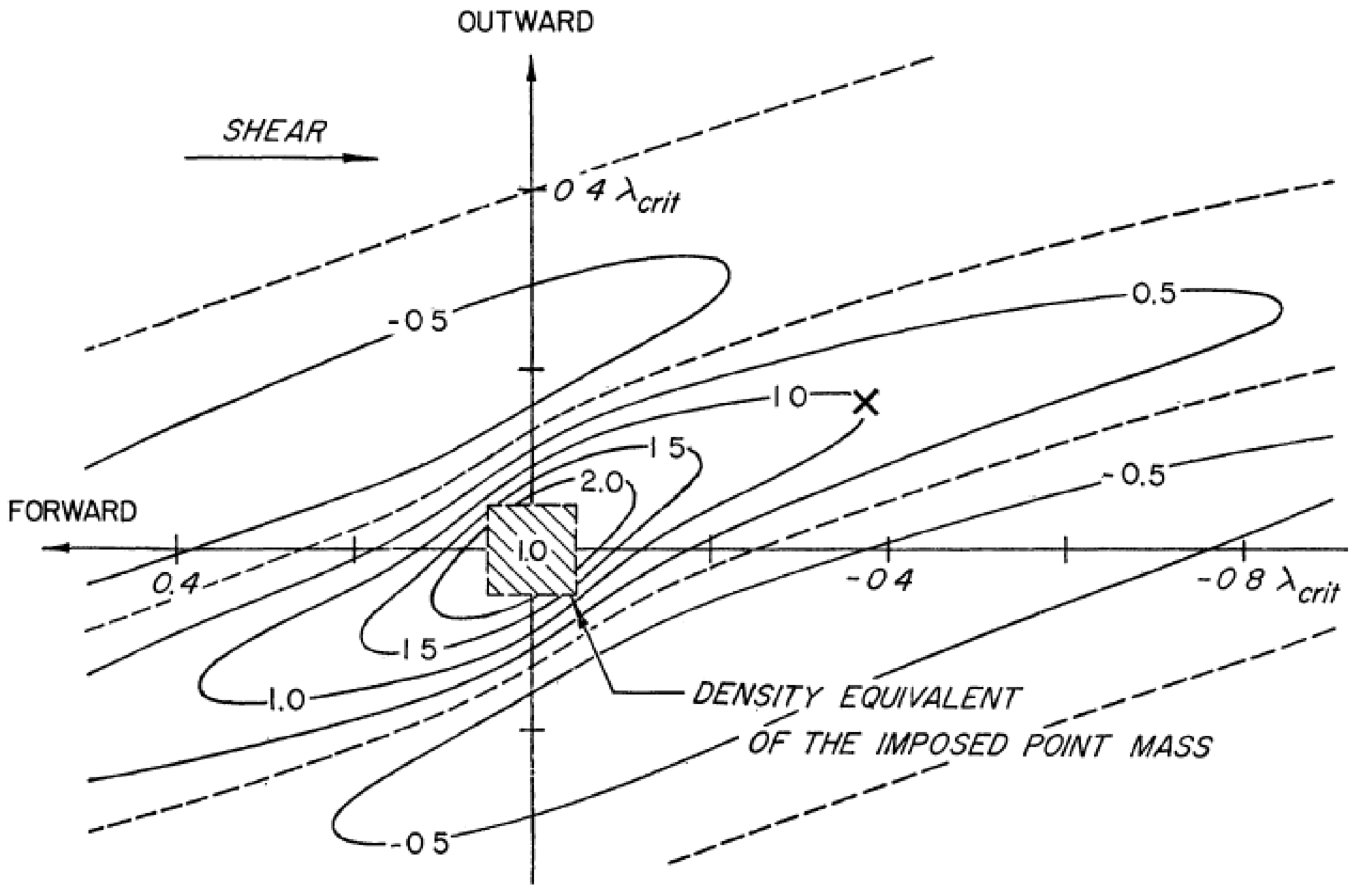






“Debye sphere” of a mass in a disc

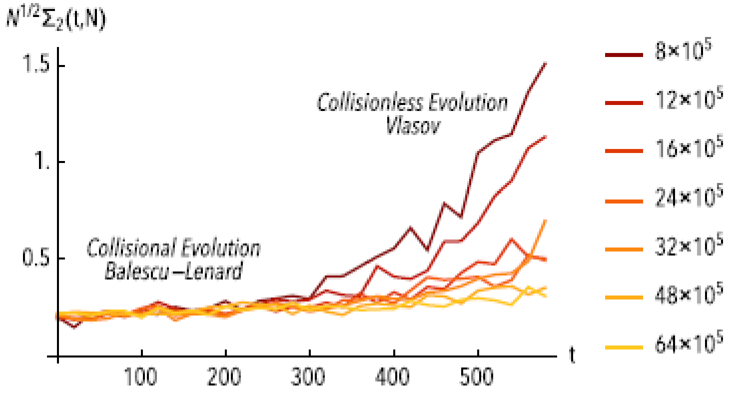
Julian & Toomre 1966 ApJ 146 810



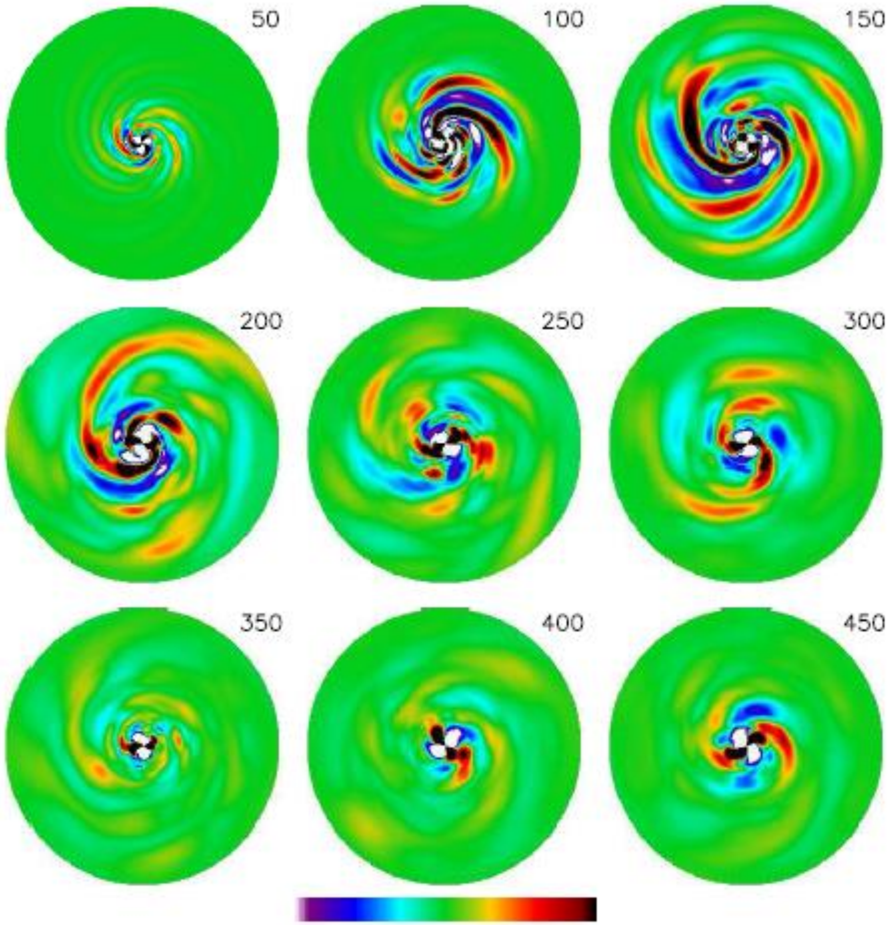
Q = 1.4 v_c = const

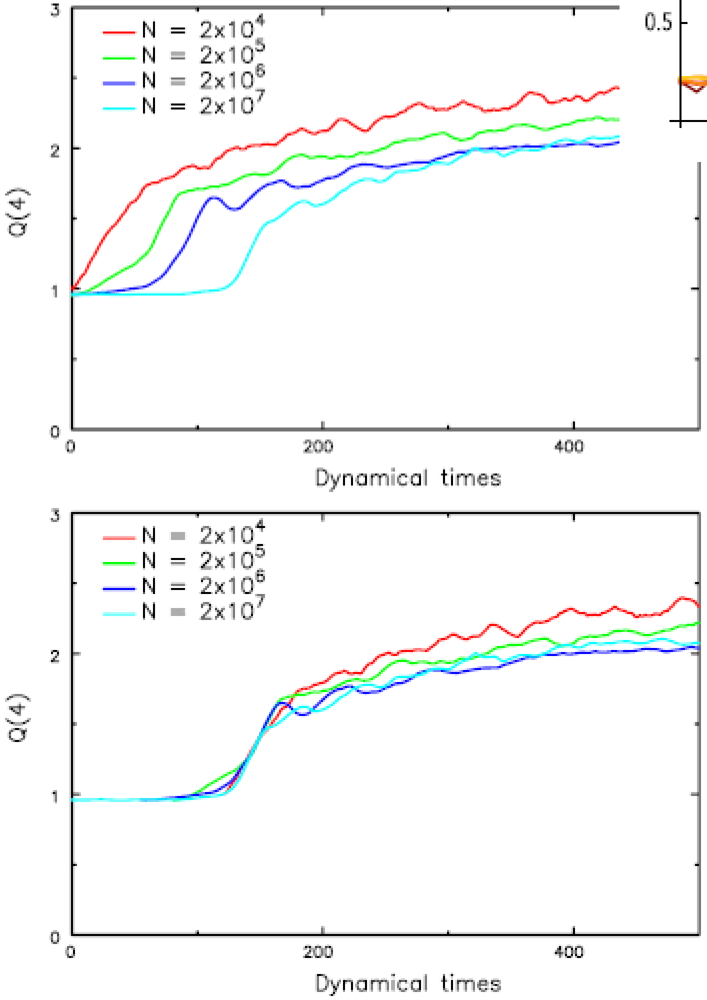
Consequences of resonant heating
(Sellwood & Carlberg 2014 ApJ 785, 137)

- Initial conditions generate leading wave, amplified and absorbed at its CR (what Fouvry et al compute)
- Later noise generates an amplified trailing wave that approaches its ILR, which lies inside ILR of first wave
- The feature in the DF generated by resonant absorption of the first wave is too narrow for the WKBJ approx to hold
- So feature reflects back to CR some of the second wave
- There the reflected portion re-amplified
- Eventually all wave E absorbed at ILR
- So the feature generated in DF at ILR of 2nd wave stronger than the feature at ILR of first wave
- Second feature is an even more effectively silvered mirror!
- Soon the disc is an effective laser in which favoured modes grow exponentially
- The Poisson noise has made the disc unstable at a *collisionless* level



Fouvry et al 2015





Sellwood & Carlberg 2014