RAL HEP Summer School September 2011

Introduction to QED and QCD

Problem sheet 5

Problem 1

Consider the one-loop graphs for the fermion-fermion-gauge vertex function in the non-abelian theory:

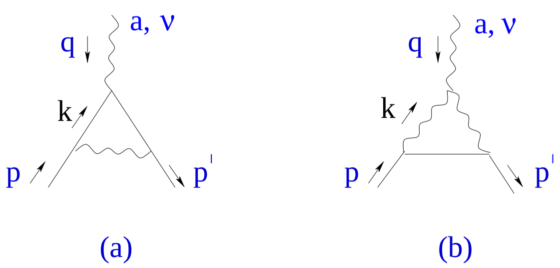

Figure 1: *One-loop corrections to fermion-fermion-gauge vertex.*

Show that the color factors for graphs (a) and (b) are given by

$$T^c T^a T^b \delta^{bc} = (C_F - \frac{1}{2}\, C_A) T^a \qquad \text{(a)} \quad ,$$

$$T^b T^c f^{abc} = \frac{i}{2}\, C_A T^a \qquad \text{(b)} \quad .$$

Problem 2

Determine the ultraviolet divergent part of each of the graphs (a) and (b) in Fig. 1, using dimensional regularization with $d = 4 - 2\varepsilon$ space-time dimensions and working in Feynman gauge:

$$\text{graph (a)} = \frac{ig^3}{(4\pi)^2} (C_F - \frac{1}{2}\, C_A) T^a \gamma^\nu \frac{1}{\varepsilon} + \ldots$$

$$\text{graph (b)} = \frac{ig^3}{(4\pi)^2} \frac{3}{2}\, C_A T^a \gamma^\nu \frac{1}{\varepsilon} + \ldots$$

Problem 3

Combining the results for graphs (a) and (b) in the previous question, determine the vertex renormalization constant Z_1:

$$Z_1 = 1 - \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} (C_F + C_A) \ .$$

Problem 4

Specialize graph (a) in Fig. 1 to the abelian case,

$$\bar{u}(p')\ ie\Gamma^\nu u(p) = e^3 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p')\ \gamma^\lambda\ (\not{k} + \not{q} + m)\ \gamma^\nu\ (\not{k} + m)\ \gamma_\lambda\ u(p)}{[(k+q)^2 - m^2 + i\varepsilon]\ [k^2 - m^2 + i\varepsilon]\ [(p-k)^2 + i\varepsilon]} \ .$$

Using relativistic invariance and gauge invariance, show that the vertex function Γ^ν can be decomposed as follows:

$$\Gamma^\nu(p, p') = F_1(q^2)\ \gamma^\nu + \frac{i}{m}\ F_2(q^2)\ \Sigma^{\nu\rho}\ q_\rho \ ,$$

where $\Sigma^{\mu\nu} = (i/4)[\gamma^\mu, \gamma^\nu]$, $q^\mu = p'^\mu - p^\mu$, and the scalar functions $F_1(q^2)$ and $F_2(q^2)$ are the electron's electric and magnetic form factors.

Problem 5

By the method outlined in the lectures one arrives at the following integral representation for $F_2(0)$,

$$F_2(0) = -ie^2 \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3\ 2\ \delta(x_1 + x_2 + x_3 - 1) \int \frac{d^4\widetilde{k}}{(2\pi)^4} \frac{-4m^2 x_3(1 - x_3)}{[\widetilde{k}^2 - m^2(1 - x_3)^2]^3} \ .$$

Show that this integral evaluates to

$$F_2(0) = \frac{\alpha}{2\pi} \ .$$

Hence the one-loop contribution to the electron's anomalous magnetic moment is given by $g - 2 = 2F_2(0) = \alpha/\pi$.