Paper A3, June 2007

Section A

Q1: 171 attempts, mean 4.6 / 6, s.d. 1.6 A standard question on hermitian operators. Most candidates were able to define what is meant by a hermitian operator and to prove that the operator for kinetic energy is hermitian. Generally answered reasonably well.

Q2: 167 attempts, mean 4.4 / 7, s.d. 2.1 A question asking for a proof of the variational principle. The vast majority of candidates were able to make reasonable progress in this question, although several candidates restricted their 'proof' to eigenstates of the Hamiltonian.

Q3: 171 attempts, mean 3.6 / 6, s.d. 1.6 A question on the time evolution of a superposition of two states. Many candidates found this question more difficult. Most wrote down the state of the system for times $t \geq 0$ correctly; of those who did not do so, many multiplied the superposition state $|\psi(0)\rangle$ by $\exp(-\mathrm{i}Et/\hbar)$ where E was variously the total energy, the average energy, or the difference in the energy of the states. Most candidates realized that in order to show that the states at $t = 0$ and $t = \tau$ were orthogonal required a demonstration that $\langle\psi(\tau)|\,\psi(0)\rangle = 0$. However, most of these attempts failed — often because the candidates could not show that an expression of the form $\exp(-\mathrm{i}\pi x)+\exp(-\mathrm{i}\pi(x-1))$ is equal to zero. A surprising number of candidates did not notice the solidus in the expression for τ and consequently derived expressions which were dimensionally incorrect.

Q4: 171 attempts, mean 4.6 / 6, s.d. 1.2 A question on spin matrices. Almost all candidates were able to find the eigenvalues and eigenvectors of s_x. Many candidates then went on to find the eigenvectors and eigenvalues for s_z, even though this was not required. Only about half the candidates were able to write down the spin state of the system of two electrons with $S = 0$, but essentially all realized that after measuring the spin of one of the electrons to be $s_z = +\hbar/2$ the probability of a measurement of the spin of the second electron yielding $s_z = -\hbar/2$ was unity.

Q5: 168 attempts, mean 4.8 / 7, s.d. 1.5 A question on the Zeeman effect in the ground state of hydrogen. Most candidates were able to make a reasonable attempt at this question. Many candidates sketched the energy levels for a fixed magnetic field, not their variation with the magnetic flux density as asked. Only around half the candidates provided reasonable discussions of how the Zeeman effect can assist the determination of the quantum numbers of atomic levels.

Q6: 169 attempts, mean 4.0 / 8, s.d. 2.0 A question on the Sudden Approximation. This question caused more difficulty. Most candidates recognized that they should express the original wave function as a superposition of the wave functions of the new potential, and most candidates were able to write down the correct overlap integral to be evaluated. Evaluating the integral proved difficult for approximately half the candidates despite the fact that the integral to be evaluated could be expressed in terms of the given integral by completing a square. A depressingly high proportion of candidates made trivial algebraic errors in finding the displacement which maximizes the given expression for P_1. The last part of the question required some insight was generally answered poorly.

Section B

Q7: 137 attempts, mean 11.9 / 20, s.d. 3.6 This question considered a stream of particles incident upon a square potential well of depth V_0. Almost all candidates explained reasonably well why the solutions to the time-independent Schrödinger equation took the suggested form; most were able to find expressions for the wave vectors k and β in the three regions, although sign errors in the result for β were common. Showing that no reflection occurred when $\beta a = n\pi$ proved more problematic; essentially all the candidates were able to use the boundary conditions to derive relations between the amplitudes of the wave functions, but many had difficulty handling the four equations which resulted. Most candidates were able to use the information given in the question to find the energies of the particles which were not reflected, and essentially all were able to quote or derive the energy eigenvalues of an infinite square well with $V = 0$ within the well. Relatively few candidates realized that the energies of those particles not reflected by the potential well of the problem correspond exactly to the energies of the eigenstates of an infinite square well with $V = -V_0$ within the well. Many of the candidates were able to derive an expression for the wave function within the potential well, under conditions of no reflection, but very few realized that this corresponded to a standing wave plus the original incident wave.

Q8: 133 attempts, mean 12.0 / 20, s.d. 4.2 In this question simple properties of a simple harmonic oscillator were derived using operator methods. Those candidates comfortable with manipulating operators generally did very well; the others less so. Most candidates were able to show that the Hamiltonian could be written in terms of the operators $\hat{A}$ and $\hat{A}^\dagger$. Most were able to prove the commutation relations asked for, although many made this more difficult than need be by writing $\hat{p}$ in terms of $\partial/\partial x$ rather than retaining $\hat{p}$ and using $[\hat{x}, \hat{p}] = \mathrm{i}\hbar$. Almost all candidates were able to show that the operator $\hat{A}$ gave rise to a ladder of states spaced in energy by $\hbar\omega$. Very few candidates explained why there should be a lowest energy; and, rather than using operator methods, many candidates found the energy of the lowest state by finding the wave function of the ground state and evaluating the expectation value of the Hamiltonian (candidates received marks for this approach, but it was more time-consuming). Most candidates were able to to write $\hat{x}$ and $\hat{p}$ in terms of $\hat{A}$ and $\hat{A}^\dagger$, but only a minority could use these relations to evaluate the expectation values required to find the uncertainty relation.

Q9: 134 attempts, mean 9.6 / 20, s.d. 3.3 A standard question on the energy shift arising from the spin-orbit interaction in hydrogen. This question was not answered well. All candidates were able to identify the operators associated with the quantum numbers l and m, but few were able to explain *why* the energy levels of the gross structure of hydrogen were independent of m. Most candidates were able to show that $\hat{L}_z$ does not commute with the perturbation, and most were able to explain why $\hat{J}^2$ and $\hat{J}_z$ did. Almost all candidates were able to derive the energy shift arising from the spin-orbit interaction, but there was in general little or no discussion of which representation was being employed in the perturbation calculation. Most candidates drew complicated diagrams in which they tried to indicate how the spin-orbit interaction affected all the levels with a given principal quantum number n; whereas the question asked for a sketch of how the interaction affected the energy a level with given n and l. Only around half the candidates realized that the spin-orbit interaction split each level of given n, l into two levels labelled by $j = l \pm 1/2$. Many candidates were unaware that a level of total angular quantum number j has a degeneracy of $2j + 1$, and this, together with incorrect energy level diagrams, meant that only a handful of candidates could show that the average energy shift is zero.

Q10: 106 attempts, mean 11.8 / 20, s.d. 3.4 This was a standard question dealing with the energy level structure of He-like ions. Most candidates were able to show that if the electron-electron interaction is ignored the Schrödinger equation can be separated into two one-electron equations. Most were able to calculate, within this approximation, the ionization energies of He and the He-like ion Ne^{8+}, although many candidates took the charge of the Ne nucleus to be +8e. Most candidates realized that the difference between their calculated ionization energies and the actual values is largely caused by neglecting the electron-electron interaction. However, very few commented on the fact that the relative error is much lower for the Ne. Most candidates explained the Exchange Principle well. The candidates' explanations for the singlet-triplet splitting in the excited configurations of He-like ions were frequently muddled. Many did not, as asked, discuss how the mean separation of the two electrons depended on the symmetry of the spatial part of the wave function, and only a fraction of those who did were able to use this information to explain why the electron-electron interaction shifts states of different symmetry by different amounts. Quite a large number of candidates thought that a pair of electrons with an anti-symmetric spatial wave function obeyed the Pauli Exclusion Principle but those with a symmetric spatial wave function did not. The candidates' sketches of the effect of the electron-electron interaction on the energy of a configuration were generally incorrect.