

## Report on C6 (2007)

39 candidates, mean 57.1, SD 18.8. As in previous years, the questions on relativity and stochastic processes were more popular and better done than the questions on many-body theory. Although the mean mark was way below target (72.8), the questions were doable in the sense that at least one candidate scored more than 22/25 on every question except Q2, which attracted only two attempt, and typically more than 10% of the candidates produced essentially perfect solutions.

Q1 Lagrangian field theory; 20 attempts, mean 15.15 sd 7.56. The distribution of marks was almost uniform on the range 2–25: four candidates scored 24 or 25, and two 2 or three.

Many candidates did a poor job of differentiating with respect to the field gradient.

Most candidates could derive the e-m tensor from translational invariance, but disappointingly many candidates thought they were using invariance under a coordinate transformation (which is assured by  $\mathcal{L}$  being a scalar) rather than that  $\mathcal{L}$  takes the same value at distinct events.

Even weaker candidates could write down suitable spinor invariants, even when they forgot to include a derivative of  $A^\mu$ .

Q2 Gauge conditions and weak gravitational fields; 24 attempts, mean 12.5, SD 6.32. Again a remarkably uniform distribution of marks, with two candidates scoring 22 or 23 and two 2 or 3.

The performance on this very straightforward question was disheartening; candidate after candidate wrote down expressions that made no sense in terms of their index structure, and then sought to fudge their way out of difficulty instead of going back and checking the working.

Many candidates were unclear that the gauge invariance in GR is associated with general coordinate transformations. A subtlety that may confuse them is that whereas in EM the force symbol  $F_{\mu\nu}$  is gauge invariant, its GR analogue,  $\Gamma_{\mu\nu}^\alpha$ , is not.

Nearly all candidates started out calculating  $\nabla_\mu g^{\mu\nu} \partial_\nu \phi$  and only an elite group grasped that the difficulties to which this gives rise are removed if one instead calculates  $g^{\mu\nu} \nabla_\mu \partial_\nu \phi$ .

A great many candidates thought that  $g^{\mu\nu} g_{\mu\nu} = 2$  rather than 4.

Only a small number of candidates provided the requested expression for the Newtonian acceleration at the edge of a uniform sphere of density  $\rho$ . Whether from failure to read the question or inability, I do not know.

Q3 Orbits in the Schwarzschild metric; 39 attempts, mean 17.72, sd 5.24. A bimodal mark distribution with the main peak centred on 21 and a subsidiary peak centred on 11. Four candidates scored 24 or 25.

A very successful question despite a typo in the last part, where  $\frac{9}{5}$  appeared instead of  $\frac{9}{2}$ . This section was generously marked in consequence. Only the best candidates reached the typo, and no candidate showed signs of frustration at being unable to derive the requested frequency range, so I don't think it had a significant impact on anyone.

A significant number of candidates determined which Christoffel symbols are non-vanishing without using a Lagrangian, on the whole arguing the case well, but losing valuable time in the process.

A surprising number of candidates failed to realize, or were very slow in realizing, that the angular velocity is obtained from the  $r$  equation of motion rather than directly from the metric.

Q4 Friedmann metric to luminosity distance; 15 attempts, mean 11.23 sd 6.73. Only one perfect attempt, most candidates being distributed around 9.

When setting the paper I feared this question would prove too easy because a significant part of it could be answered with a good understanding of B3. So the low average score is worrying.

Given the elementary nature of the explanatory part, I marked fairly strictly, for example docking a mark if fundamental observers were required to see an isotropic CMB: there had to be mention of the dipole or something like “at rest w.r.t. the CMB”.

Understanding of the effects of curvature on distance and area was mediocre. Only one candidate provided a sufficiently clear derivation of  $\delta t_2/\delta t_1 = a_2/a_1$  but I accepted clear statements of the frequency-ratio result if accompanied by an explanation of how it gives  $\delta t_2/\delta t_1$ . The candidates found this section hard because

they don't really understand the basic concepts of cosmology, and when they write down an expression such as  $dt = a dr / \sqrt{1 - Kr^2}$  they aren't relating the maths to a well defined physical situation.

In the last part some candidates understood the effect of photons being redshifted, and others the effect of their arriving over a dilated interval, but no candidate grasped that both effects are present. Distressingly many candidates couldn't even handle properly the implication of the differences in the proper areas of the spheres.

Q5; 35 attempts, mean 15.7, SD 3.90. Marks tightly bunched between 8 and 21, with two attempts scoring over 21.

Generally pretty well attempted without any obvious specific problem areas (albeit with various degrees of execution, some deficiencies apparently attributable to carelessness).

The last section was more testing of understanding with anticipated selection of better candidates.

Q6; 8 attempts, mean 9.5, SD 4.27

Most marks in the range 7-11 with one at 2 and one at 18.

Performance on the description of mean field approximation in statistical physics and the section on the Ising model were both generally good. Performance on the metallic ferromagnet was less good (both had been discussed in the lectures). Performance on the brief description of the Hartree and Hartree-Fock was average. The section on Landau theory was less attempted and tested the good student.

Note that all sections of this question had been dealt with in lectures but in separate subsections. The allocation of the last three questions on the exam covered two subsections of the course ("Many Body Quantum Mechanics" and "Phase Transitions and Critical Phenomena") given with an equal number of lectures in each. Consequently I tried to provide one question each completely on each subsection and one linking the two subsections. This is the linkage question. Most of it could be answered without recognising the link (provided the candidate could switch area of lecture recall) but with a few marks for recognising the link (hence the opportunities in the first and last sections). Mostly the connection was not commented upon. The Landau theory required relating different parts of the lectures.

Q7; 2 attempts, mean 7.5, SD 2.5

This was the question drawn completely on the "Phase Transitions and Critical Phenomena" part of the course, specifically on critical behaviour probed by real-space RG. It was not a popular choice. The two students attempting both had erratic performances on other questions, some high marks, some low; hence they may not be good comparators.

The material was closely related to lectures and should have been easy. The introductory section was descriptive bookwork. Section 2 is close to lecture and should have been done as an example already. Section 3 is mainly simple mathematical manipulation together with some interpretation.

Performance was disappointing.

Q8; 13 attempts, 9.54, SD 6.33. Most candidates scored 9 or less, but two attempts scored 22 and 24.

This question relates to the "Many Body Quantum Mechanics" part of the syllabus.

It is simply related to the lectured material but the performance of the majority of attempting candidates was disappointing; I would like to think because of carelessness.

The first section is straight bookwork and generally successfully answered.

The second section is straightforward but does require students to note that its solution requires a Bogoliubov transformation and is not simply a diagonalization in conserved-particle space. This has been well dealt with in lectures and is a crucial part of the course.

The last part is essentially a give-away in which substitution of given results from section 2 solves section 3 with a small level of expansion and interpretation. But few got to it.

That 2 students could get respectively 24 and 22 demonstrates that the problem was not too difficult, but the greater weighting of marks was much lower, around 6-9, possibly suggesting it was off-putting, but I believe was also partly due to carelessness in reading and possibly also suffering from being the last question.