Functions of a complex variable (S1)

Answers for Problem Sheet 3

1. (a) $-1-2\sum_{n=1}^{+\infty} z^n$, $|z|<1$ (b) $1+2\sum_{n=1}^{+\infty} z^{-n}$

2. (a) $\sum_{n=0}^{+\infty}(-1)^n[(1-3^{-n-1})/2]z^n$ (b) $-1/6-(1/2)[\sum_{n=1}^{+\infty}(-1)^n 3^{-n-1}z^n+\sum_{n=1}^{+\infty}(-1)^n z^{-n}]$

   (c) $(1/2)\sum_{n=2}^{+\infty}(-1)^n(3^{n-1}-1)z^{-n}$ (d) $\sum_{n=0}^{+\infty}(-1)^n 2^{-n-1}(z+1)^{n-1}$, $0<|z+1|<2$

3. (a) $\sum_{n=0}^{+\infty} z^{n-2}$ (b) $-\sum_{n=0}^{+\infty} z^{-n-3}$

4. (a) $\sum_{n=0}^{+\infty}(-1)^n z^{3n+1}$ (b) $\sum_{n=0}^{+\infty}(-1)^n z^{-3n-2}$

5. $1/z+1-z/2-5z^2/6+\dots$

6. $1-5(z+2)^{-1}-(1/6)(z+2)^{-2}+(5/6)(z+2)^{-3}+\dots$, $|z+2|>0$

7. $(1/9)(z-3)^{-2}-(2/27)(z-3)^{-1}+(1/27)-(4/243)(z-3)+\dots$, $0<|z-3|<3$

8. (a) $z=i/[\pi(2n+1)]$, $n=0,\pm1,\dots$: simple poles; $z=0$: non-isolated singularity (b) no

9. (a) pole of order 2 (b) essential singularity

   (c) holomorphic (d) non-isolated singularity

10. i): (a) $1/z^3-1/(6z)+7z/360+\dots$, $0<|z|<\pi$ (b) pole of order 3 (c) $-i\pi/3$

    ii): (a) $z+1/z+1/(2z^3)+\dots$, $|z|>0$ (b) essential singularity (c) $2\pi i$

    iii): (a) $1/z^5-1/(2z^3)+1/(24z)-z/720+\dots$, $|z|>0$ (b) pole of order 5 (c) $i\pi/12$

    iv): (a) $1+1/(2z^2)+1/(24z^4)+\dots$, $|z|>0$ (b) essential singularity (c) 0

11. (a) $z=1/2$ pole of order 2; $\mathrm{Res}_{z=1/2}\,\mathrm{f}=-1/4$

    (b) $z=0$ essential singularity; $\mathrm{Res}_{z=0}\,\mathrm{f}=0$

    (c) $z=n\pi$ $(n=0,\pm1,\dots)$ poles of order 1; $\mathrm{Res}_{z=n\pi}\,\mathrm{f}=1$

12. (a) 0 (b) $-2\pi i$ (c) $i\pi/e$

13. (a) $\pi/6$ (b) $\pi/(2e^3)$ (c) $2\pi/3$

14. $\pi\, e^{-|\lambda|}$

15. (a) 2π (b) 0 (c) -2π (d) 0

16. $\pi\sqrt{2}$

17. (a) $\pi^2/6$ (b) $(\pi a\ \coth\pi a-1)/(2a^2)$

18. (a) $-\pi^3/4$ (b) *i)* 0 *ii)* $\pi^3/8$

19. $2i\sqrt{\pi}$