

## Functions of a complex variable (S1)

### Answers for Problem Sheet 4

1. (a)  $2\pi/(3\sqrt{3})$       (b)  $(\pi/n)/\sin(\pi/n)$
2. (a)  $\pi$       (b)  $\pi$
3. (a)  $-i\pi/3$       (b)  $7\pi i/180$       (c)  $0$       (d)  $2\pi i/3$
4. (a)  $J = 0$  ;  $P = 1$  ,  $N = 1$       (b)  $J = -2$  ;  $P = 2$  ,  $N = 0$
5. (a)  $3$       (b)  $6$
6.  $1$
7. (a)  $\pi^2/12$       (b)  $\pi^3/32$
8. (a)  $\pi/\sqrt{2}$       (b)  $2\pi/\sqrt{3}$
9. (a)  $-\pi/4$       (b)  $\pi \ln 2$
10.  $1$  for  $t > 0$ ;  $0$  for  $t < 0$
11. (a)  $\tilde{f}(\omega) = 2 \sin \omega / \omega$       (b)  $\tilde{f}(\omega) = \sin^2(\omega/2) / (\omega/2)^2$
12. (a)  $F(z) = 1/(1+z^2)$       (b)  $F(z) = 2z/(1+z^2)^2$       (c)  $F(z) = z/(z^2-1)$
13.  $1/\sqrt{\pi t}$
14. (a)  $\partial \tilde{u} / \partial t = -\lambda p^2 \tilde{u}$  ,  $\tilde{u}(p, 0) = \tilde{h}(p)$   
 (b)  $\tilde{u}(p, t) = \tilde{h}(p) e^{-\lambda p^2 t}$   
 (c)  $u(x, t) = (1/\sqrt{4\pi\lambda t}) \int_{-\infty}^{\infty} dy e^{-(x-y)^2/(4\lambda t)} h(y)$
15. (a)  $\tilde{y} + \tilde{y}/z^2 = 2/(z^2+4)$  ;  $\tilde{y}(z) = 2z^2/[(z^2+1)(z^2+4)]$   
 (b)  $y(t) = (4/3) \sin 2t - (2/3) \sin t$