Paper C6 2006

49 candidates (including P&P), mean 59.6%, SD 15.0%

Q1. 19 attampts, mean mark 14.3, SD 6.5, highest mark 25

Spinor invariants & Lagrangian field theory

Too many students were muddled about order changes when taking adjoints, so $(M\eta)^\dagger$ became $M^\dagger\eta^\dagger$ instead of $\eta^\dagger M^\dagger$. Nevertheless, correct expressions were frequently reached because there was a matching flexibility about the ordering of terms. Few students recognised that $\sigma_{\mathbf{n}} \equiv \mathbf{n}\cdot\sigma$ has exactly the same algebraic properties as say σ_x. Lagrangian densities were always chosen so that their vector indices were properly summed over, but too often, despite the first part of the problem, no attention was paid to ensuring that the (implicit) spinor indices were likewise summed over. Thus $\partial_\mu\psi\partial^\mu\psi$ would appear. Also too many "interaction" terms contained only A^μ and no ψ.

Q2. 28 attempts, mean mark 12.9, SD 7.5, highest mark 25

Gravitational lensing.

Remarkably few students were clear that the speed of light in vacuo is always c and that the non-unit refractive index is merely a convenient artefact obtained because ds is not a proper distance and dt is not a proper time. Nearly everybody could derive the deflection angle from Fermat's principle. The big stumbling block proved to be obtaining the Newtonian gravitational force generated by a density distribution $\rho \propto 1/r^2$. The electrostatic analogue of this problem is at Prelims level, so this failure was disappointing.

Q 3. 39 attempts, mean mark 16.3, SD 4.2, highest mark 25

Orbits in the Schwarzschild metric.

This question worked well. Students had no difficulty deriving the equations of motion. Most obtained the period readily, although a significant number were distracted by the t and ϕ equations of motion, and of these many fell into the trap of evaluating the integration constant in the t equation by considering $r \to \infty$, which is impossible in this context. The lifetime exercise distinguished the best students because only they appreciated that proper time needed to be obtained for an orbiting observer rather than one stationary at fixed r, or using inadequate formulae from special relativity.

Q4. 41 attempts, mean mark 16.7, SD 4.7, highest mark 25

Cosmology

Surprisingly few students could derive the condition $\Gamma^\mu_{00} = 0$ in the first part, but the majority of these could show that a metric that lacks $dtdx$ terms satisfies this condition. It was a pity that the space-time dependence of g_{ij} had not been made explicit: a fraction of students assumed that it was a constant. Quite a number of students didn't even attempt this first part of the question.

The derivation of both the Christoffel symbols and the Ricci tensor of the flat cosmology was well done by nearly everyone.

Too many students were unclear what equation expresses conservation of energy: patial derivatives of T^{00} often appeared; a few students didn't even propose derivatives; several got bogged down on the other side of Einstein's equations. A handful of students obtained the given conservation equation by considering the adiabatic expansion of a sphere of gas in the absence of gravity. With some reluctance I gave these students full marks: the argument is clearly bogus given that it neglects both potential and kinetic energy, but I suspect taught in the Astrophysics sub-department.

Q5. 22 attempts, mean mark 15/25, standard deviation 4/25, highest mark 22/25.

Question about boson creation and annihilation operators and application to interacting Bose gas.

Generally well done. Weaker candidates typically failed to derive the last displayed result, usually because they did not consider the normalisation of the state $|M, N\rangle$.

This question unfortunately contained two misprints:

(i) At the end of the second paragraph, candidates were asked to show that $[b(\mathbf{r}), (b_l^\dagger)^N] = N\phi(\mathbf{r})$, while the correct result is $[b(\mathbf{r}), (b_l^\dagger)^N] = N\phi(\mathbf{r})(b_l^\dagger)^{N-1}$;

(ii) In the final paragraph, candidates were asked to consider ‘fixed $M = N$’, while the intended restriction was ‘fixed $M + N$’.

In connection with (i): some of the candidates attempting this question did derive the correct result; those that arrived instead at the incorrect result given in the question did so by getting first to the correct result, and then dropping the factor of $(b_l^\dagger)^{N-1}$ without explanation. Full marks for this part were given in either case. A careful reading of answers did not uncover any candidates who had difficulties with later parts of the question because of the misprint (i).

In connection with (ii): the misprint spoiled the physical point of the question but not the mathematical problem. In the circumstances, only one mark was allocated for an answer to the last sentence (‘In what sense may one conclude ... ’) and no candidate made any suggestions.

Q6. 13 attempts, mean mark 10.9/25, standard deviation 4.3/25, highest mark 17/25.

Quantum mechanics question about a radiation mode coupled to a two-level system.

Some adequate attempts but none very good and some rather poor. Better candidates recognised that the first 6 marks were for a standard derivation, but weaker ones simply wrote down the answer or passed over this part. Most candidates could evaluate $[\mathcal{H}, B + F]$ correctly but few got to the end of the calculation of the eigenvalues of $\mathcal{H}$ and there were no good attempts at the final 7 marks.

Q7. 19 attempts, mean mark 16.6/25, standard deviation 3.3/25, highest mark 23/25.

Descriptive material about the liquid-vapour transition and a calculation on a one-dimensional model for this transition, using transfer matrices.

Most candidates could sketch a phase diagram but only a minority knew what to expect for dependence of density on chemical potential.

All candidates attempting the question could set out the essentials of the transfer matrix method, but only a minority were clear about how to represent the observable n_i as a 2×2 matrix within this approach. All tried (mainly successfully) to find the eigenvalues of $\mathbf{T}$, and marks were awarded for this even though it was not strictly necessary.

Q8. 15 attempts, mean mark 12.6/25, standard deviation 4.9/25, highest mark 23/25.

Treatment of a master equation for a simple three-state system.

While there were some very good answers to this question, there were also some disappointingly weak ones. A significant fraction of candidates attempting the question confused the problem for one with evolution through discrete time steps. Some candidates also failed to write down the correct Master equation: in these cases, the cause appeared to be poor understanding of background material, rather than mis-interpretation of the question. Those who had the correct Master equation could derive the current but only a couple of candidates solved the final part, on time dependence.