Quantum Matter 2, Problem Sheet 2

1 Entanglement growth after a quantum quench

Consider a tight-binding model of free, spinless fermions with Hamiltonian

$$H = -J \sum_{j=1}^{L} c_j^\dagger c_{j+1} + c_j^\dagger c_{j+1} \ , \qquad J > 0. \tag{1}$$

The system is initialized in the product state

$$|\Psi(0)\rangle = \prod_{j=1}^{L/2} c_{2j}^\dagger |0\rangle. \tag{2}$$

Determine the correlation matrix of the system in the thermodynamic limit as a function of time after the quench. Use this result to numerically compute the bi-partite entanglement entropy of a subsystem consisting of ℓ neighbouring sites $S_\ell(t)$. What is the asymptotic value reached at late times for large values of ℓ? Explain.

2 Interaction effects after quantum quenches

Consider a model of interacting, spinless fermions with Hamiltonian

$$H = -J \sum_{j=1}^{L} c_j^\dagger c_{j+1} + c_j^\dagger c_{j+1} + \lambda \sum_j n_j n_{j+1} \ . \tag{3}$$

(a) Using symmetry arguments show that the momentum space Green's function can be written in the form

$$G(p, q, t) = \langle \Psi(t)| c^\dagger(p) c(q) |\Psi(t)\rangle = g_+(p, t)\delta_{p,q} + g_-(p, t)\delta_{q,p+\pi}. \tag{4}$$

(b) Determine the equations of motion for the momentum space single-particle Green's function after a quantum quench from the state (2) using time-dependent, self-consistent mean-field theory. Describe in detail how you would solve these equations numerically.

Hints: Decouple the interaction term in *position space* introducing an appropriate number of time-dependent mean fields. This results in a quadratic Hamiltonian, from which you can obtain the equations of motion for $c(p, t)$ and hence for $G(p, q, t)$.