Basics of Quantum Mechanics

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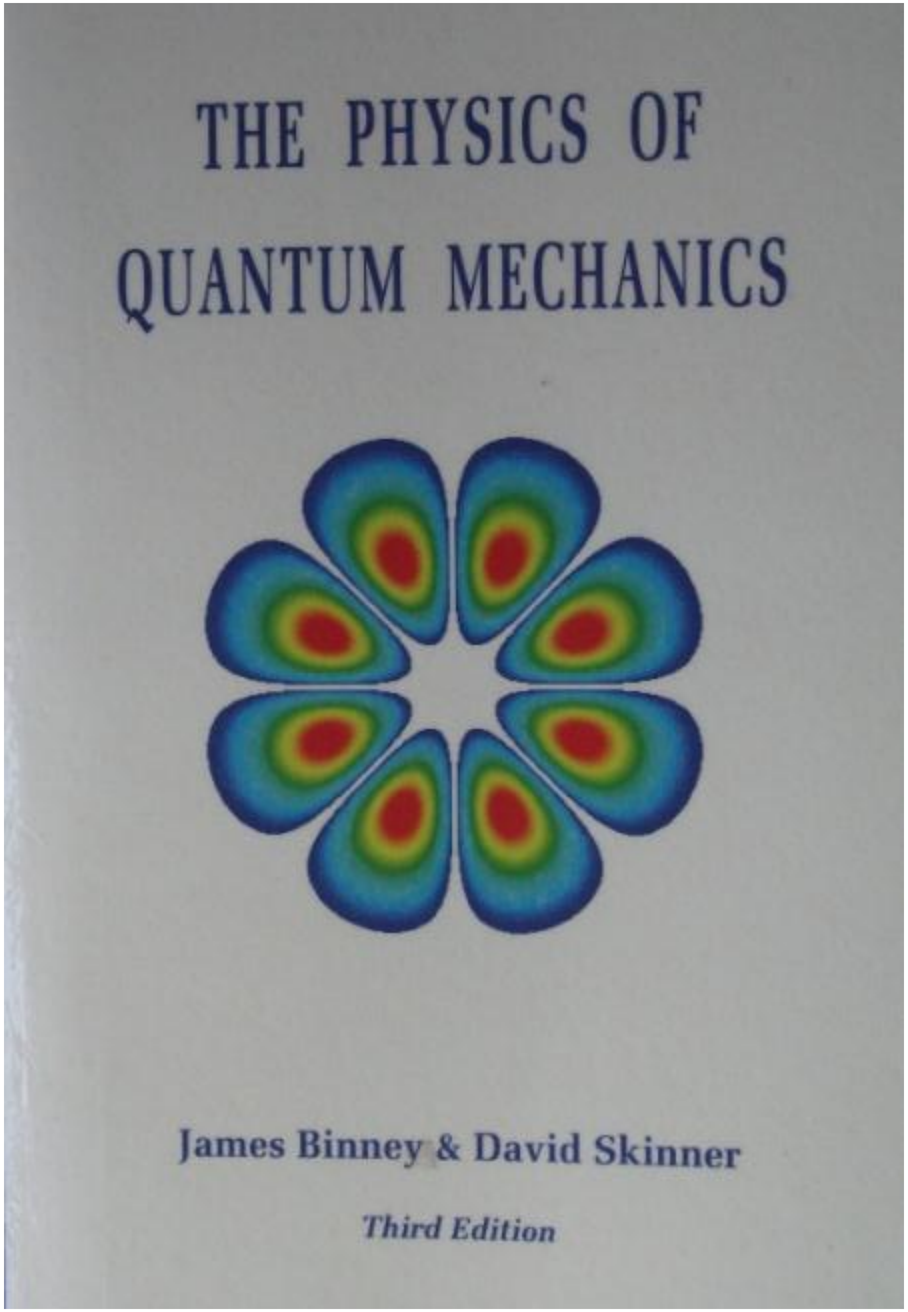


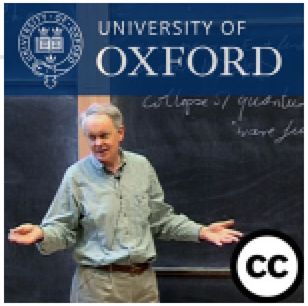
- The book

Available at Clarendon Reception for £20
Also for free download at
http://www-thphys.physics.ox.ac.uk/people/JamesBinney/QBhome.htm

- The film: podcasts can be reached from

http://www-thphys.physics.ox.ac.uk/people/JamesBinney/lectures.html

Quantum Mechanics



In this series of physics lectures, Professor J.J. Binney explains how probabilities are obtained from quantum amplitudes, why they give rise to quantum interference, the concept of a complete set of amplitudes and how this defines a "quantum state". A book of the course can be obtained from http://bit.ly/binneybook

001 Introduction to Quantum Mechanics, Probability Amplitudes and Quantum States
11 December 2009 12:47

First lecture of the Quantum Mechanics course given by Professor James Binney in Michaelmas Term 2009.

Media files
binney01.mp4 (MP4 Video, 288 MB)

002 Dirac Notation and the Energy Representation
11 December 2009 12:48

Second lecture of the Quantum Mechanics course given by Professor James Binney in Michaelmas Term 2009.

Media files
binney02.mp4 (MP4 Video, 300 MB)

003 Operators and Measurement
11 December 2009 12:49

Third lecture of the Quantum Mechanics course given by Professor James Binney in Michaelmas Term 2009.

Physics

- It's about predicting the future from knowledge of the present
- We do it with numbers
- Knowledge of the present derives from measurements
- Measurements are prone to error – our knowledge is imperfect $\Rightarrow$ physics is ultimately probabilistic
  - eg ladder
  - eg pendulum
- To push physics to its limits you must quote probabilities
  - eg R=14 $\pm$ 0.1 Ohms

Measurement 1

- To measure you must disturb
- The disturbance may be too small to matter
  - measure a star's position!
- But often the disturbance matters
  - eg measuring V across a circuit component
- Small things are more strongly disturbed by measuring kit than large ones
- Atoms, electrons, etc are significantly disturbed
- Ideal measurements are *reproducible*:
  - if I say "the momentum p of this electron is 3 GeV/c" I'm claiming that if you measure p with precision, you'll get 3 GeV/c

Measurement 2

- Key to QM is the idea that any system has states in which the outcome of a measurement is certain – these states are abstractions but crucial abstractions
  - eg $|E_1>$ is state in which a measurement of energy will yield E_1 J
  - eg |+> is a state in which a measurement of the z-component of spin angular momentum will yield $+\frac{1}{2}\hbar$ (kg m^2/s)
  - eg $|E_1+>$ is a state in which the results of measuring either E and s_z are certain
  - eg |p> is a state in which a precision measurement of momentum is certain to yield p GeV/c
- In a generic state $|\psi>$, the result of measuring E is uncertain
- But after a high-precision measurement the result of measuring E again is certain (reproducibility!)
- So the act of measuring E jogs the system from the generic state $|\psi>$ into one of the special states $|E_i>$

Measurement 3

- If we do a high-precision measurement of p when the system is in the state $|\psi>$ we jog it into a state $|p_i>$ in which the result of measuring p again is certain
- In general a precision measurement of E when the system is in the state $|p_i>$ yields an uncertain result – we can only calculate probabilities P_{ji} of finding E_j
- Once we have found E_j and jogged the system into the state $|E_j>$ the result of measuring p is uncertain because the system is no longer in one of the special states in which the outcome of a precision p measurement is certain
- That is, each thing you can measure jogs the system into one of a different set of states, so it's not possible to get the system into a state in which the outcome of any precision measurement is certain
  - measurements are generally incompatible
  - *dynamical variables are questions you can ask, not intrinsic properties*

Quantum physics

- We take on board that
  - we have to calculate probability distributions P(x) not just expectation values <x>
  - measurements disturb the system & leave it in a state that differs from the pre-measurement state
- Q physics tackles these tasks using the idealisation of reproducible measurements
- So far everything has been straightforward & inevitable
  - this is just grown-up physics
- But it's clear that Q physics is going to be mathematically more challenging than C physics because calculating a whole (non-negative) function P(x) is much harder than calculating one number <x>

Quantum amplitudes

- Q physics is built on a wonderful mystery:
  - It (& it alone) obtains a probability P from a complex number A the *quantum amplitude* for P:
  - $P=|A|^2$
- Nobody knows why this is the correct thing to do
- No application of this formalism has been successful outside Q physics
- The whole mathematical formalism of Q physics follows naturally & easily once you accept the use of quantum amplitudes
- The formalism is immensely convenient
  - It allows us to calculate probability distributions much more easily than in C physics
- Aren’t we lucky: in our hour of need a powerful new formalism comes to our rescue!

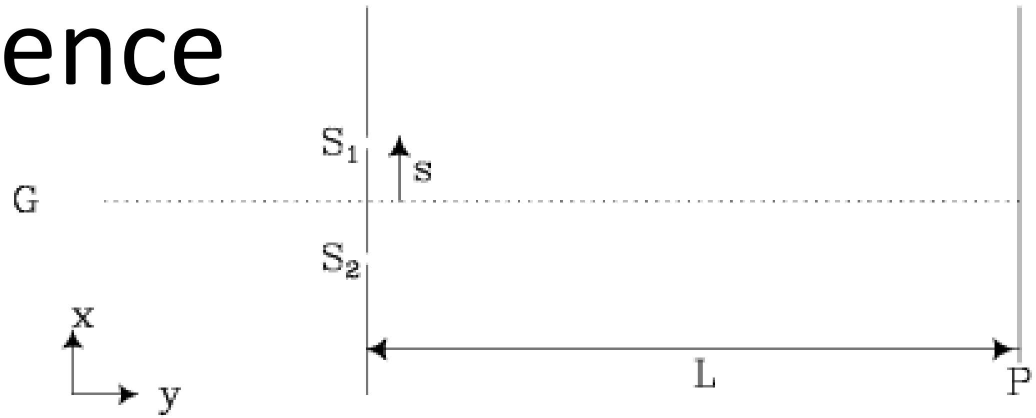
Quantum interference

- Quantum amplitudes have a key, logic-defying property:
  – If something can happen in 2 mutually exclusive ways, 1 and 2, and the amplitude for it to happen by route 1 is A_1 and by route 2 is A_2 then the probability for it to happen by either 1 or 2 is

    $$P_{1+2} = |A_{1+2}|^2 = |A_1+A_2|^2 = |A_1|^2 + |A_2|^2 + (A_1^* A_2 + A_1 A_2^*)$$
    $$= P_1 + P_2 + 2\mathrm{Re}(A_1^* A_2)$$

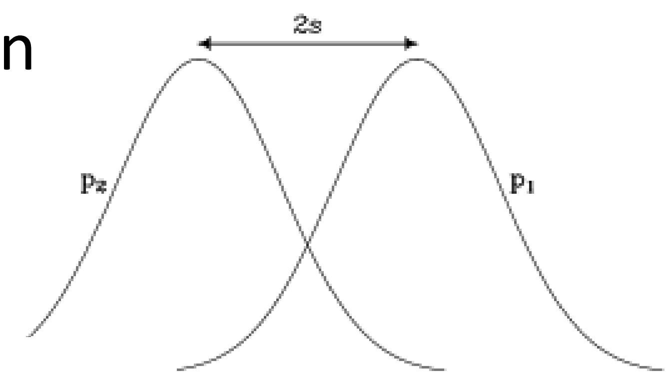
- That is*: we add amplitudes not probabilities*
- The extra term is a manifestation of “quantum interference”

2-slit interference



$$P(x) = |A_1(x) + A_2(x)|^2 = |A_1(x)|^2 + |A_2(x)|^2 + 2\Re(A_1(x)A_2^*(x))$$

- Expect $|A_1|^2$ to be roughly Gaussian
- Write $A_i = |A_i|e^{i\phi_i} = \sqrt{p_i}e^{i\phi_i}$



$$p(x) = p_1(x) + p_2(x) + I(x)$$

$$I(x) = 2\sqrt{p_1(x)p_2(x)}\cos(\phi_1(x) - \phi_2(x))$$

- Near centre line $p_1(x) \simeq p_2(x)$ and P(x) oscillates from 0 to $4P_1(x)$

Quantum states 1

- There are certain things we can measure
- "observables" – a terrible name
- With each observable Q there is a list of possible values q_i returned by a precise measurement of Q
- The set of q_i is called the *spectrum* of Q
  - eg spectrum of x coordinate is $(-\infty, \infty)$
  - eg spectrum of KE is $(0,\infty)$
  - eg spectrum of any component of angular momentum is {…, $(k-1)\hbar$, $k\hbar$, $(k+1)\hbar$,..), where k=0 or ½ and $\hbar = 1.05 \times 10^{-34}$ J s
- Elements of the spectrum are called *allowed values* of Q

Quantum states 2

- With each element of the spectrum q_i there is a probability amplitude A_i that a precise measurement will return that value and a state $|q_i>$ in which the system will be left after the measurement
- QM is the science of calculating from the set $\{A_i\}$ the amplitudes, say $\{a_j\}$, for getting a value such as b_j on measuring another observable B
- A complete set of amplitudes contains sufficient amplitudes to enable the amplitudes for any measurement to be predicted
- Conventionally a complete set is a minimal set:
  - None of its members can be calculated from a knowledge of the other members alone
- A complete set of amplitudes characterises the current state of the system as precisely as is physically possible
- That state, $|\psi>$, is pointed to by the complex numbers $\{A_i\}$ in just the way a geometric point **a** is pointed to by its coordinates (a_x,a_y,a_z)
  - So $|\psi> \leftrightarrow \{A_i\}$ just as $\mathbf{a} \leftrightarrow \{a_i\}$
- $|\psi>$ is a vector with complex components

Quantum states 3

- Just as many different sets of coordinates (a_x,a_y,a_z) or (a_r,a_θ,a_ϕ) all pick out the same geometrical point **a**, so many sets of amplitudes pick out the same physical state $|\psi>$
- By designating a state $|\psi>$ (“ket psi”) we keep open our options as to which complete set of amplitudes we will use for calculations
- In C physics choosing the appropriate coordinate system is often the key to solving a given problem
- In Q physics choosing the appropriate set of amplitudes is often the key
  - eg we can specify the state $|\psi>$ of an electron by giving the amplitudes a(p) to measure momentum p or the amplitudes ψ(x) to measure location x
  - ψ(x) is called the *wavefunction* and its values are quantum amplitudes

Dirac notation 1

- We already discussed the physical significance of the sum of 2 amplitudes
- So if $|\psi\rangle=(A_1,A_2,..)$ and $|\phi\rangle=(B_1,B_2,..)$ are 2 states of the same system, we should consider
  - $|\psi\rangle+|\phi\rangle \leftrightarrow (A_1+B_1, A_2+B_2,..)$
  - Standard rule for adding vectors
- Because probabilities for all possibilities must sum to 1, we require $\sum_i |A_i|^2=1$ and $\sum_i |B_i|^2=1$, & we need to normalise $|\psi\rangle+|\phi\rangle$ by multiplying by $\alpha = 1/(\sum_i|A_i + B_i|^2)^{1/2}$
- So a new physical state is $|\psi'\rangle = \alpha(|\psi\rangle+|\phi\rangle)$
- Objects that you can add & multiply by numbers constitute a vector space
- It's often useful to choose a basis $\{|i\rangle\}$ for a vector space:
- Any state $|\psi\rangle=\sum_i a_i|i\rangle$ for some amplitudes a_i

Dirac notation 2

- With every vector space V we get the dual space V' for free:
  - V' is the space of all linear (complex-valued) functions on V
- We denote members of V' by $<f|$ ("bra f") & then $<f|\psi>$ is a (complex) number, the value taken by the linear function $<f|$ on the vector $|\psi>$
  - In traditional notation $f(|\psi>)$
- If $|i>$ is a basis for V, a basis for V' is provided by the functions $<j|$ defined by the rule
  - $<j|i> = \delta_{ij}$
- Given $|\psi> = \sum_i a_i|i>$ we choose to define
  - $<\psi| = \sum_j a_j^*<j|$ so that
  - $<\psi|\psi> = \sum_{ij} a_j^* a_i <j|i> = \sum_i |a_i|^2 = 1$
- If $|\phi> = \sum_j b_j|j>$ then
  - $<\phi|\psi> = \sum_i b_i^* a_i = (<\psi|\phi>)^*$

Energy representation

- For a particle trapped in a potential well the spectrum of energy E is discrete so there are states $|E_i>$ in which a measurement of E has a certain outcome
- These states form a basis for V so any state
  - $|\psi> = \sum_i A_i |E_i>$
- If we “bra through” by $<E_j|$ we have
  - $<E_j|\psi> = A_j$
- This is a key rule & explains the importance of bras: they enable us to extract experimentally important amplitudes from the system’s state