Introduction to Quantum Mechanics HT 2010
Problems 6 (weeks 4–5)

6.1 Some things about hydrogen's gross structure that it's important to know (ignore spin throughout):

a) What quantum numbers characterise stationary states of hydrogen?
b) What combinations of values of these numbers are permitted?
c) Give the formula for the energy of a stationary state in terms of the Rydberg $\mathcal{R}$. What is the value of $\mathcal{R}$ in eV?
d) How many stationary states are there in the first excited level and in the second excited level?
e) What is the wavefunction of the ground state?
f) Write down an expression for the mass of the reduced particle.
g) We can apply hydrogenic formulae to any two charged particles that are electrostatically bound. How does the ground-state energy then scale with (i) the mass of the reduced particle, and (ii) the charge Ze on the nucleus? (iii) How does the radial scale of the system scale with Z?

6.2 In the Bohr atom, electrons move on classical circular orbits that have angular momenta $l\hbar$, where $l = 1, 2, \ldots$. Show that the radius of the first Bohr orbit is a_0 and that the model predicts the correct energy spectrum. In fact the ground state of hydrogen has zero angular momentum. Why did Bohr get correct answers from an incorrect hypothesis?

6.3 Show that the speed of a classical electron in the lowest Bohr orbit (Problem 6.2) is $v = \alpha c$, where $\alpha = e^2/4\pi\epsilon_0\hbar c$ is the fine-structure constant. What is the corresponding speed for a hydrogen-like Fe ion (atomic number $Z = 26$)?

6.4 Show that Bohr's hypothesis (that a particle's angular momentum must be an integer multiple of $\hbar$), when applied to the three-dimensional harmonic oscillator, predicts energy levels $E = l\hbar\omega$ with $l = 1, 2, \ldots$. Is there an experiment that would falsify this prediction?

6.5 Show that the electric field experienced by an electron in the ground state of hydrogen is of order $5 \times 10^{11}\,\mathrm{V\,m^{-1}}$. Can comparable macroscopic fields be generated in the laboratory?

6.6 Positronium consists of an electron and a positron (both spin-half and of equal mass) in orbit around one another. What are its energy levels? By what factor is a positronium atom bigger than a hydrogen atom?

6.7 The emission spectrum of the He^+ ion contains the Pickering series of spectral lines that is analogous to the Lyman, Balmer and Paschen series in the spectrum of hydrogen.

Balmer $i = 1, 2, \ldots$	0.456806	0.616682	0.690685	0.730884
Pickering $i = 2, 4, \ldots$	0.456987	0.616933	0.690967	0.731183

The table gives the frequencies (in 10^{15} Hz) of the first four lines of the Balmer series and the first four even-numbered lines of the Pickering series. The frequencies of these lines in the Pickering series are almost coincident with the frequencies of lines of the Balmer series. Explain this finding. Provide a quantitative explanation of the small offset between these nearly coincident lines in terms of the reduced mass of the electron in the two systems. (In 1896 E.C. Pickering identified the odd-numbered lines in his series in the spectrum of the star ζ Puppis. Helium had yet to be discovered and he believed that the lines were being produced by hydrogen. Naturally he confused the even-numbered lines of his series with ordinary Balmer lines.)

6.8 Show that for hydrogen the matrix element $\langle 2, 0, 0|z|2, 1, 0\rangle = -3a_0$. On account of the non-zero value of this matrix element, when an electric field is applied to a hydrogen atom in its first excited state, the atom's energy is linear in the field strength (§9.1.2).