IPhT, Saclay, April 2010

QCD Hard Processes in the Forward Region at the LHC

F. Hautmann (Oxford)

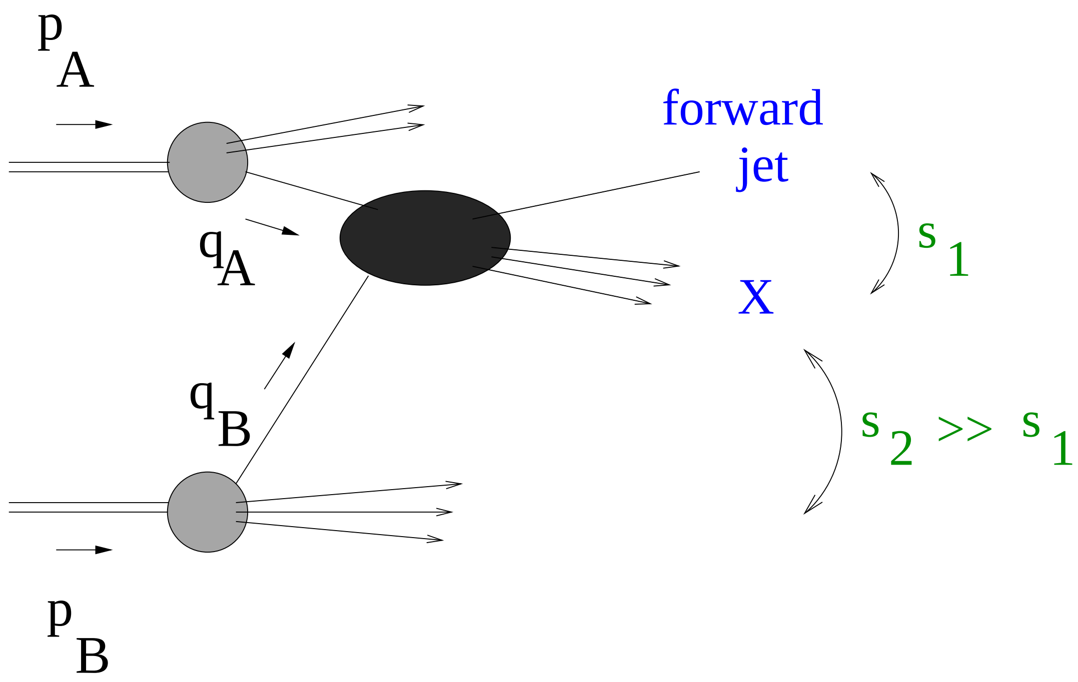
I. Introduction: high-p_T events at forward rapidities

II. QCD factorization at high energy and unintegrated pdf's

III. Applications to physics of parton showers and jets

IV. Prospects for LHC multi-particle final states

I. High-p_T production in the forward region at the LHC



▷ phase space opening up for large $\sqrt{s}$

▷ unprecedented coverage of large rapidities (calorimeters+proton taggers)

$\Downarrow$

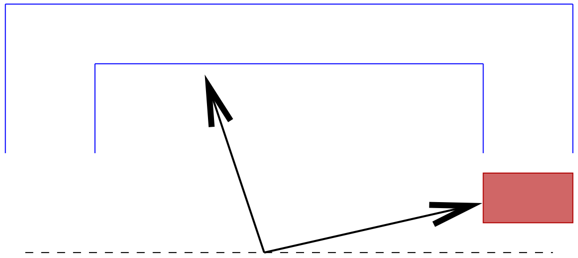
• physics of hard processes with multiple hard scales

and highly asymmetric parton kinematics $q_A \cdot p_B \gg q_B \cdot p_A$

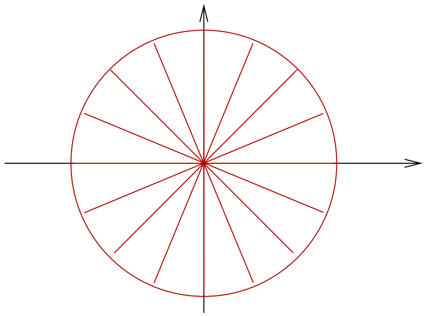
• polar angles small but far enough from beam axis

• measure azimuthal plane correlations

$$p_\perp \gtrsim 20 \text{ GeV}, \ \Delta\eta \gtrsim 4 \div 6$$



central + forward detectors



azimuthal plane

▷ ATLAS, CMS, LHCb
+ CASTOR experiments

[Z. Ajaltouni et al., HERA-LHC Proc. arXiv:0903.3861;

M. Grothe, arXiv:0901.0998; D. d'Enterria, arXiv:0806.0883;

X. Aslanoglou et al., CERN-CMS-NOTE-2008-022 (2008)]

▷ Multi-scale problem ⇒

⇒ all-order summation of high-energy logarithmic corrections long recognized to be necessary for reliable QCD predictions

Mueller & Navelet, 1987; Del Duca, Peskin & Tang, 1993; Stirling, 1994

⇒ efforts toward improved Monte Carlos / semi-analytic approaches

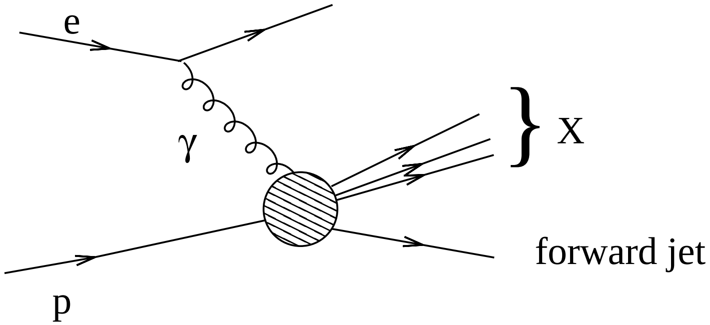
Andersen, arXiv:0906.1965; Andersen and Sabio Vera, 2003;

Andersen, Del Duca, Frixione, Schmidt and Stirling, 2001;

Schwennsen, hep-ph/0703198; Bartels, Sabio Vera and Schwennsen, 2006;

Ewerz, Orr, Stirling and Webber, 2000; Orr and Stirling, 1998

- DIS case ⇒



- neither PYTHIA Monte Carlo nor NLO calculations are able to describe forward jet ep data

[A. Knutsson, LUNFD6-NFFL-7225-2007 (2007); L. Jönsson, AIP Conf. Proc. 828 (2006) 175]

- High-energy factorization at fixed transverse momentum

$$\frac{d\sigma}{dQ_t^2 d\varphi} = \sum_a \int \phi_{a/A} \otimes \frac{d\widehat{\sigma}}{dQ_t^2 d\varphi} \otimes \phi_{g^*/B}$$

▷ needed to resum consistently both logs of rapidity and logs of hard scale

Catani et al., 1991; Ciafaloni, 1998

Deak, Jung, Kutak & H, arXiv:0908.0538

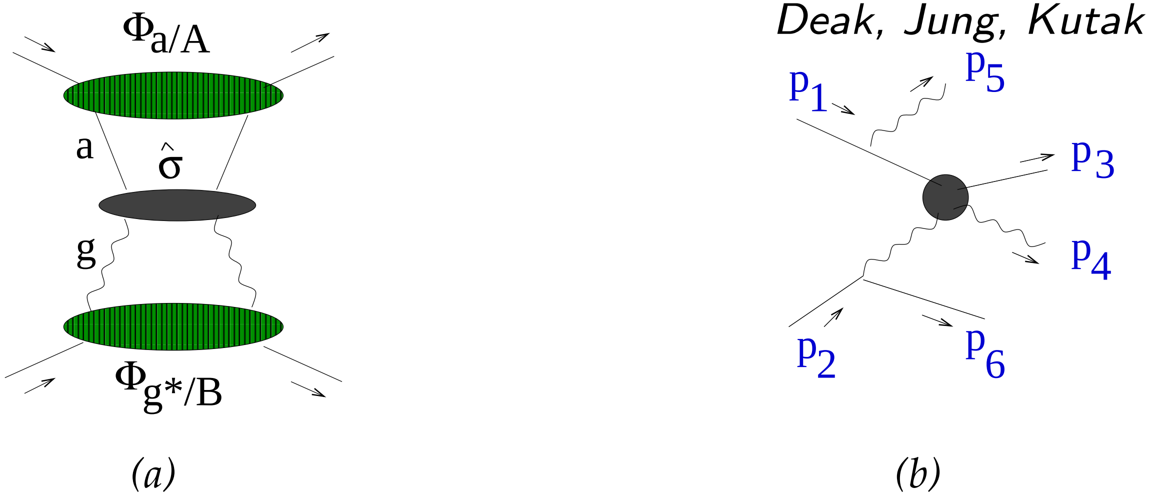


Figure 1: (a) Factorized structure of the cross section; (b) a typical contribution to the qg channel matrix element.

◊ ϕ_a near-collinear, large-x; ϕ_{g^*} $k_\perp$-dependent, small-x

◊ $\widehat{\sigma}$ off-shell continuation of hard-scattering matrix elements

Remarks

♢ Note difference from classic Mueller-Navelet approach to forward hard processes

$$\sigma^{(MN)} = \sum_a \int \phi_{a/A} \otimes V_{jet1} \otimes \mathcal{G}_{gg} \otimes V_{jet2} \otimes \phi_{b/B}$$

- non-collinear corrections to ϕ distributions
- no "vertex jet function" V_{jet}
- jets produced by either hard ME or parton shower (taking account of $k_\perp$)

OUTLINE

i) Generalized factorization and unintegrated pdf's

ii) Parton evolution by branching methods

iii) Applications to collider processes with multi-jets

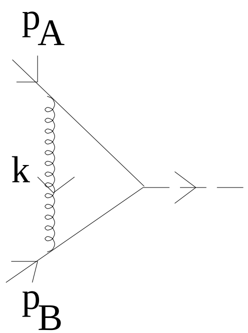
II. Generalized factorization formulas

Example: Sudakov form factor of quarks

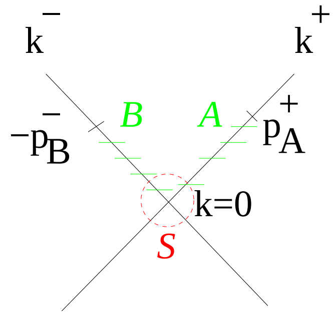
Collins & H, PLB 472 (2000) 129

SCET: Hoang, Manohar et al., arXiv:0901.1332

- Theory well-known. Enters Drell-Yan production, W-boson $p_\perp$ distribution, etc.



(a)



(b)

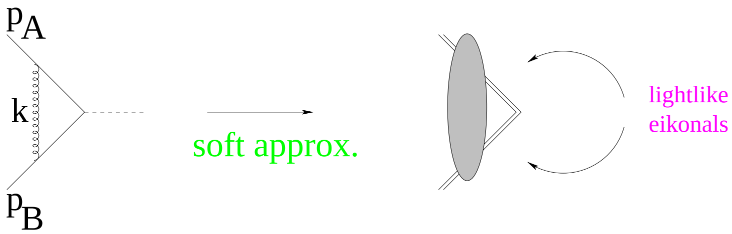
Look for decomposition of the amplitude Γ

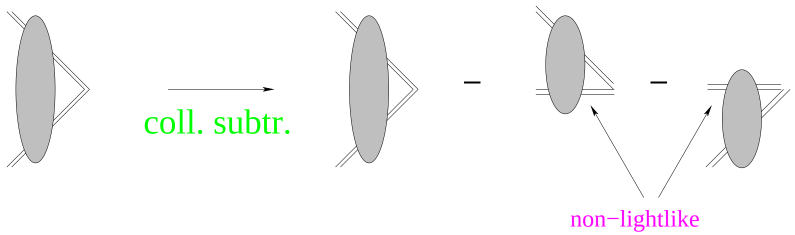
$$\Gamma = \sum_{\text{regions } R} M_\Gamma(R) \ + \ \text{nonleading}$$

such that i) term for hard region be integrable; ii) splitting be defined gauge-invariantly

$$\sigma[\Gamma] = \int [dk] \ S \ \otimes \ C_A \ \otimes \ C_B \ \otimes \ H \ + \ \text{nonleading}$$

Example: Soft-region term S





$$u_A = (u_A^+, u_A^-, 0_\perp),\ u_B = (u_B^+, u_B^-, 0_\perp) \quad (\eta = u_A^+/u_A^-)$$

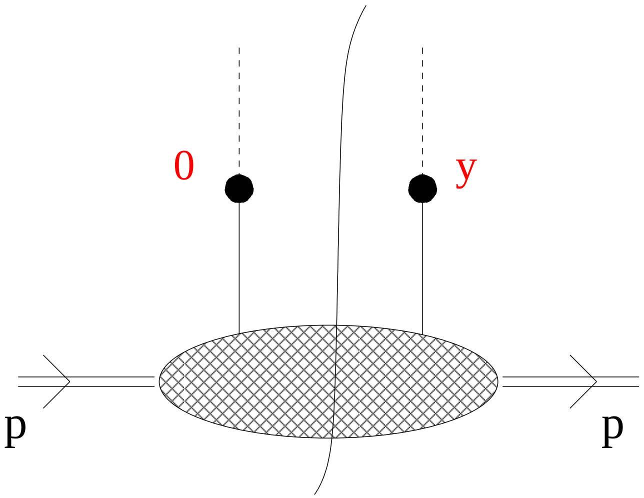
$$S = \frac{\overbrace{\langle 0|V_q(\hat{p}_A)\, V_{\bar{q}}(\hat{p}_B)|0\rangle}^{\text{unsubtracted soft}}}{\underbrace{\langle 0|V_q(\hat{p}_A)\, V_{\bar{q}}(u_B)|0\rangle\, \langle 0|V_q(u_A)\, V_{\bar{q}}(\hat{p}_B)|0\rangle}_{\text{collinear subtractions}}} \overbrace{\langle 0|V_q(u_A)|0\rangle\, \langle 0|V_{\bar{q}}(u_B)|0\rangle}^{\text{residual external lines}}$$

with $V_q(n) = \mathcal{P}\exp\left(ig\int_{-\infty}^{0} dz\, A(z\,n)\cdot n\right)$, $V_{\bar{q}}(n) = \mathcal{P}\exp\left(-ig\int_{-\infty}^{0} dz\, A(z\,n)\cdot n\right)$

$\Rightarrow \partial S/\partial\eta = K \otimes S$ CSS evolution equations [Collins-Soper-Sterman]

↖ resums $\alpha_s^n \ln^m M/p_T$

UNINTEGRATED PARTON DISTRIBUTIONS



$p = (p^+ , m^2 / 2p^+ , 0_\perp)$

$$\widetilde{f}(y) = \langle \, P \mid \overline{\psi}(y) \; V_y^\dagger(n) \; \gamma^+ \; V_0(n) \; \psi(0) \mid P \, \rangle \quad , \qquad y = (0, y^-, y_\perp)$$

$V_y(n) = \mathcal{P} \exp \left(i g_s \int_0^\infty d\tau \; n \cdot A(y + \tau \; n) \right)$ eikonal Wilson line in direction n

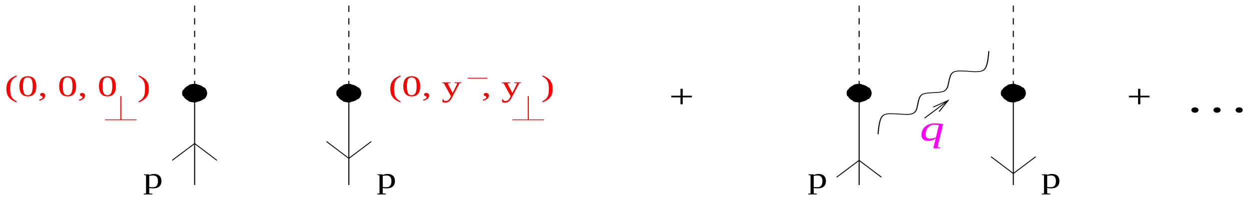
- works at tree level [Mulders, 2002; Belitsky et al., 2003]
- subtler at level of radiative corrections [Collins & Zu; H; Cherednikov et al.]

  $\hookrightarrow x \to 1 \Rightarrow$ explicit regularization method (unlike inclusive case)
- non-abelian Coulomb phase $\longrightarrow$ spectator effects possibly non-decoupl.

  [Mulders, Bomhof; Collins, Qiu; Brodsky et al]

II.A LIGHTCONE DIVERGENCES

◇ Suppose a gluon is absorbed or emitted by eikonal line: $n = (0, 1, 0_\perp)$



$$f_{(1)} = P_R(x, k_\perp) - \delta(1-x)\,\delta(k_\perp) \int dx' dk'_\perp P_R(x', k'_\perp)$$

where $$P_R = \frac{\alpha_s C_F}{\pi^2} \left[\frac{1}{1-x} \frac{1}{k_\perp^2 + \rho^2} + \{\text{regular at } x \to 1\} \right]$$ ρ=IR regulator

↑ *endpoint singularity* $(q^+ \longrightarrow 0, \ \forall\ k_\perp)$ [Brodsky et al, 2001; Collins, 2002]

◇ Physical observables:

$$\mathcal{O} = \int dx\, dk_\perp\, f_{(1)}(x, k_\perp)\, \varphi(x, k_\perp)$$
$$= \int dx\, dk_\perp\, [\varphi(x, k_\perp) - \varphi(1, 0_\perp)]\, P_R(x, k_\perp)$$

inclusive case: φ independent of $k_\perp \Rightarrow 1/(1-x)_+$ from real + virtual

general case: endpoint divergences (incomplete KLN cancellation)

CUT-OFF APPROACH

▷ cut-off in Monte-Carlo generators using u-pdf's

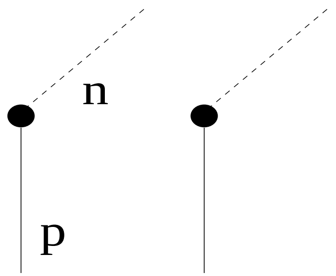
S. Jadach and M. Skrzypek, hep-ph/0905.1399 (DGLAP)

S. Höche, F. Krauss and T. Teubner, EPJC 58 (2008) 17 (KMR/BFKL)

LDCMC Lönnblad & Sjödahl, 2005; Gustafson, Lönnblad & Miu, 2002 (LDC)

CASCADE Jung, 2004, 2002; Jung and Salam, 2001 (CCFM)

▷ cut-off from gauge link in non-lightlike direction n:



$$\eta = (p \cdot n)^2 / n^2$$

Collins, Rogers & Stasto, PRD 77 (2008) 085009

Ji, Ma & Yuan, PRD 71 (2005) 034005; JHEP 0507 (2005) 020

earlier work from 80's and 90's: Collins et al; Korchemsky et al

finite $\eta \Rightarrow$ singularity is cut off at $1 - x \gtrsim \sqrt{k_\perp / 4\eta}$

∗ Note: Subtractive regularization is possible alternative to cut-off [Collins & H, 2001]

II.B UPDF's BY SUBTRACTIVE APPROACH

• Endpoint divergences $x \to 1$ from incomplete KLN cancellation

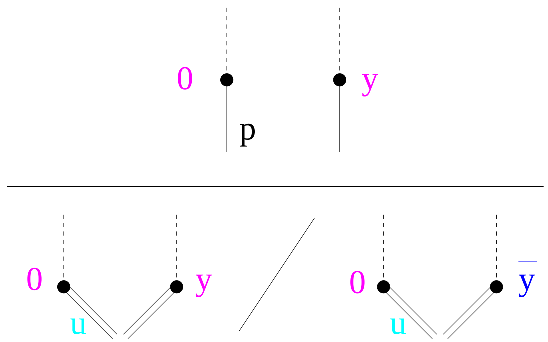
Subtractive method: more systematic than cut-off. Widely used in NLO calculations.

Formulation suitable for eikonal-operator matrix elements: Collins & H, 2001.

[See also "SCET" analog: Manohar and Stewart, 2007; J. Chiu et al, arXiv:0905.1141]

• gauge link still evaluated at n lightlike, but multiplied by "subtraction factors"

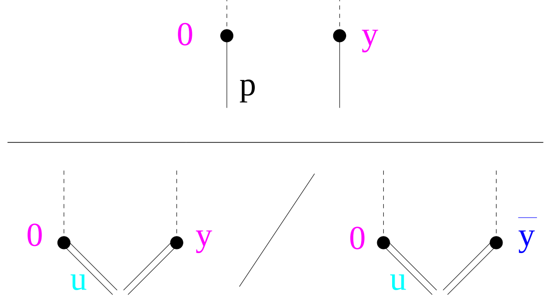
$$\widetilde{f}^{(\mathrm{subtr})}(y^-, y_\perp) = \frac{\overbrace{\langle P|\overline{\psi}(y) V_y^\dagger(n) \gamma^+ V_0(n) \psi(0)|P\rangle}^{\text{original matrix element}}}{\underbrace{\langle 0|V_y(u) V_y^\dagger(n) V_0(n) V_0^\dagger(u)|0\rangle \ / \ \langle 0|V_{\bar{y}}(u) V_{\bar{y}}{}^\dagger(n) V_0(n) V_0^\dagger(u)|0\rangle}_{\text{counterterms}}}$$



$\bar{y} = (0, y^-, 0_\perp)$; u = auxiliary non-lightlike eikonal $(u^+, u^-, 0_\perp)$

H, PLB 655 (2007) 26

♢ u serves to regularize the endpoint; drops out of distribution integrated over $k_\perp$



One loop expansion:

$$f_{(1)}^{(\mathrm{subtr})}(x,k_\perp) \;=\; P_R(x,k_\perp) - \delta(1-x)\,\delta(k_\perp)\int dx'\,dk'_\perp\,P_R(x',k'_\perp) \quad (\leftarrow\text{from numerator})$$

$$-\;\; W_R(x,k_\perp,\zeta) + \delta(k_\perp)\int dk'_\perp\,W_R(x,k'_\perp,\zeta) \quad (\leftarrow\text{from vev's})$$

with $P_R = \alpha_s C_F/\pi^2 \left\{1/[(1-x)\;(k_\perp^2 + m^2(1-x)^2)] + \ldots\right\}$ = real emission prob.

$W_R = \alpha_s C_F/\pi^2 \left\{1/[(1-x)\;(k_\perp^2 + 4\zeta(1-x)^2)] + \ldots\right\}$ = counterterm

- ζ-dependence cancels upon integration in $k_\perp$ $\quad [\zeta = (p^{+2}/2)u^-/u^+]$

$$\Rightarrow \mathcal{O} \;=\; \int dx\;dk_\perp\;f_{(1)}^{(\mathrm{subtr})}(x,k_\perp)\;\varphi(x,k_\perp)$$

$$=\; \int dx\;dk_\perp\;\{P_R\;[\varphi(x,0_\perp)-\varphi(1,0_\perp)] + (P_R - W_R)\;[\varphi(x,k_\perp)-\varphi(x,0_\perp)]\}$$

- first term: usual $1/(1-x)_+$ distribution
- second term: singularity in P_R cancelled by W_R

Note: counterterms at one loop give contributions to $f(x, k_\perp)$

$$-W_R(x, k_\perp, \zeta) + \delta(1-x)\,\delta(k_\perp) \int dx' dk'_\perp W_R$$

and

$$+\delta(k_\perp) \int dk'_\perp W_R(x, k'_\perp, \zeta) - \delta(1-x)\,\delta(k_\perp) \int dx' dk'_\perp W_R$$

ζ angle of eikonal u; W_R computed to order α_s

▷ virtual correction to gauge link does not depend on $y_\perp$

Korchemsky et al, 1992

▷ relation with cusp anomalous dimension in *Cherednikov et al* *arXiv:0904.2727; arXiv:0802.2821*

▷ one-loop counterterm gives extension for k$_\perp \neq 0$ of the plus-distribution regularization

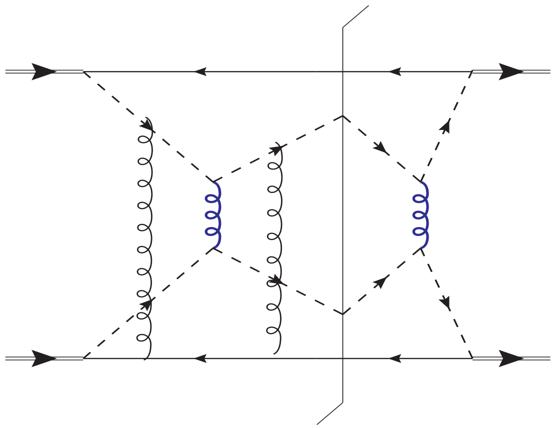
II.C COULOMB PHASE EFFECTS

- soft gluon exchange with spectator partons

Mert Aybat & Sterman, PLB671 (2009) 46

Boer, Brodsky & Hwang, PRD 67 (2003) 054003

⇒ factorization breaking in higher loops?



Collins, arXiv:0708.4410

Vogelsang and Yuan, arXiv:0708.4398

Bomhof and Mulders, arXiv:0709.1390

♢ likely suppressed for small-x, small-$\Delta\phi$

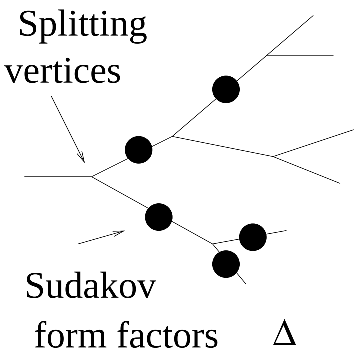
♢ could affect physical picture near large x, back-to-back region

- Note: Coulomb/radiative mixing terms also appear to break coherence in di-jet cross sections with gap in rapidity

Forshaw & Seymour, arXiv:0901.3037

Forshaw, Kyrieleis & Seymour, hep-ph/0604094

III. QCD EVOLUTION BY PARTON SHOWERING METHODS

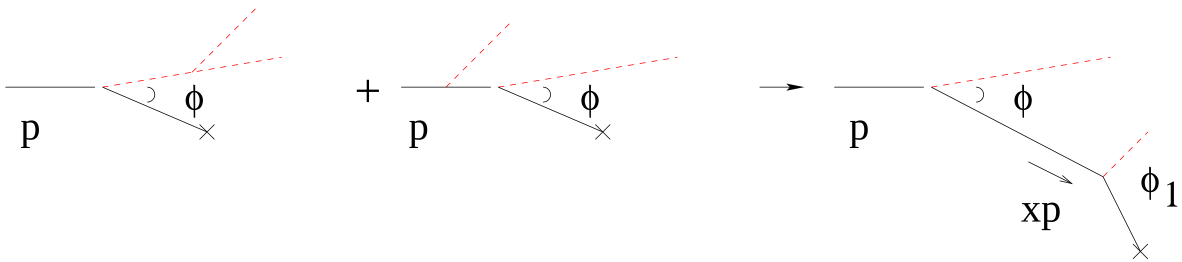


$$d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \; \alpha_S(q^2) \; P(z) \; \Delta(q^2, q_0^2)$$

$\hookrightarrow$ collinear, incoherent emission

◇ Soft emission $\longrightarrow$ interferences $\longrightarrow$ ordering in decay angles

$\hookrightarrow$ gluon coherence for $x \sim 1$



- ex.: HERWIG, new PYTHIA

◇ Gluon coherence for $x \ll 1 \Rightarrow$ corrections to angular ordering:

$\hookrightarrow$ MC based on $k_\perp$-dependent unintegrated pdfs and MEs

COHERENCE IN HIGH-ENERGY LIMIT

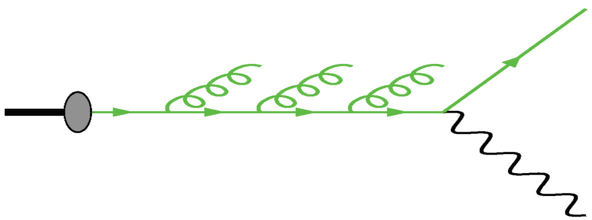
Arguments on soft vector emission current from external legs $\longrightarrow$

- leading IR singularities

[J.C. Taylor, 1980; Gribov-Low (QED)]

- fully appropriate in single-scale hard processes

Dokshitzer, Khoze, Mueller and Troian, RMP (1988); Webber, A. Rev. Nucl. Part. (1986)



multi-scale: $s = q_1^2 \gg \cdots \gg q_n^2 \gg \Lambda^2$

[e.g.: LHC final states with multi-jets]

$\Downarrow$

▷ internal emissions non-negligible

▷ current also factorizable at high-energy: *[Ciafaloni 1998; 1988]*

$$\left|M^{(n+1)}(k,p)\right|^2 = \left\{ [M^{(n)}(k+q,p)]^\dagger \, [\mathbf{J}^{(R)}]^2 \, M^{(n)}(k+q,p) - [M^{(n)}(k,p)]^\dagger \, [\mathbf{J}^{(V)}]^2 \, M^{(n)}(k,p) \right\} \; .$$

BUT... ▷

▷ ... • $\mathbf{J}$ depends on total transverse momentum transmitted

$\Rightarrow$ matrix elements and pdf at fixed $k_\perp$ ("unintegrated")

• virtual corrections not fully represented by Δ form factor

$\Rightarrow$ modified branching probability $P(z, k_\perp)$ as well

▷ enhanced terms $\mathcal{O}(\alpha_S^k \ln^m s/p_T^2)$

♢ Note: superleading logs $m > k$ cancel in fully inclusive quantities

e.g: high-energy corrections to anomalous dimensions γ^{ij}

at most single-logarithmic

$$\gamma^{ij}(\alpha_s, \omega) = \frac{\alpha_s}{\omega^p} \ c_0^{ij} \ \left[1 + \sum_{n=1}^{\infty} c_n^{ij} \ \left(\frac{\alpha_s}{\omega} \right)^n + \mathcal{O}\left(\alpha_s \ (\frac{\alpha_s}{\omega})^{n-1} \right) \right]$$

ω - moment conjugate to $\ln s$ *BFKL; Jaroszewicz; Catani et al.*

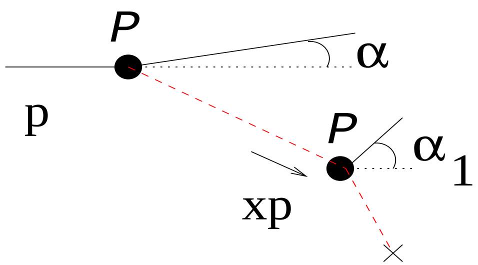
♢ but cancellations do not apply in exclusive final-state correlations

K$_\perp$-DEPENDENT PARTON BRANCHING

- MC for (almost-)NLO QCD evolution at unintegrated level
  proposed in Jadach & Skrzypek, arXiv:0905.1399 [hep-ph]

- $\{x\to 0\} \oplus \{x \to 1\}$ gluon branching eq. (leading-logarithms, all orders in α_s)
  CCFM evolution equation [Marchesini et al., 1990's]

$$\mathcal{G}(x, k_T, \mu) = \mathcal{G}_0(x, k_T, \mu) + \int \frac{dz}{z} \int \frac{dq^2}{q^2} \, \Theta(\mu - zq) \times \underbrace{\Delta(\mu, zq)}_{\text{Sudakov}} \; \underbrace{\mathcal{P}(z, q, k_T)}_{\text{unintegr. splitting}} \; \mathcal{G}\left(\frac{x}{z}, k_T + (1-z)q, q\right)$$



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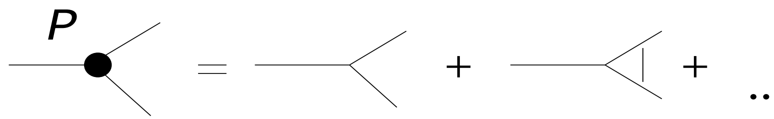


▷ Monte Carlo implementations CASCADE, LDCMC, ...

- unintegrated quark with k_T-dependent branching
  $\hookrightarrow$ ongoing work

Merging PS and ME

Both PS distributions and hard ME depend on $k_\perp$

- Merging in high-energy limit can be done using

$$\gamma \, \frac{1}{k_\perp^2} \left(\frac{k_\perp^2}{\mu^2} \right)^\gamma \overset{\gamma \ll 1}{=} \delta(k_\perp^2) + \gamma \left(\frac{1}{k_\perp^2} \right)_{\rm R} + \gamma^2 \left(\frac{1}{k_\perp^2} \ln \frac{k_\perp^2}{\mu^2} \right)_{\rm R} + \dots$$

$$\text{where} \int dk_\perp \, (G(k_\perp, \mu))_{\rm R} \, \varphi(k_\perp) = \int dk_\perp \, G(k_\perp, \mu)[\varphi(k_\perp) - \Theta(\mu - k_\perp) \, \varphi(0)]$$

Unintegrated quark evolution

[Jung & H, in progress]

- sea: flavor-singlet evolution coupled to gluons at small x via

$$\mathcal{P}_{g \to q}(z; q, k) = P_{qg,\rm GLAP}(z) \left(1 + \sum_{n=0}^{\infty} b_n(z) (k^2/q^2)^n \right)$$

all b_n known; $\mathcal{P}_{g \to q}$ computed in closed form (positive-definite)

[Catani & H, 1994; Ciafaloni et al., 2005-2006]

- valence: independent evolution (dominated by soft gluons x $\to$ 1)

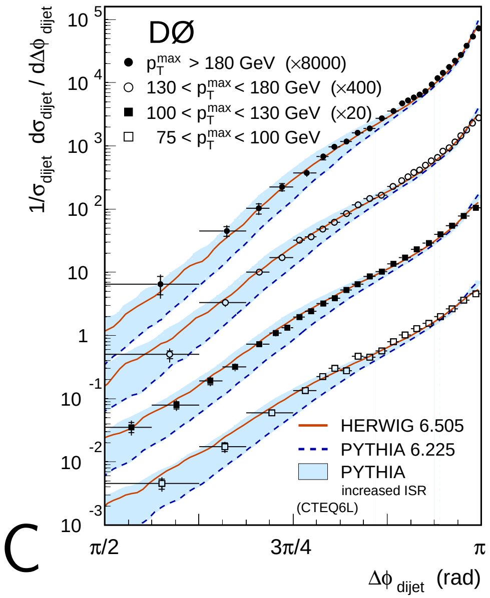
III.A ANGULAR CORRELATIONS IN $p\bar{p}$ AND ep MULTI-JETS

$\Delta\phi$ correlation between two hardest jets

▷ Tevatron $\Delta\phi$ dominated by leading-order processes

- good description by HERWIG as well as by NLO
  - used for MC parameter tuning in PYTHIA

[M.G. Albrow et al., TEV4LHC Proc., hep-ph/0610012]



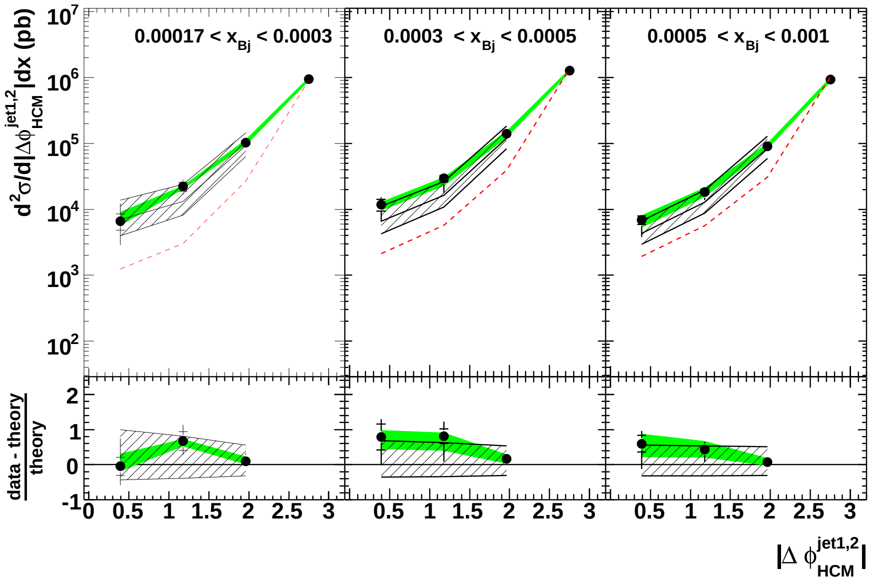
▷ HERA $\Delta\phi$ not well described by standard MC

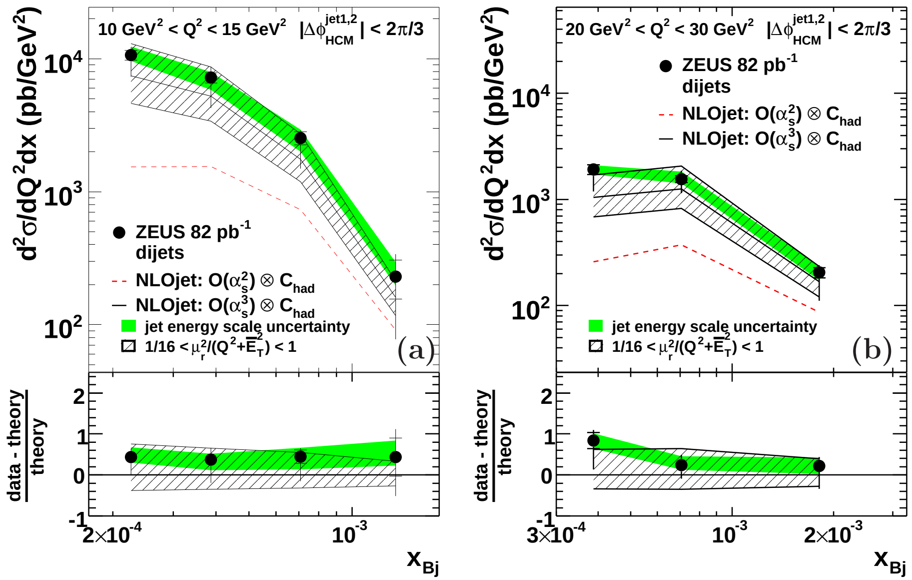
[S. Chekanov et al., arXiv:0705.1931] $\hookrightarrow$ see next

▷ <u>accessible at the LHC relatively early</u>

$\hookrightarrow$ how do MC describe multiple radiation?

DI-JET EP CORRELATIONS: COMPARISON WITH NLO RESULTS





(left) Azimuth dependence and (right) Bjorken-x dependence of di-jet distributions

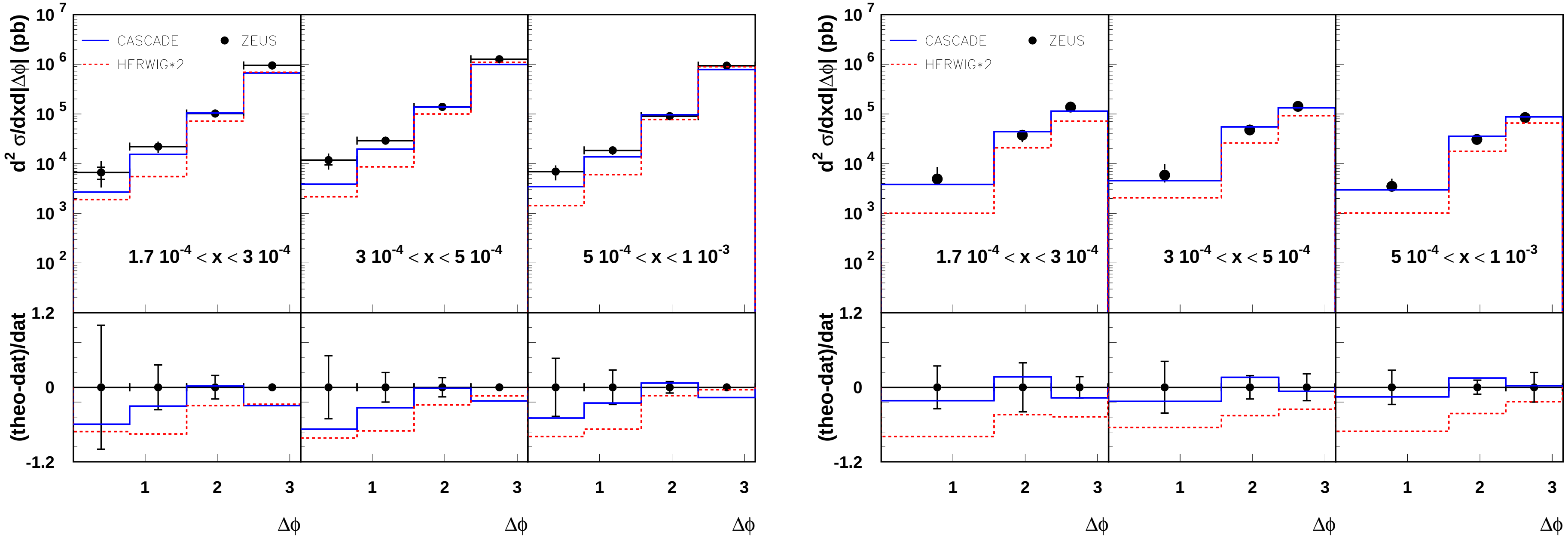
$$Q^2 > 10 \text{ GeV}^2 \quad , \qquad 10^{-4} < x < 10^{-2}$$

[S. Chekanov et al., arXiv:0705.1931]

◇ large variation from order-α_s^2 to order-α_s^3 prediction as $\Delta\phi$ and x decrease

⇒ sizeable theory uncertainty at NLO (underestimated by "μ error band")

ANGULAR JET CORRELATIONS FROM $K_\perp$-SHOWER (CASCADE) AND COLLINEAR-SHOWER (HERWIG)

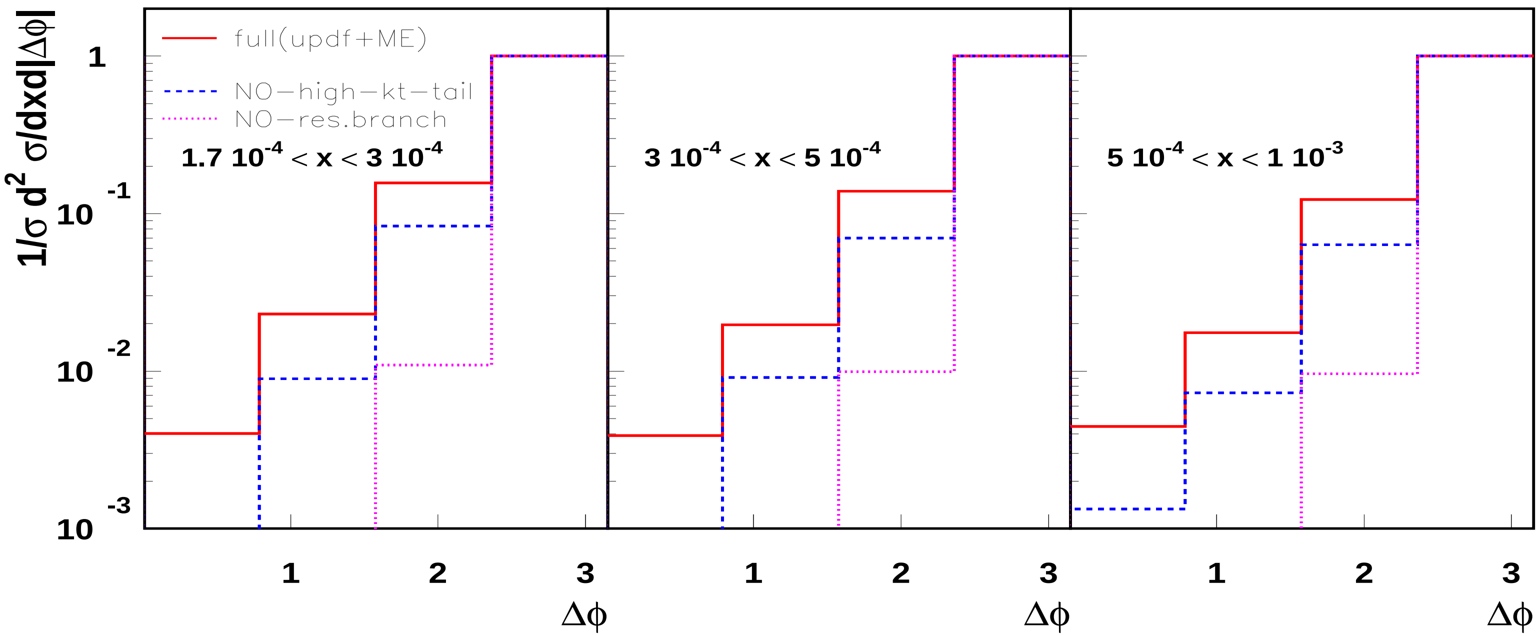


(left) di-jet cross section; (right) three-jet cross section

Jung & H, JHEP 0810 (2008) 113

- quantitative effects of small-x coherence sizeable
- largest differences at small $\Delta\phi$
- good description of shapes by $k_\perp$-shower
- HERWIG normalized to 2-jet region by K-factor

Normalize to the back-to-back cross section:



—— updf $\oplus$ ME

- - - updf $\oplus$ ME$_{collin.}$: $\mathcal{M} \to \mathcal{M}_{collin.}(k_T) = \mathcal{M}(0_\perp)\ \Theta(\mu - k_T)$

$\cdots\cdots$ no resolved branching : $\mathcal{A} \to \mathcal{A}_{no-res.}(x, k_T, \mu) = \mathcal{A}_0(x, k_T, Q_0)\ \Delta(\mu, Q_0)$

▷ high-k$_\perp$, coherent effect essential for correlation at small $\Delta\phi$

(cfr., e.g., MC by *Höche, Krauss & Teubner, EPJC 58 (2008) 17*:
u-pdf but no ME correction)

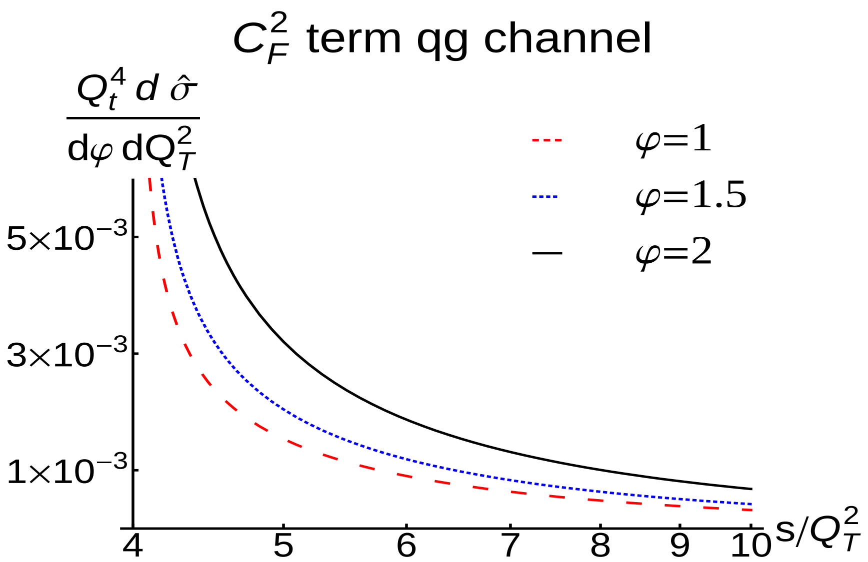
IV. FORWARD JET HADRO-PRODUCTION CROSS SECTIONS

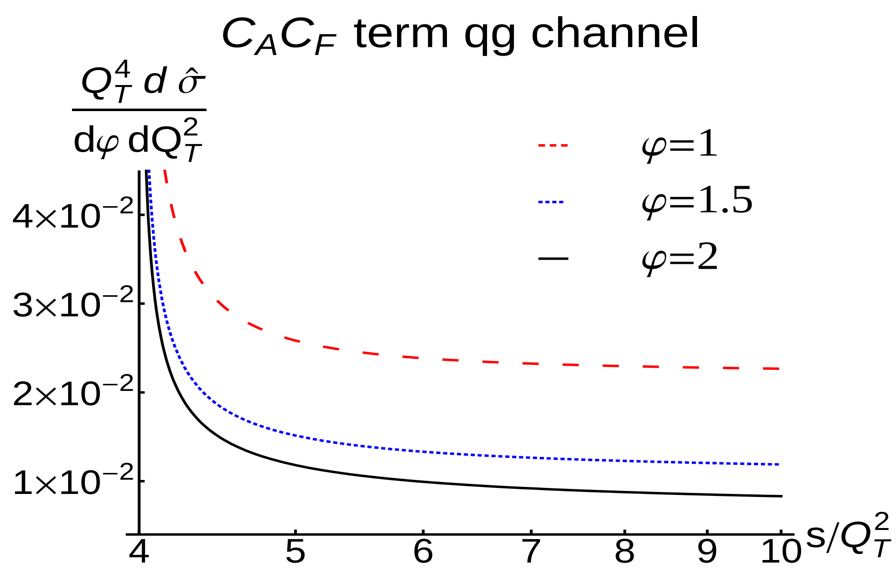
- Matrix elements for fully exclusive events with forward jets

[Deak, Jung, Kutak & H, 2009]

- Both quark and gluon channels found to be important for realistic phenomenology

Q_t = final-state transverse energy (in terms of two leading jets p_t's)



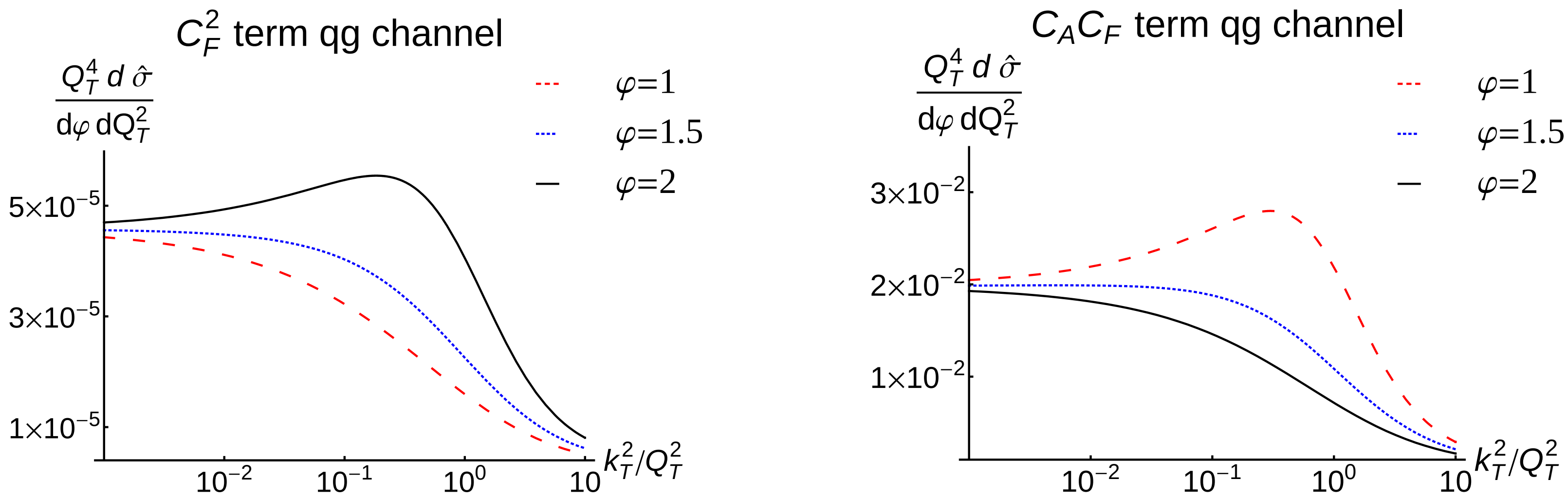


▷ $C_F C_A$ contribution to qg dominates large $\hat{s}/Q_t^2$ (constant at large energy)

BEHAVIOR AT LARGE $K_\perp$

k_t = transverse momentum carried away by extra jets

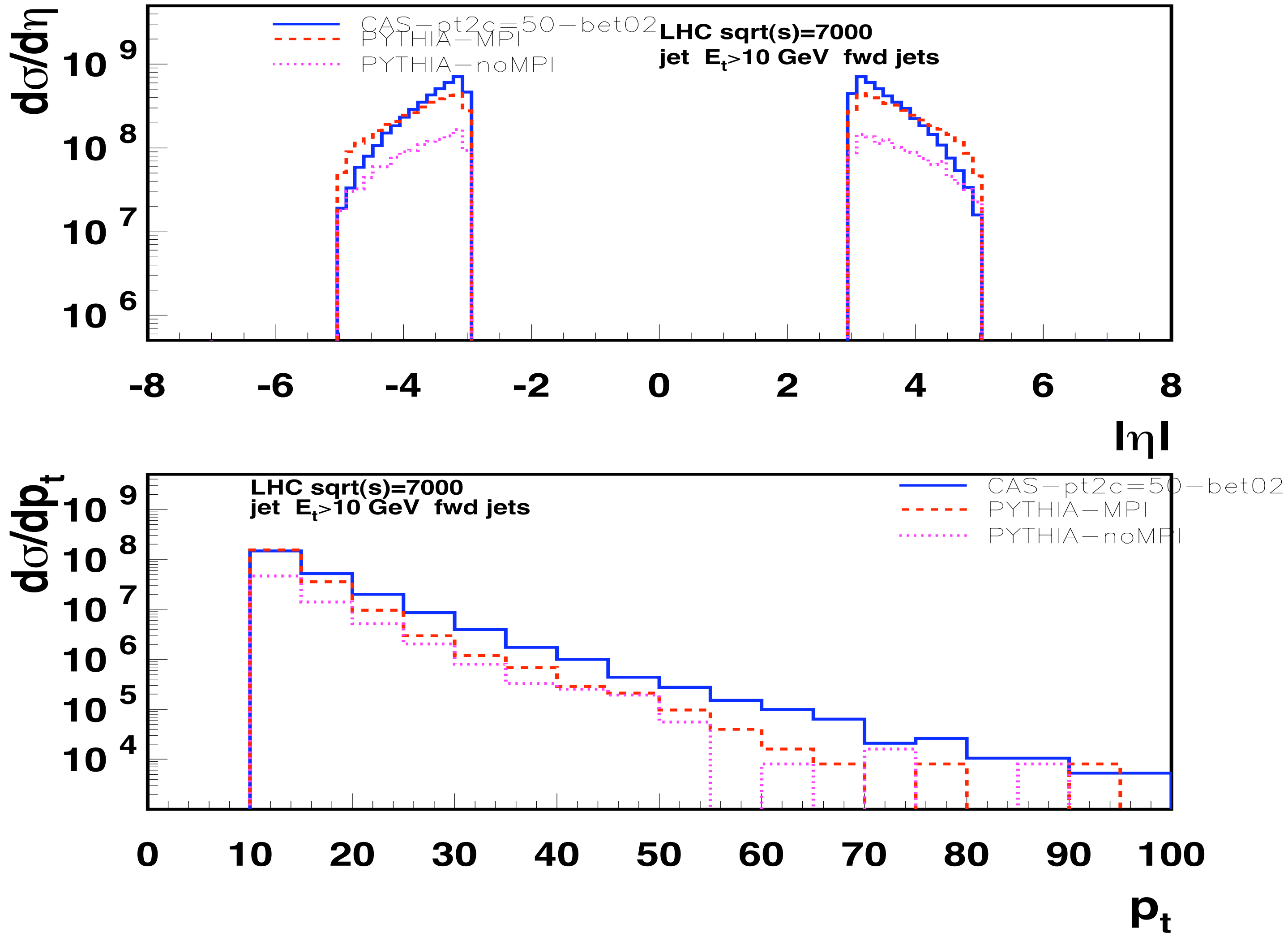
$k_t / Q_t \to 0$ leading order process



[Deak, Jung, Kutak & H, in progress]

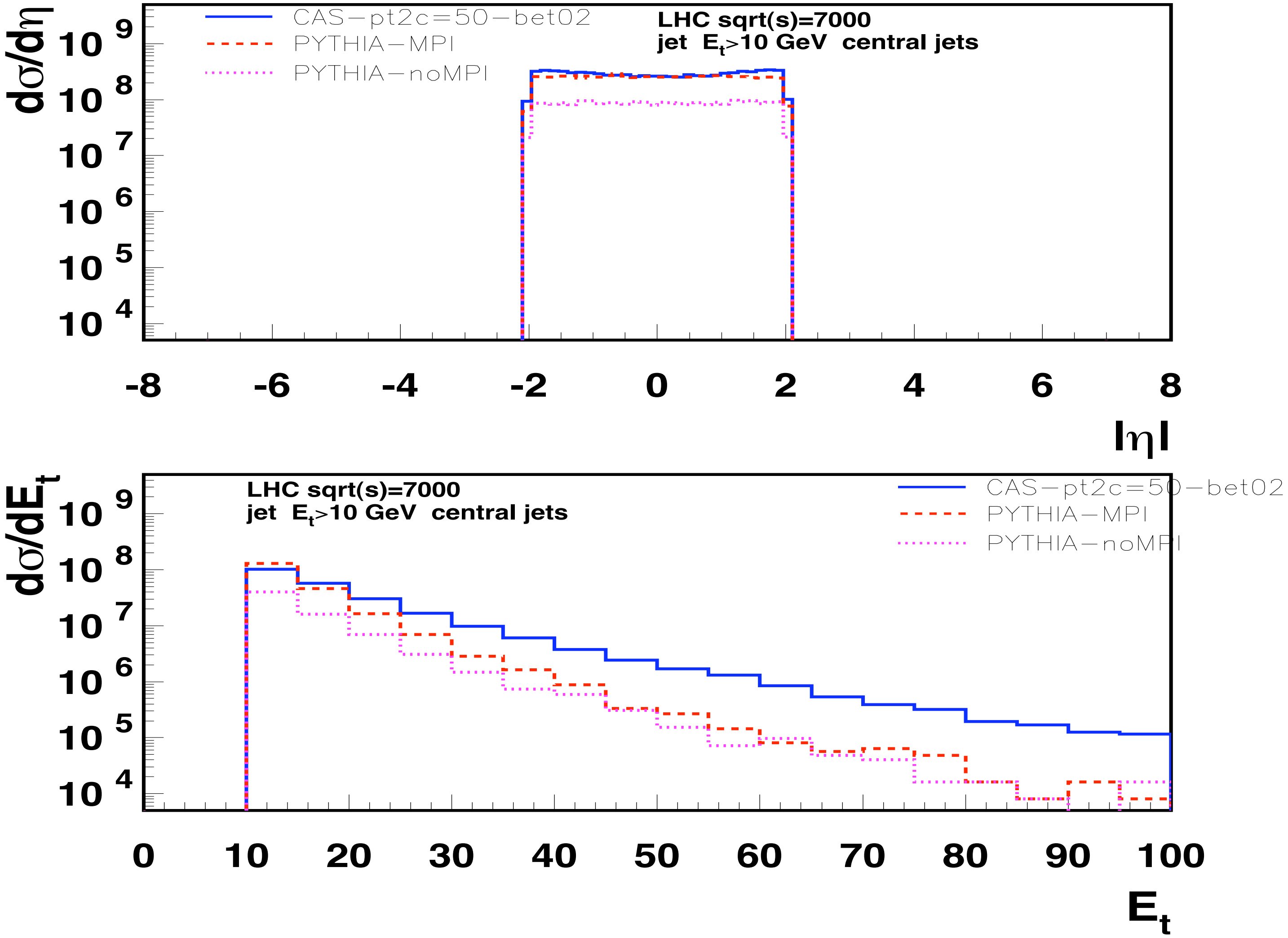
- measures transverse momentum distribution of third jet
- dynamical cut-off at $k_t \sim Q_t$ set by coherence effects
- non-negligible terms from finite k_t tail

1 central $\oplus$ 1 forward jet



$k_\perp$-shower vs. collinear shower (with & without multi-parton interactions)

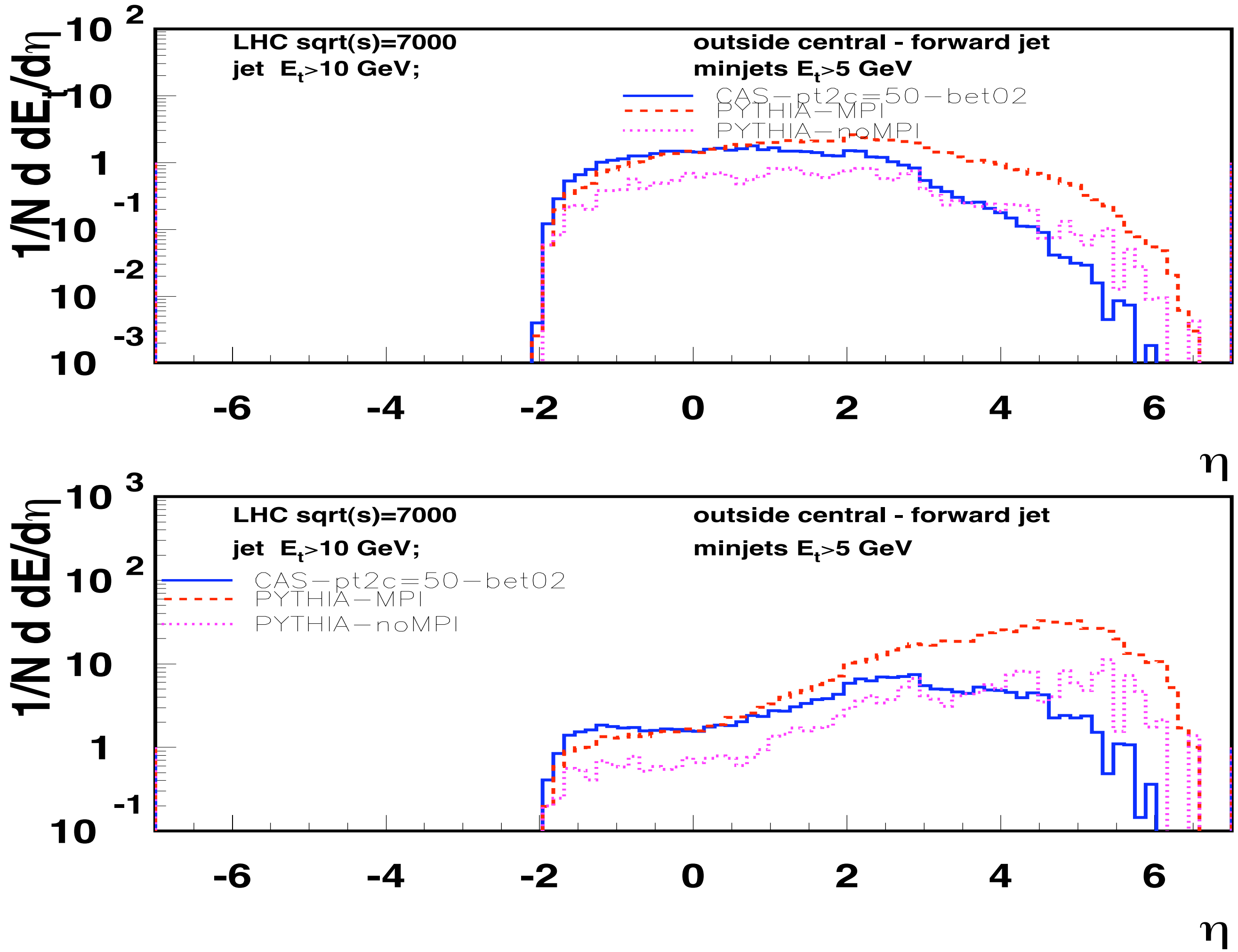
1 central ⊕ 1 forward jet



harder spectrum in central region due to small-x shower

Transverse energy flow: outside region

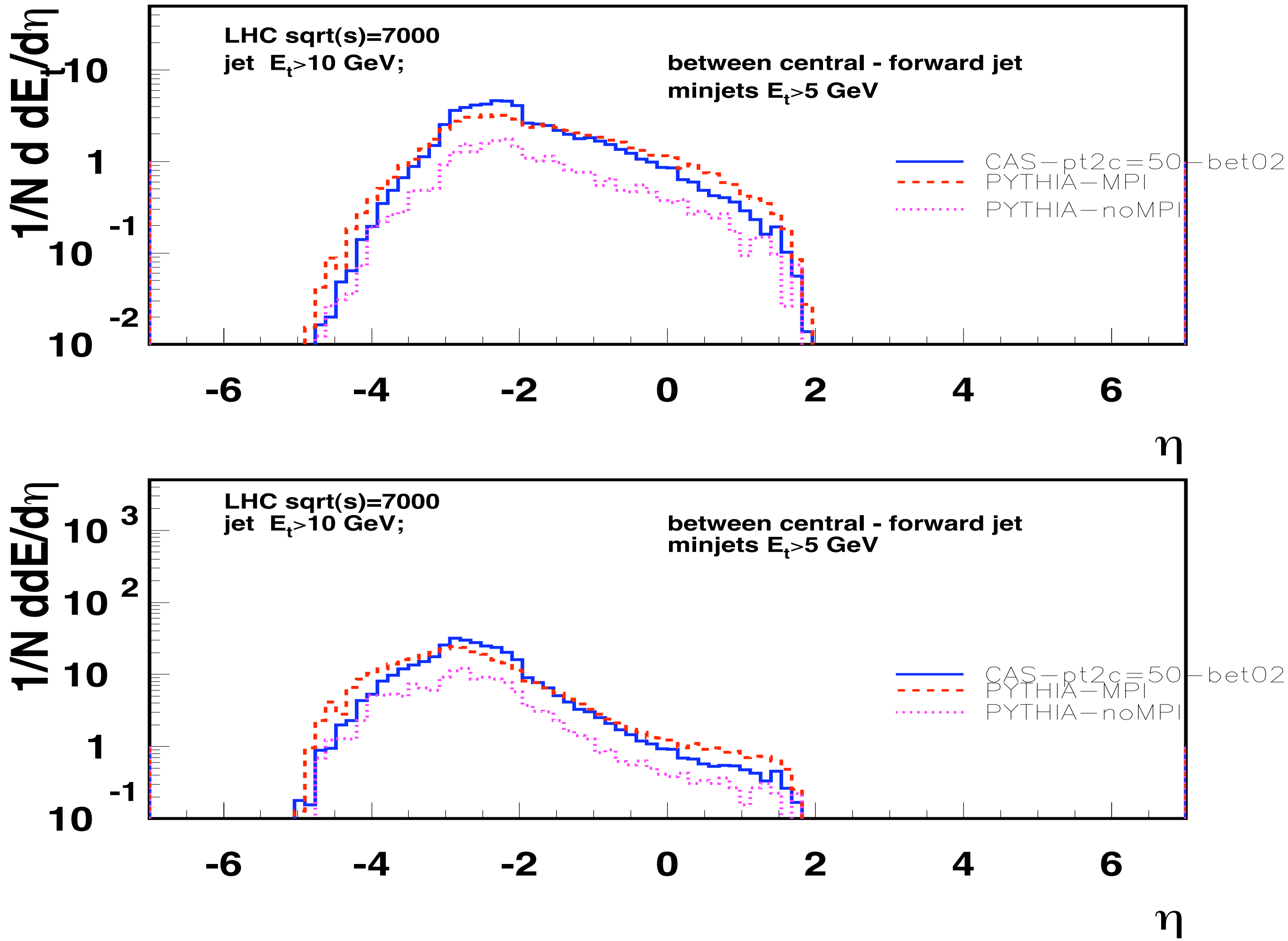
[Deak et al., in progress]



- at large (opposite) rapidities, full branching well approximated by collinear ordering
- higher energy flow only from multiple interactions

Transverse energy flow: between region

[Deak et al., in progress]



- higher mini-jet activity in the between region from corrections to collinear ordering

V. FURTHER APPLICATIONS TO MULTI-JET FINAL STATES

- production of b-flavor + jets — what size NLO uncertainties at LHC energies?

  [see MC@NLO; Nason et al.]

  ▷ sizeable corrections from $g \to b\bar{b}$ coupling to spacelike jet

  ▷ coherence effects to $b\bar{b} + 2\ jets$ for $m_b \ll p_T^{(b\bar{b})} \ll p_T^{(jet)}$

- even more complicated multi-scale effects in $b\bar{b} + W/Z$ production

  [HERA-LHC Proc. arXiv:0903.3861; Mangano, 1993]

- Tevatron b-jets angular correlations

  ($\hookrightarrow$ CDF $\Delta\phi$ data)

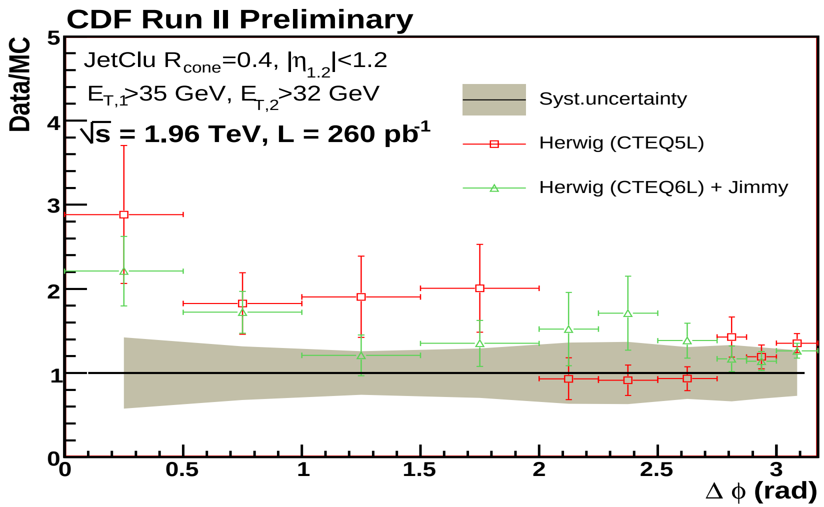
- final states with Higgs

  $\to$ possibly 10 $\div$ 20 % effects in p_T spectrum from $x \ll 1$ terms?

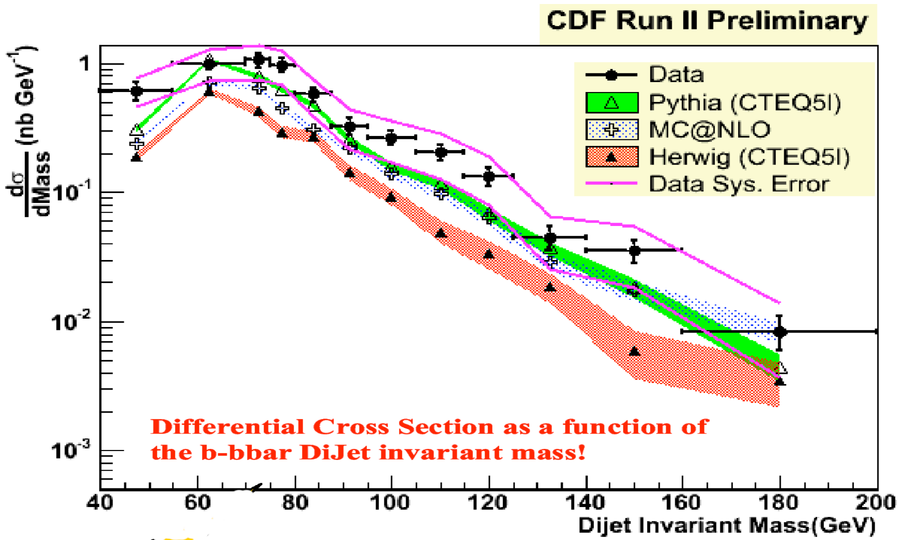
  [Kulesza, Sterman & Vogelsang, 2004]

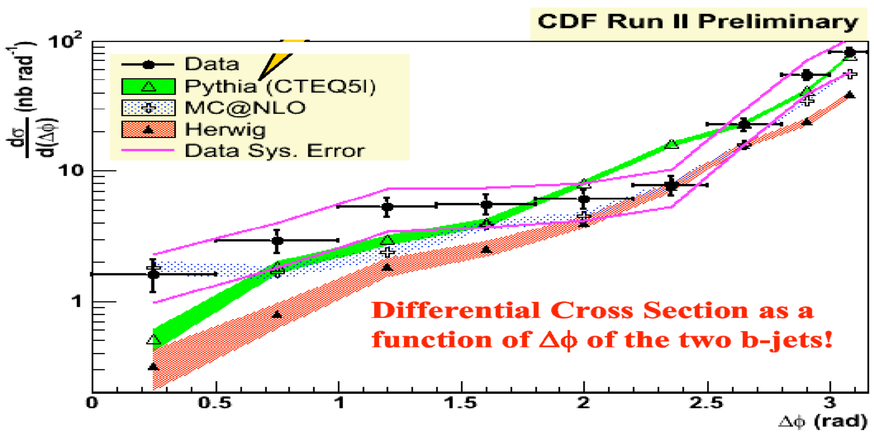
  [Marzani, Ball, Del Duca et al., 2008; H, 2002]

Tevatron b-jets correlations



[CDF Coll., FNAL-8939 (2007)]



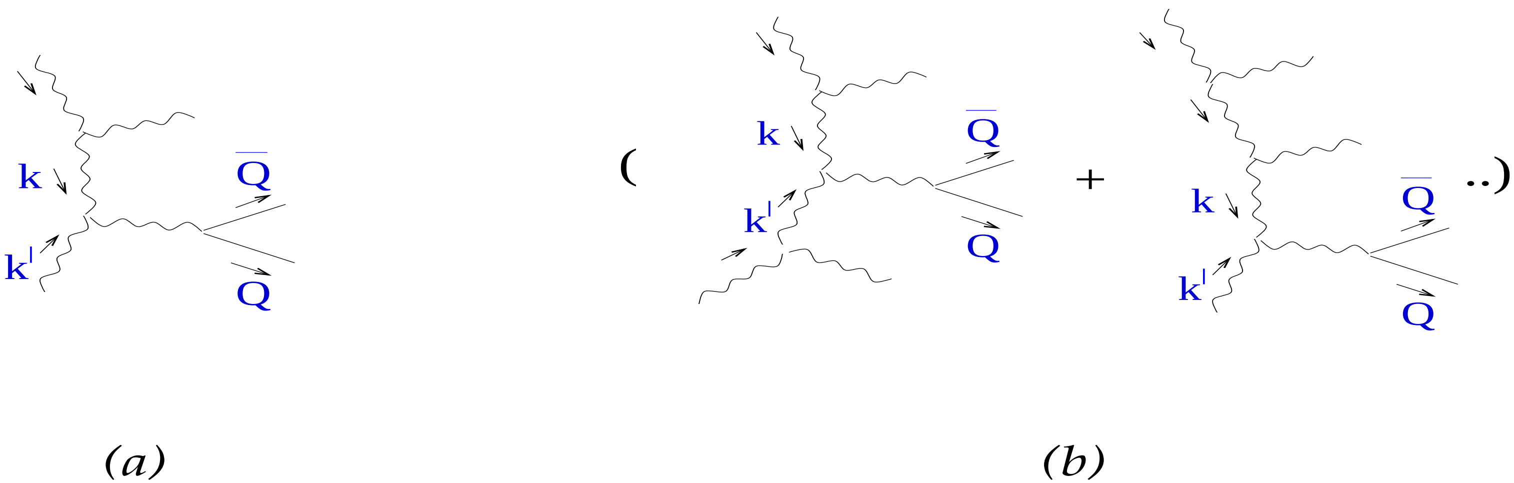


- Herwig description not satisfactory
- $k_\perp$ distribution of underlying event?

Heavy flavor production: high-energy behavior

$$\sigma_{gg,N} \simeq C \left(\frac{m_Q^2}{K_T^2} \right)^{N+1} \ln(1 + K_T^2/(4m_Q^2))$$

⇒ strong triple-pole singularity in moments conjugate to k_T
from $m_Q^2 \ll (k_T + k'_T)^2 \ll k_T^2 \simeq k'^2_T$



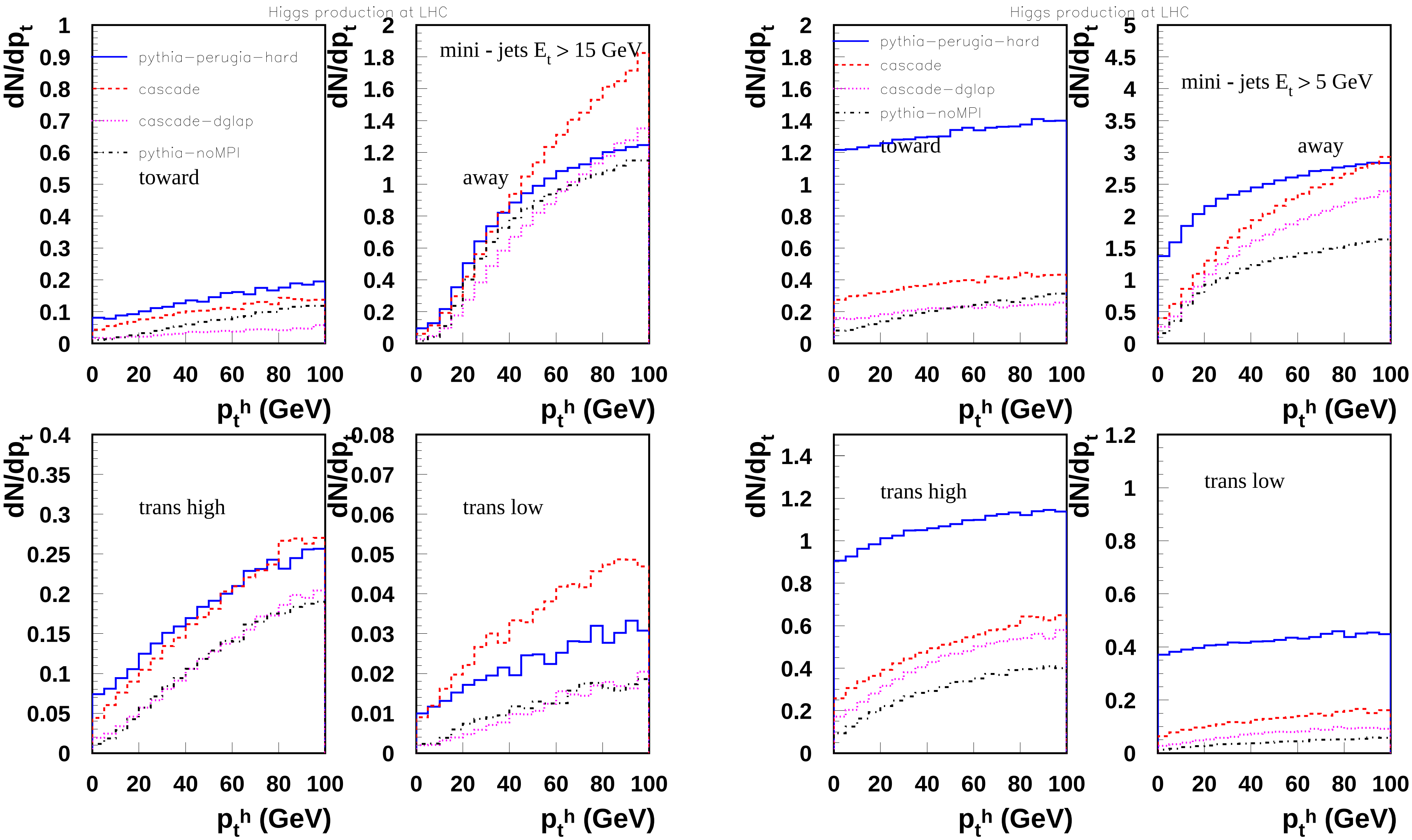
(*a*) heavy quark hadroproduction from gluon showering;

(*b*) next correction from extra jet emission

• not included by collinear showers (even at NLO [MC@NLO])

• obtainable by $k_\perp$-shower (compare "multi-parton interactions"?)

(Mini)-jet multiplicity vs. Higgs p_T in different regions of ϕ



CONCLUSIONS

- Forward + central detectors at the LHC allow correlations of high-p_T probes to be measured across large rapidity intervals
  - ▷ new particle discovery processes
  - ▷ new aspects of standard model physics

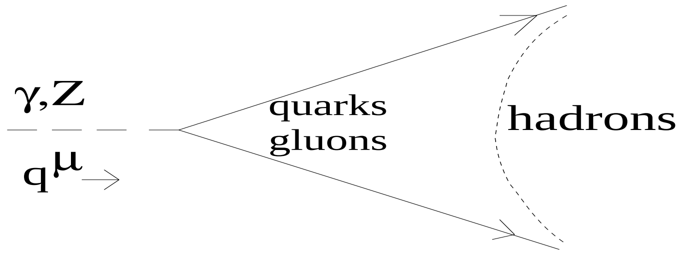
- Branching methods based on u-pdfs and $k_\perp$-MEs useful to
  - ▷ simulate high-energy parton showers
  - ▷ investigate possibly new effects from QCD physics

- Systematic theoretical studies of u-pdf's ongoing
  - ▷ relevant to turn these Monte-Carlo's into general-purpose tools

EXTRA SLIDES

a) e^+e^- annihilation

$$\sqrt{Q^2} = \sqrt{q^\mu q_\mu} \gg \text{“hadronic scale”} \approx 1 \text{ fermi}^{-1}$$



Heuristic separation of short-time and long-time dynamics:

- $(\Delta t)_{hadroniz.} \gg (\delta t)_{partonic} \sim 1/\sqrt{Q^2} \;\Rightarrow$

$$P(e^+e^- \to h) = P(e^+e^- \to q\bar{q}) P(q\bar{q} \to h)$$

- Completeness $\sum_h P(i \to h) = 1 \;\Rightarrow$

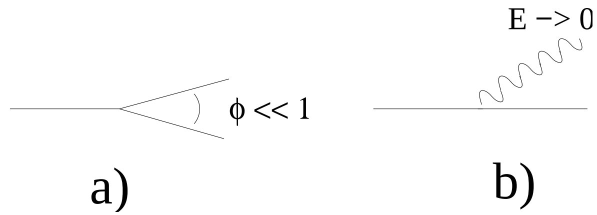
$$\sigma_{\rm tot}(e^+e^- \to h) \equiv \sum_h P(e^+e^- \to h)$$

$$= P(e^+e^- \to q\bar{q}) \sum_h P(q\bar{q} \to h) = P(e^+e^- \to q\bar{q})$$

▷ almost right — but not quite: rhs is IR-divergent in PT... $\hookrightarrow$

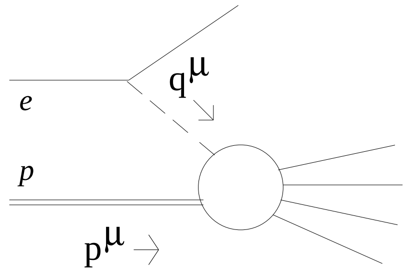
$\hookrightarrow$ particle number nonconserved $\Rightarrow$ add in multi-particle states ($q\bar{q}g$ to 1st order)

- $\sigma(e^+e^-\to q\bar{q}) + \sigma(e^+e^-\to q\bar{q}g)$ insensitive to long-time interactions, i.e.



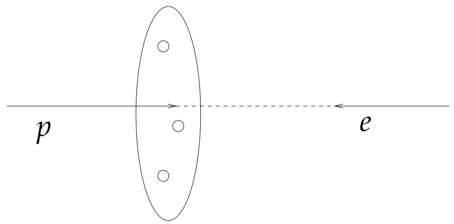
to collinear and soft parton emission

b) $\underline{DIS}$



- necessarily sensitive to long timescales, BUT

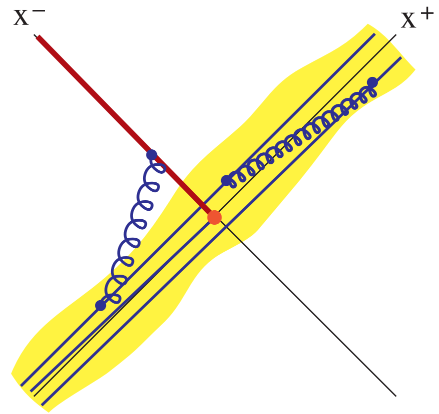
σ can be written as $\sigma(Q,m) = C(Q, \text{parton momenta } > \mu) \otimes f(\text{parton momenta } < \mu, m)$



scattering in infinite-momentum frame

p Lorentz-contracted, parton interactions time-dilated: $\delta t_{\text{scatter}} \ll \tau_{\text{parton}}$

- f gives recoil color flow:



$$f(x,\mu) = \int \frac{dy^-}{2\pi}\, e^{-ixp^+y^-}\ \widetilde{f}(y)$$

$$\widetilde{f}(y) = \langle\ P \mid \overline{\psi}(y)\ V_y^\dagger(n)\ \gamma^+\ V_0(n)\ \psi(0) \mid P\ \rangle \quad , \qquad y = (0, y^-, 0)$$

$V_y(n) = \mathcal{P} \exp\left(ig_s \int_0^\infty d\tau\ n \cdot A(y + \tau\ n)\right)$ eikonal line in direction $n = (0, 1, 0_\perp)$

♢ fast moving color charge along light-like line

♢ renormalization of operator product ⇒ evolution equations

$$\frac{d}{d\ln\mu}\,\sigma = 0 \quad \Rightarrow \quad \frac{d}{d\ln\mu}\ \ln f = \gamma = -\frac{d}{d\ln\mu}\ \ln C \quad [\text{DGLAP}]$$

$\hookrightarrow$ resums $(\alpha_s \ln Q/\Lambda_{\text{QCD}})^n$

- More general processes require parton field correlations at distances off the lightcone [Example: Sudakov form factor $\hookrightarrow$]

◇ Explicit one-loop expression for soft term:

$$S_1 = \frac{-i\,g^2}{(2\,\pi)^4}\int dk^+\,dk^-\,d^2k_\perp\,\frac{1}{(k^2-m_g^2+i\,\varepsilon)}\,[\overbrace{\frac{1}{(k^- - i\,\varepsilon)\,(k^+ + i\,\varepsilon)}}^{\text{soft approximation}}$$

$$-\underbrace{\frac{1}{(k^- - i\,\varepsilon)}\,\frac{u_B^-}{(u_B^-\,k^+ + u_B^+\,k^- + i\,\varepsilon)}}_{\substack{\text{collinear}-\text{to}-p_A\ \text{counterterm}\ (k^-\to 0)\\ \text{note: } |u_B^+/u_B^-|\ \text{cuts off small } k^+}} - \underbrace{\frac{u_A^+}{(u_A^+\,k^- + u_A^-\,k^+ - i\,\varepsilon)}\,\frac{1}{(k^+ + i\,\varepsilon)}}_{\substack{\text{collinear}-\text{to}-p_B\ \text{counterterm}\ (k^+\to 0)\\ \text{note: } |u_A^-/u_A^+|\ \text{cuts off small } k^-}}]$$

Here $u_A = (u_A^+, u_A^-, 0_\perp)$, $u_B = (u_B^+, u_B^-, 0_\perp)$ are directions of non-lightlike eikonals

◇ S_1 is one-loop expansion of eikonal-operator vev's product:

$$S = \frac{\overbrace{\langle 0|V_q(\hat{p}_A)\,V_{\bar{q}}(\hat{p}_B)|0\rangle}^{\text{unsubtracted soft}}}{\underbrace{\langle 0|V_q(\hat{p}_A)\,V_{\bar{q}}(u_B)|0\rangle\,\langle 0|V_q(u_A)\,V_{\bar{q}}(\hat{p}_B)|0\rangle}_{\text{collinear subtractions}}}\ \overbrace{\langle 0|V_q(u_A)|0\rangle\,\langle 0|V_{\bar{q}}(u_B)|0\rangle}^{\text{residual external lines}}$$

with $V_q(n) = \mathcal{P}\exp\left(ig\int_{-\infty}^{0} dz\,A(z\,n)\cdot n\right)$, $V_{\bar{q}}(n) = \mathcal{P}\exp\left(-ig\int_{-\infty}^{0} dz\,A(z\,n)\cdot n\right)$

One-loop result for hard-region term:

Collins + H

hep-ph/0009286

$$M_\Gamma(H) = \frac{-g^2}{8\,\pi^2} \int \frac{d^2 k_\perp}{k_\perp^2} \left\{ \ln\left(\frac{k_\perp^2}{Q^2}\right) + i\pi \, + \, \frac{1 - k_\perp^2/Q^2}{r} \left[\ln\left(\frac{1+r}{1-r}\right) - i\pi \right] \right\}$$

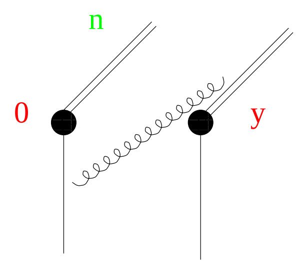
$$\text{where } Q^2 = 2\, p_A^+ \, p_B^- \, , \quad r = \begin{cases} \sqrt{1 - 4\, k_\perp^2/Q^2} & \text{if } 4k_\perp^2/Q^2 \le 1 \quad , \\ i\,\sqrt{4\, k_\perp^2/Q^2 - 1} & \text{if } 4k_\perp^2/Q^2 > 1 \quad . \end{cases}$$

- $M(H)$ purely ultraviolet (regardless of whether or not observable is IR-safe)
- obtained by defining IR counterterms through gauge-invariant operators

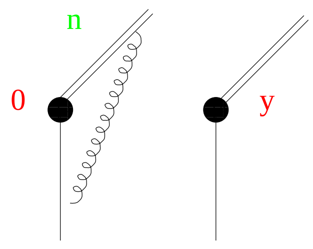
Order-α_s analysis

[H, hep-ph/0702196]

- ▷ Expand Wilson-line matrix element to one loop
- ▷ Gauge link at infinity does not contribute in covariant gauge
- ▷ $d = 4 - 2\varepsilon$ for UV divergences



(a)



(b)

$$\widetilde{f}_{(a)+(b)}(y) = \frac{\alpha_s C_F}{4^{d/2-2}\pi^{d/2-1}} \, p^+ \int_0^1 dv \, \frac{v}{1-v} \left[e^{ip\cdot yv} \, 2^{d/2-1} \left(\frac{\rho^2}{\mu^2}\right)^{d/4-1} \right.$$

$$\left. \times \quad \frac{1}{(-y^2\mu^2)^{d/4-1}} K_{d/2-2}(\sqrt{-\rho^2 y^2}) - e^{ip\cdot y} \, \Gamma(2-\frac{d}{2}) \, (\frac{\mu^2}{\rho^2})^{2-d/2} \right]$$

K = modified Bessel function; Γ = Euler gamma function

$$\rho^2 = (1-v)^2 m^2 + v\lambda^2$$

- $v \longrightarrow 1$: endpoint singularity
- can relate result to ordinary pdf by expanding in y^2 $\hookrightarrow$

$\hookrightarrow$ Separate long-distance terms in $\ln(\mu^2/\rho^2)$
and short-distance terms in $\ln(y^2\mu^2)$

[nonlocal operator technique
of Balitsky & Braun, 1991]

$$
\begin{aligned}
\widetilde{f}_{(a)+(b)} \simeq\ & \frac{\alpha_s C_F}{4^{d/2-2}\pi^{d/2-1}}\, p^+ \int_0^1 dv\, \frac{v}{1-v} \left\{ \left[e^{ip\cdot yv} - e^{ip\cdot y}\right]\, \Gamma(2-\frac{d}{2})\, (\frac{\mu^2}{\rho^2})^{2-d/2} \right. \\
& + e^{ip\cdot yv}\, 4^{d/2-2}\, \Gamma(\frac{d}{2}-2)\, (-y^2\mu^2)^{2-d/2} \\
& + \sum_{k=1}^{\infty} \frac{\Gamma(2-d/2)\,\Gamma(d/2-1)}{k!\, 4^k\, \Gamma(k+d/2-1)}\, e^{ip\cdot yv}\, (\frac{\rho^2}{\mu^2})^{d/2+k-2} (-y^2\mu^2)^k \\
& \left. + \sum_{k=1}^{\infty} \frac{4^{d/2-2-k}\, \Gamma(d/2-2)\,\Gamma(3-d/2)}{k!\,\Gamma(k+3-d/2)}\, e^{ip\cdot yv}\, (\frac{\rho^2}{\mu^2})^{k} (-y^2\mu^2)^{2-d/2+k} \right\}
\end{aligned}
$$

- First line in rhs: $\longrightarrow$ ordinary pdf ($v=1$ singularity cancels)
- Next terms: $y_\perp \neq 0$ (sing. present even at $d \neq 4$ and finite ρ)

• Distributions at fixed $k_\perp$ are no longer protected by KLN mechanism against uncancelled lightcone divergences

• Only after supplying matrix element with a regularization prescription is distribution well defined.

• Note: regularization of endpoint divergences may also affect distributions integrated over $k_\perp$ and UV subtractions

Ex. : $$\int dk_\perp \ f(x, k_\perp, \mu) \ \Theta(\mu - k_\perp) \ \stackrel{?}{=} \ f^{\overline{\rm MS}}(x, \mu)$$

$=$ holds only at tree level: full relation involves coefficient function R

$$\int^\mu dk_\perp \ f(x, k_\perp, \mu) \ = \ R(x) \ \otimes \ f^{\overline{\rm MS}}(x, \mu)$$

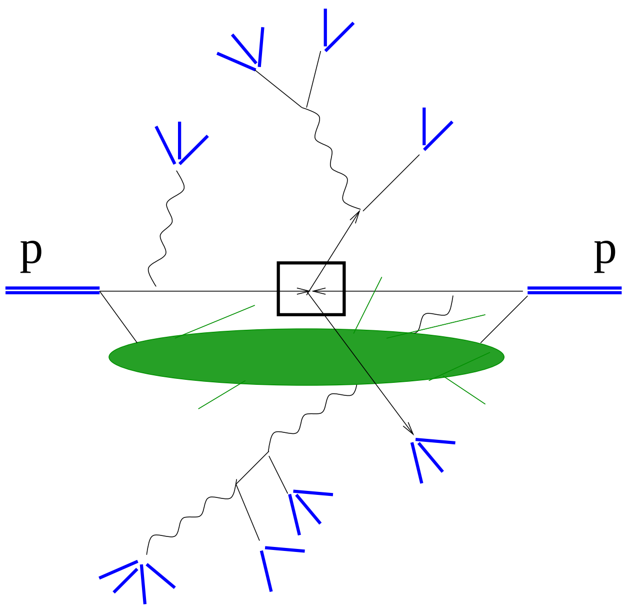
◇ R calculable as a power series in α_s, $R(x) = \delta(1-x) + \sum_k \ r_k \ \alpha_s^k$:

— $(\phi^3)_6$ *[Collins & Zu, 2005]*

— $f_g(x \to 0)$ *[Catani et al, 1994]*

• Applications: Cut-off regularization vs. Subtractive regularization

Structure of LHC events:



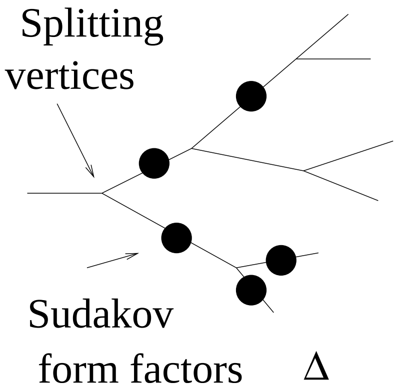
Hard process

Parton shower

Hadronization

Underlying event

Parton shower: generate branching according to



$$d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \; \alpha_S(q^2) \; P(z) \; \Delta(q^2, q_0^2)$$

- based on dominance of collinear evolution of jets

- Factorization of QCD cross sections in collinear limit
  $\longrightarrow$ probabilistic (Markov) picture

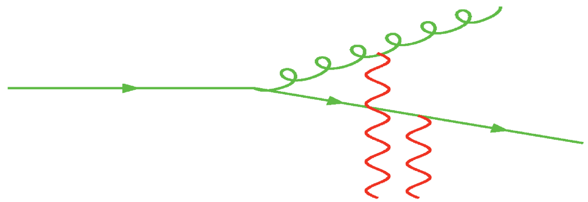
- summation of logarithmically enhanced radiative contributions
  $(\alpha_S \; \ln p_T/\Lambda)^n$

- soft gluon radiation by coherent branching

$\hookrightarrow$

III.A SOFT-GLUON COHERENCE

▷ soft gluons radiated over long times $\longrightarrow$ quantum interferences



- Factorization in soft limit: *[J.C. Taylor, 1980; Gribov-Low (QED)]*

$$|M_{n+1}^{a_1 \ldots a_n a}(p_1, p_n, q)\rangle = \mathbf{J}^a |M_n^{a_1 \ldots a_n}(p_1, p_n)\rangle \ , \quad \mathbf{J}^{a\mu} = \sum_i \mathbf{Q}_i^a \frac{p_i^\mu}{p_i \cdot q} \ , \ \mathbf{Q} = \text{charge}$$

interference terms $\downarrow$

$$d\sigma_{n+1} = d\sigma_n \frac{d^3 q}{(q^0)^3} \sum_{i,j} \mathbf{Q}_i \cdot \mathbf{Q}_j \ w_{ij} \quad , \qquad w_{ij} = \frac{(q^0)^2 \ p_i \cdot p_j}{(p_i \cdot q)(p_j \cdot q)}$$

— not positive definite, non-Markov..?

$\longrightarrow$ spoils probabilistic picture? NO, owing to soft-gluon coherence $\hookrightarrow$

Dokshitzer, Khoze, Mueller and Troian, RMP (1988)

Webber, Ann. Rev. Nucl. Part. Sci. (1986)

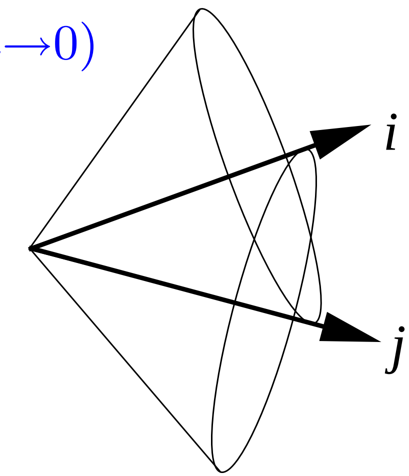
SINGLE EMISSION

- separate singularities along emitters' directions

$$\frac{(q^0)^2 p_i \cdot p_j}{(p_i \cdot q)(p_j \cdot q)} \equiv \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}}$$

$$= \frac{1}{2}\left(\frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} - \frac{1}{\zeta_{jq}} + \frac{1}{\zeta_{iq}}\right) + \frac{1}{2}\left(\frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} - \frac{1}{\zeta_{iq}} + \frac{1}{\zeta_{jq}}\right)$$

$$\text{where } \zeta_{nk} \equiv \frac{p_n \cdot p_k}{p_n^0 p_k^0} \simeq 1 - \cos\theta_{nk} \quad (m \to 0)$$



$\longrightarrow$ by azimuthal average

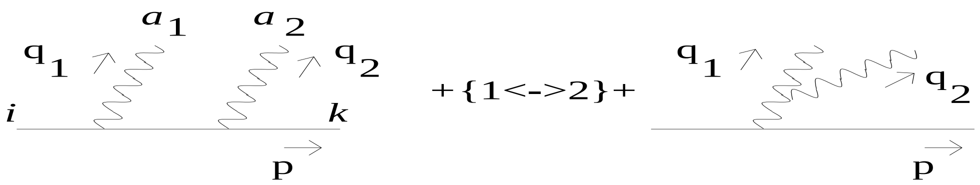
$$\langle \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} \rangle = \frac{1}{\zeta_{iq}}\Theta(\zeta_{ij} - \zeta_{iq}) + \frac{1}{\zeta_{jq}}\Theta(\zeta_{ij} - \zeta_{jq})$$

$\diamondsuit$ large-angle emissions of soft gluons sum coherently outside angular-ordered cones

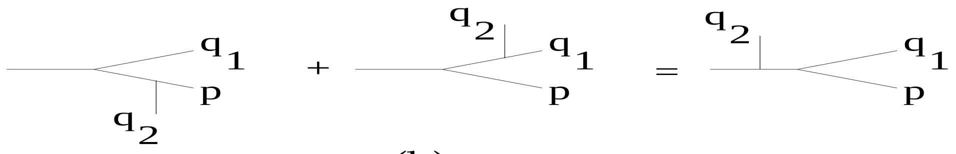
MULTIPLE EMISSION

- 2 soft gluon emission: q_1, q_2 with $q_2^0 \ll q_1^0$

$$\mathbf{J}_1^{\mu a_1} = \mathbf{Q}_p^{a_1} \frac{p^\mu}{p \cdot q_1} \quad , \quad \mathbf{J}_2^{\mu a_2} = \mathbf{Q}_p^{a_2} \frac{p^\mu}{p \cdot q_2} + \mathbf{Q}_{q_1}^{a_2} \frac{q_1^\mu}{q_1 \cdot q_2}$$



(a)



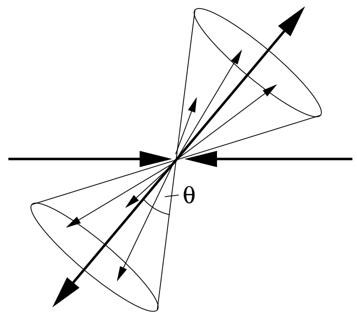
(b)

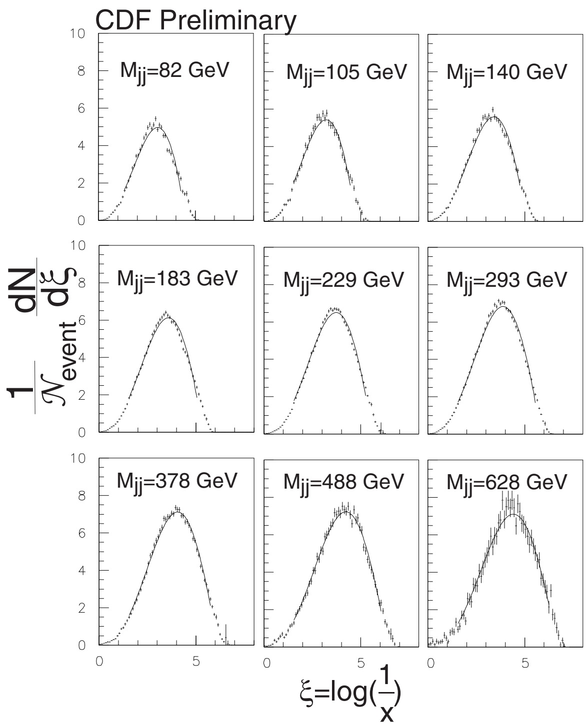
$$\mathcal{M}_{ki}^{a_1 a_2} = g_s^2 \, \langle a_1 \; k | \, \mathbf{J}_2 \cdot \varepsilon_2 \, | a' \; i' \rangle \, \langle \; i' \; | \mathbf{J}_1 \cdot \varepsilon_1 | \; i \; \rangle$$

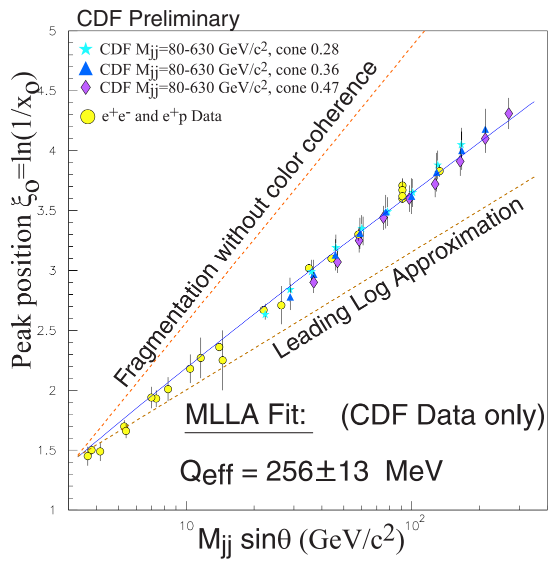
$$= g_s^2 \, \frac{p \cdot \varepsilon_1}{p \cdot q_1} \left(\frac{p \cdot \varepsilon_2}{p \cdot q_2} \, t^{a_2} t^{a_1} + \frac{q_1 \cdot \varepsilon_2}{q_1 \cdot q_2} \, [t^{a_1}, t^{a_2}] \right)_{ki}$$

- small angle: bremsstrahlung cones
- large angle ($\theta_{pq_2} \gg \theta_{pq_1}$): sees total charge $\mathbf{Q}_p + \mathbf{Q}_{q_1}$

- Extensive collider data studies emphasize the phenomenological relevance of coherence effects. Example: $p\bar{p}$ dijets







[B. Webber, CERN seminar, 2008]

III.D Unintegrated quark evolution

[Jung & H, in progress]

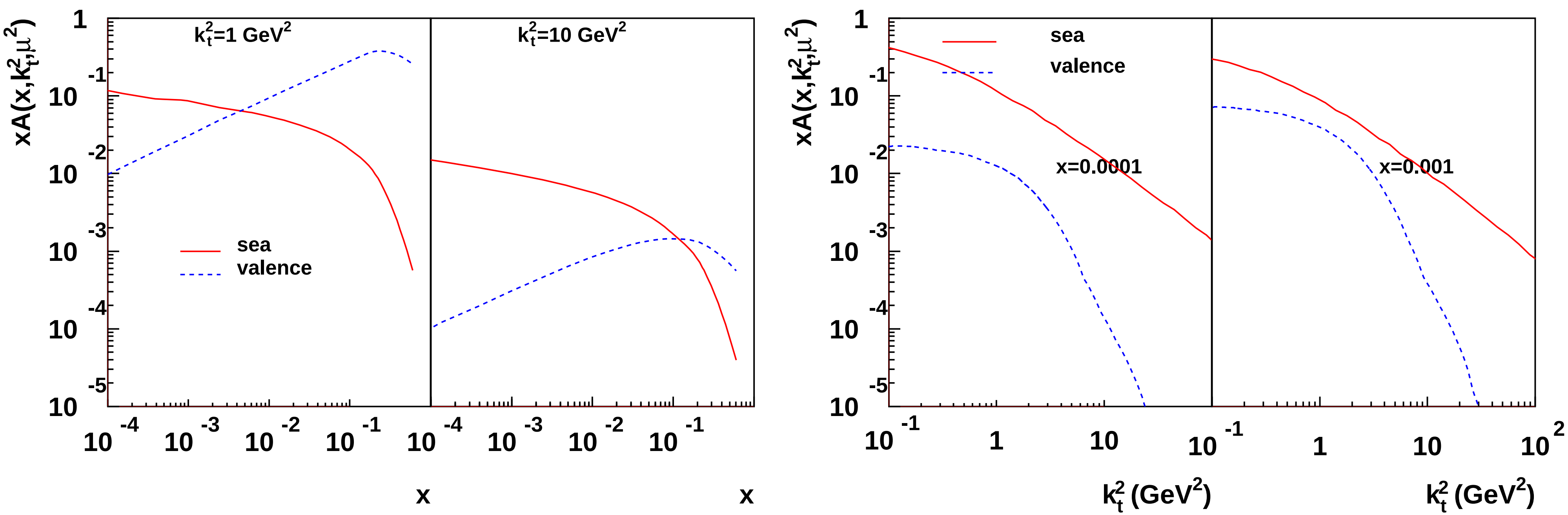
- sea: flavor-singlet evolution coupled to gluons at small x via

$$\mathcal{P}_{g\to q}(z;q,k) = P_{qg,\mathrm{GLAP}}(z)\ \left(1 + \sum_{n=0}^{\infty}\ b_n(z)(k^2/q^2)^n\right)$$

all b_n known; $\mathcal{P}_{g\to q}$ computed in closed form (positive-definite)

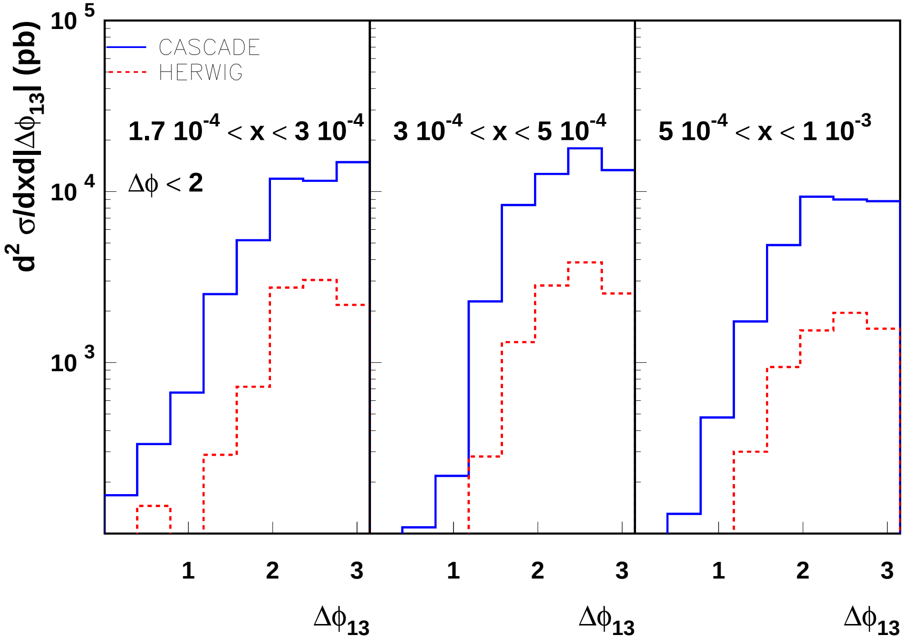
[Catani & H, 1994; Ciafaloni et al., 2005-2006]

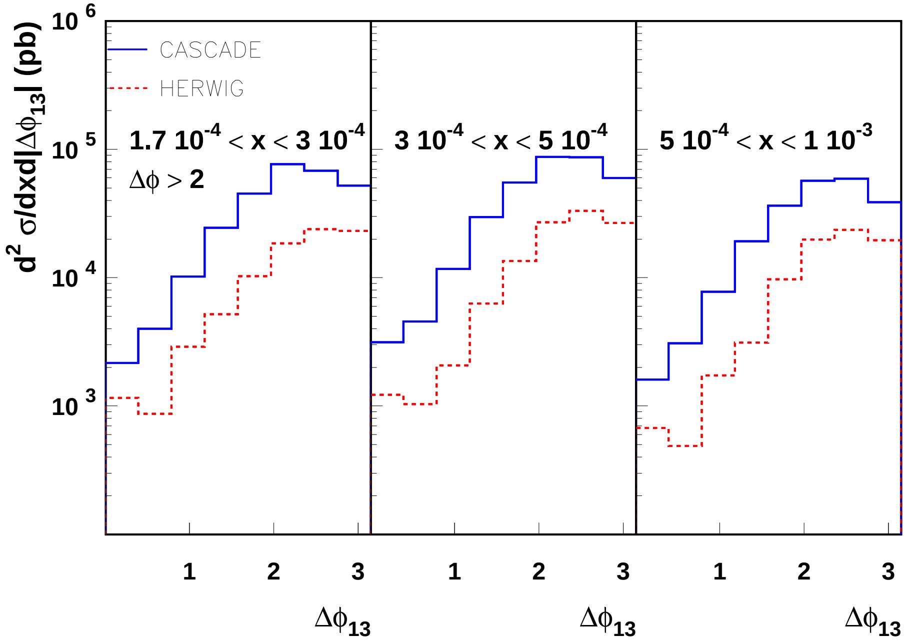
μ = 2 kt



- valence: independent evolution (dominated by soft gluons x → 1)

AZIMUTHAL DISTRIBUTION OF THE THIRD JET





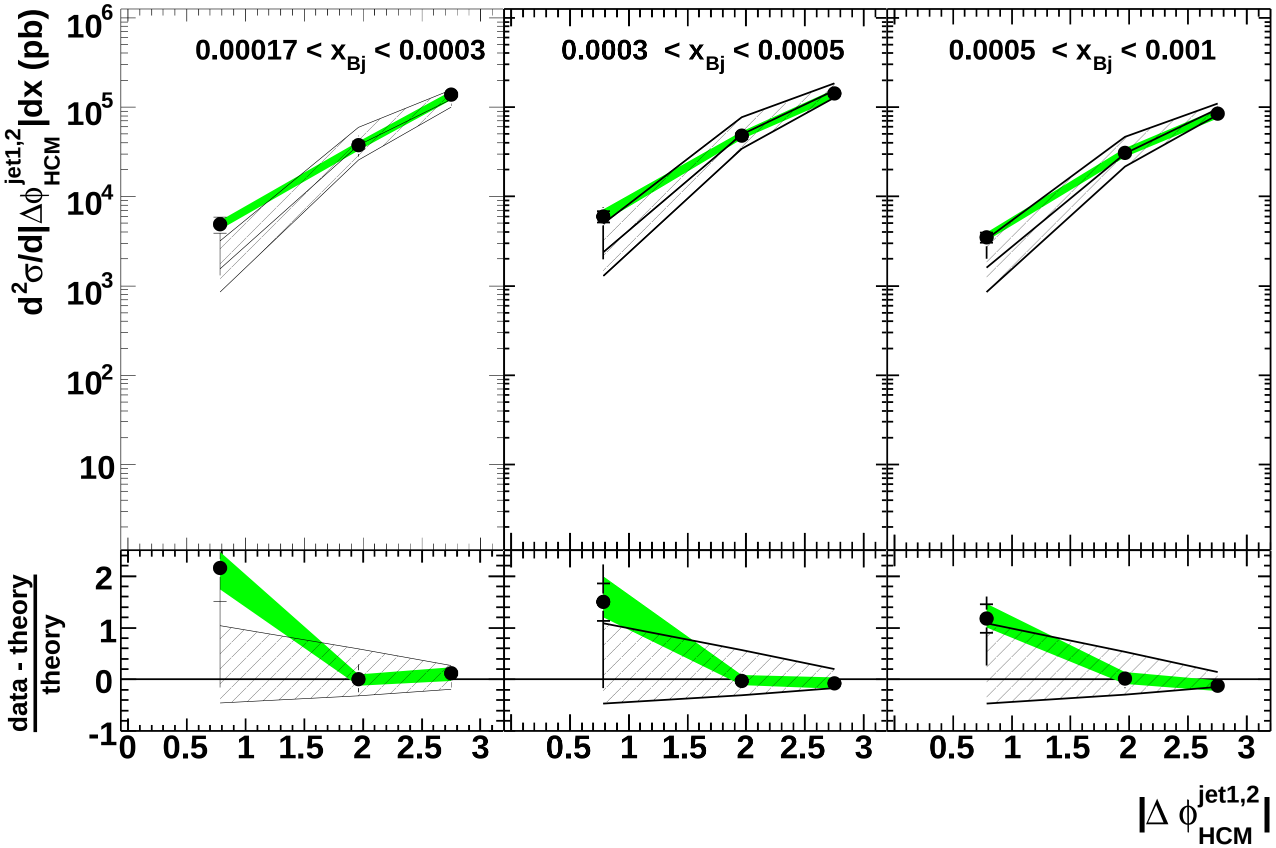
Cross section in the azimuthal angle between the hardest and the third jet
for small (left) and large (right) azimuthal separations between the leading jets

Jung & H, arXiv:0805.1049 [hep-ph]

- small $\Delta\phi \Rightarrow$ non-negligible initial $k_\perp \Rightarrow$ larger corrections to collinear ordering
- curves become closer at large $\Delta\phi$

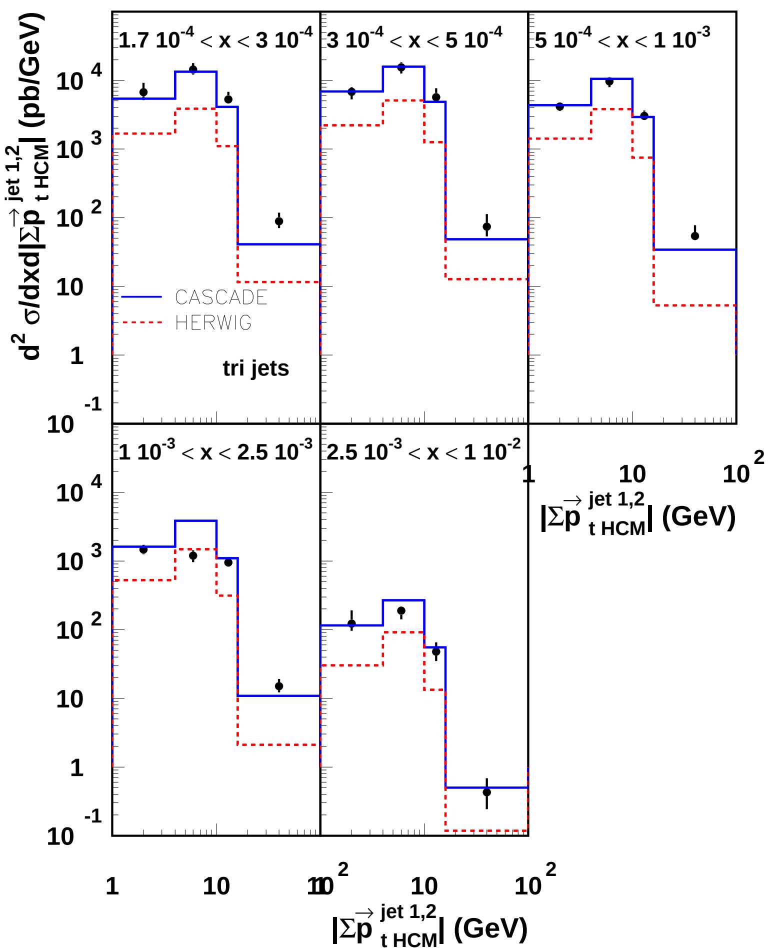
AZIMUTHAL DISTRIBUTION IN EP 3-JET CROSS SECTION

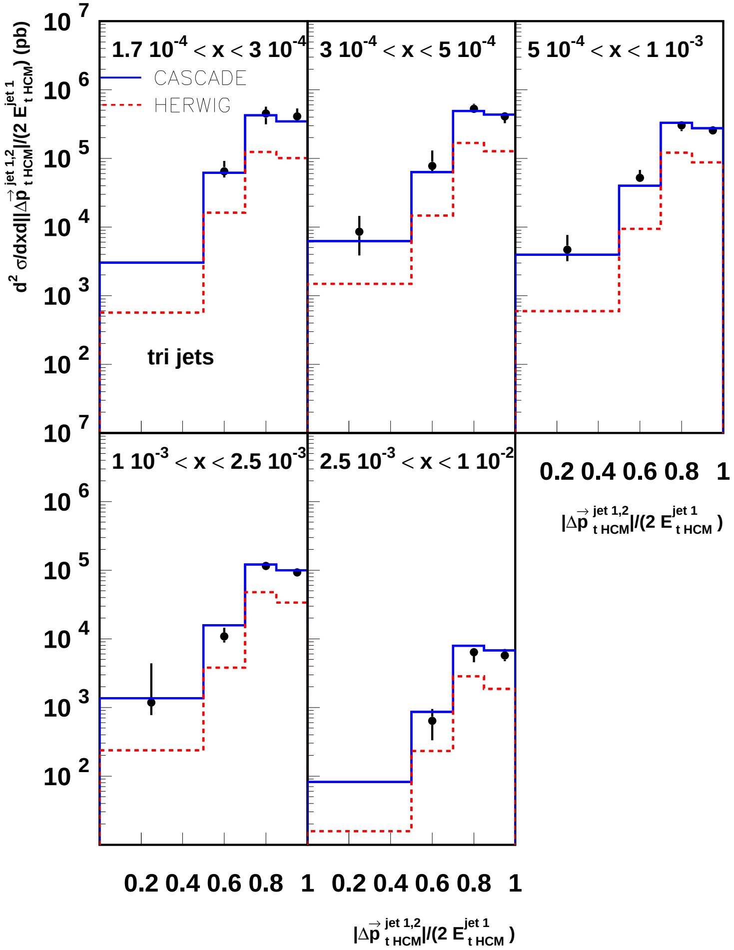
[ZEUS, 2007]



- grey dashed band: NLO result [NLOJET++]
- NLO results more stable for more inclusive distributions

MOMENTUM CORRELATIONS [Jung & H, arXiv:0805.1049]





- correlations in the transverse momentum imbalance between the leading jets