Introduction to QED and QCD

Problem sheet 4

Problem 1

Write the amplitude for Compton scattering $e(p)+\gamma(k) \to e(p')+\gamma(k')$, $\mathcal{M}_{fi} = M_{\mu\nu}\varepsilon'^{\mu}(k')\varepsilon^{\nu}(k)$, and verify that it is gauge-invariant.

Problem 2

By performing the polarization sum over the Compton amplitude as described in the lectures one arrives at the unpolarized matrix element square

$$\overline{|\mathcal{M}_{fi}|^2} = 2e^4 \left[\frac{p\cdot k}{p\cdot k'} + \frac{p\cdot k'}{p\cdot k} + 2m^2 \left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'} \right) + m^4 \left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'} \right)^2 \right] \quad .$$

Evaluate the process in the center-of-mass system (Fig. 1). Show that in the high energy limit $s = (p+k)^2 \gg m^2$ the photon angular distribution is dominated by backward scattering $\theta \approx \pi$ and is given by

$$\frac{d\sigma}{d\Omega} \simeq \frac{\alpha^2}{s(1+\cos\theta)+2m^2} \quad .$$

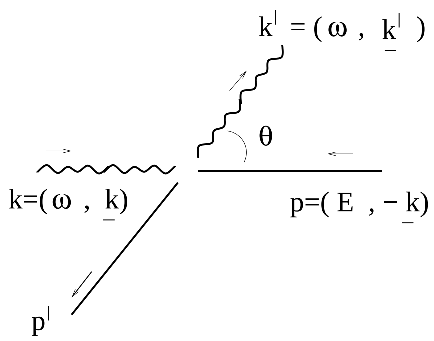

Figure 1: *Compton scattering in the center-of-mass system.*

Problem 3

The production of photon pairs from e^+e^- annihilation is related to Compton scattering via crossing. Obtain the invariant matrix element square for photon pair production from the Compton matrix element above. Work in the center-of-mass system (Fig. 2) and show that the angular distribution of photon pairs in the high energy limit $E \gg m$ is given by

$$\frac{d\sigma}{d\Omega} \simeq \frac{\alpha^2}{4E^2} \frac{1+\cos^2\theta}{\sin^2\theta} \quad .$$

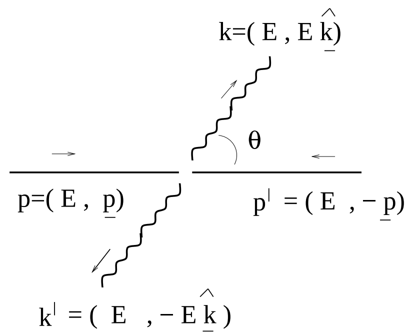

Figure 2: e^+e^- *annihilation into photon pairs in the center-of-mass system.*

Problem 4

Evaluate Compton scattering of scalar particles (Fig. 3). Consider the laboratory frame in

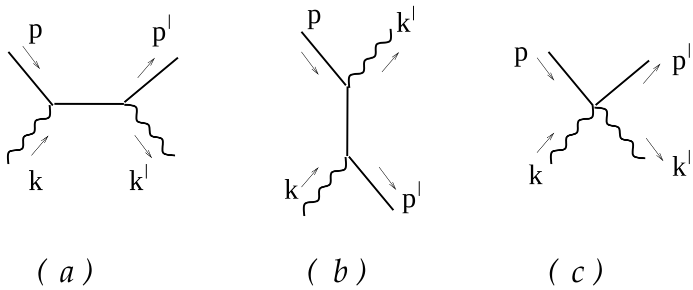

Figure 3: *Compton scattering of scalar particles.*

which the scalar particle is initially at rest. Show that the cross section at fixed photon polarizations is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{m^2} \left(\frac{\omega'}{\omega}\right)^2 (\varepsilon \cdot \varepsilon')^2 \quad ,$$

where m is the particle's mass, ε and ε' are the initial and final photon polarizations, and ω and ω' are the initial and final photon energies. Determine the unpolarized cross section by evaluating the average over polarizations of $(\varepsilon \cdot \varepsilon')^2$.