Introduction to QED and QCD

Problem sheet 1

Problem 1

Obtain the continuity equation for the Schrödinger equation and for the Klein-Gordon equation.

Problem 2

Suppose introducing a potential V in the Klein-Gordon equation as follows,

$$\left(i \frac{\partial}{\partial t} - V \right)^2 \phi = (-\boldsymbol{\nabla}^2 + m^2) \, \phi \ ,$$

and consider a wave incident on the potential step in Fig 1. Apply the continuity of ϕ and $\partial\phi/\partial x$ at $x = 0$. Show that for $V > m + E_p$ positive group velocity leads to negative density, or particle pair creation in the barrier.

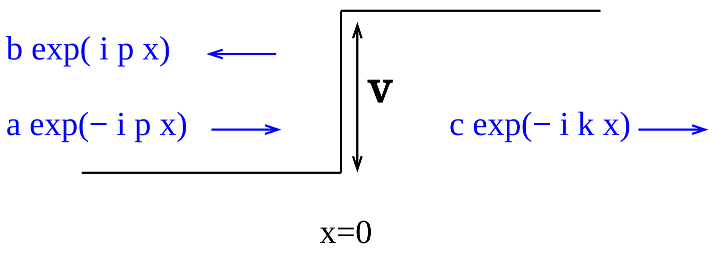

Figure 1: *Potential step.*

Problem 3

Show that the Dirac γ matrices

$$\gamma^0 = \beta, \qquad \gamma^k = \beta\alpha^k$$

obey the hermiticity relation

$$(\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0 \quad .$$

Problem 4

Using the anticommutation relations for the Dirac γ matrices, show that in four dimensions

$$\gamma^\mu \gamma_\mu = 4 \quad ,$$

$$\gamma^\mu \gamma^\nu \gamma_\mu = -2\gamma^\nu \quad ,$$

$$\gamma^\mu \gamma^\nu \gamma^\lambda \gamma_\mu = 4 g^{\nu\lambda} \quad ,$$

$$\gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\rho \gamma_\mu = -2\gamma^\rho \gamma^\lambda \gamma^\nu \quad .$$

Determine how the previous identities are modified in dimension d based on the fact that $g^{\mu\nu} g_{\mu\nu} = \delta^\mu_\mu = d$.

Problem 5

Consider Compton scattering in which a photon with 4-momentum $k^\mu = (\omega, \mathbf{k})$, $\omega = |\mathbf{k}|$, strikes a particle of mass m at rest and is scattered at angle θ with final 4-momentum $k'^\mu = (\omega', \mathbf{k}')$, $\omega' = |\mathbf{k}'|$ (Fig. 2). Verify that the final photon's energy ω' is given by

$$\omega' = \frac{\omega}{1 + (\omega/m)(1 - \cos\theta)} \quad .$$

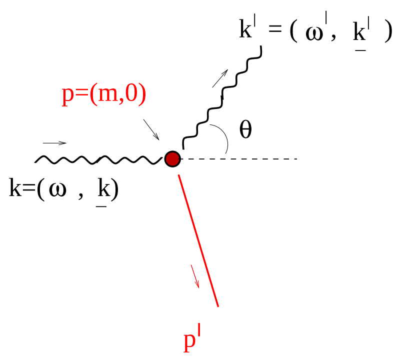

Figure 2: *Photon-particle Compton scattering.*