Endless Love:
On the termination of a number game

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Introduction

The "LOVES" game, played in playgrounds and classrooms across at least the UK, is a word-and-number game which attempts to assign a two-digit number to two character strings, based on the occurrence frequencies of letters within the strings.

Rules

To begin, the two strings are concatenated, the occurrence frequencies of each letter in the word "LOVES" are found, and expressed as a series of integers in the interval $[0, 9]$. Throughout the game, numbers greater than 9 are expressed as decimal strings. For example, the number 12 becomes the string $\{1, 2\}$.

An additive procedure is then applied to the number string iteratively. Given a string $S(t)$ of $n(t)$ digits, the integer sequence at the next step $I(t+1)$ is found with:

$$I(t+1)_i = S(t)_i + S(t)_{i+1},$$

with $I(t+1)$ containing $n(t) - 1$ integers. To form the number string $S(t+1)$, any numbers in $I(t+1)$ greater than 9 are expanded into their representative strings as described above, which may result in $n(t+1)$ being greater than $n(t) - 1$.

As an example run, consider "Alice LOVES Bob". The "LOVES" is placed by convention to separate the two strings and does not contribute to the letter count. The initial letter frequencies are $\{1, 1, 0, 1, 0\}$. The subsequent iterations yield $\{2, 1, 1, 1\} \rightarrow \{3, 2, 2\} \rightarrow \{5, 4\}$, whereupon the iteration terminates, having produced a length-2 string. The conclusion of the game is to represent the two subjects' love as a percentage made from the final string: "Alice loves Bob 54%!".

Termination

We are concerned with whether or not the iteration terminates, and if it does, the number of steps required to reach the length-2 string. To this end, all start strings in the interval $[00000, 99999]$ were investigated. This clearly neglects character string with more than 9 occurrences of any of the "LOVES" letters, but suffices for a preliminary investigation.

Fig. 1 shows the occurrence of all terminating outcomes. There are significant peaks at 18% and 99%. Fig. 2 shows the convergence time for all start strings, ordered as decimal numbers.

Out of 10^5 runs, 2401 did not terminate, giving

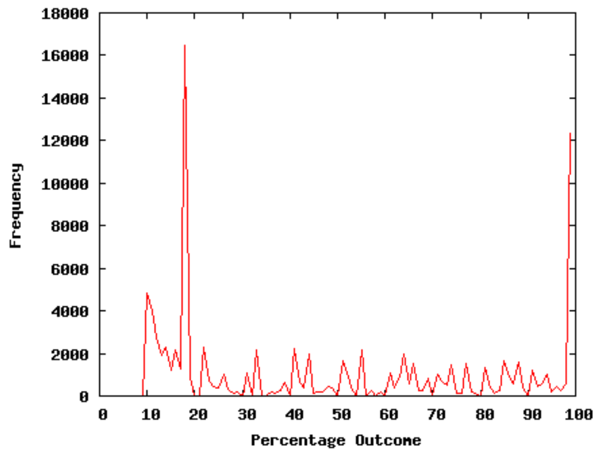

Figure 1: Frequency of a given percentage outcome occurring, over the start string interval $[00000, 99999]$. The peaks are at 18 and 99.

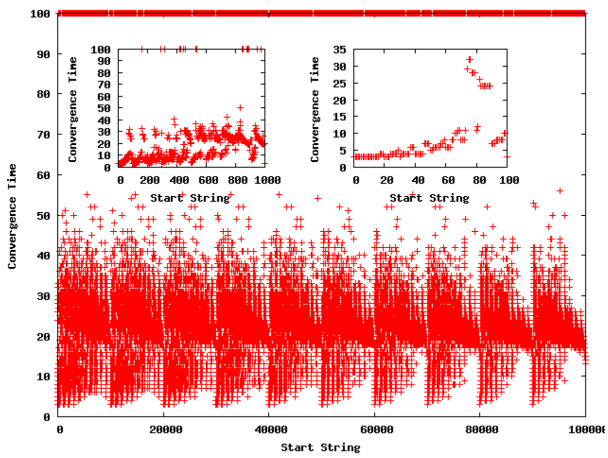

Figure 2: Convergence time given a particular start string. Insets show same plot on different scales. Convergence time of 100 indicates non-termination.

a 2.4% chance of non-termination. From these 2401, 360 show a steady-state period-2 cycle (e.g. 991 $\rightarrow$ 1810 $\rightarrow$ 991) and 240 show a steady-state 3-cycle (e.g. 33336 $\rightarrow$ 6669 $\rightarrow$ 121215 $\rightarrow$ 33336). The remaining 1921 expanded without bound.

It seems that 2-cycles are characterised by the $99x$ form and 3-cycles by the $1212x$ form. However, given different values of x, these forms may also result in the boundless growth class.

The problem of determining a rule to list all non-terminating strings is open. If $NT(n)$ denotes the number of non-terminating strings less than n, where strings are represented by decimal numbers, $NT(n)$ is numerically observed to grow linearly with n, with $NT(n) \simeq 0.025n$.

Example

Consider "Veronica LOVES Jessee". The initial string is $\{0, 1, 1, 4, 2\}$. Subsequent iterations yield $\{1, 2, 5, 6\} \rightarrow \{3, 7, 1, 1\} \rightarrow \{1, 0, 8, 2\} \rightarrow \{1, 8, 1, 0\} \rightarrow \{9, 9, 1\} \rightarrow \{1, 8, 1, 0\} \rightarrow ....$